

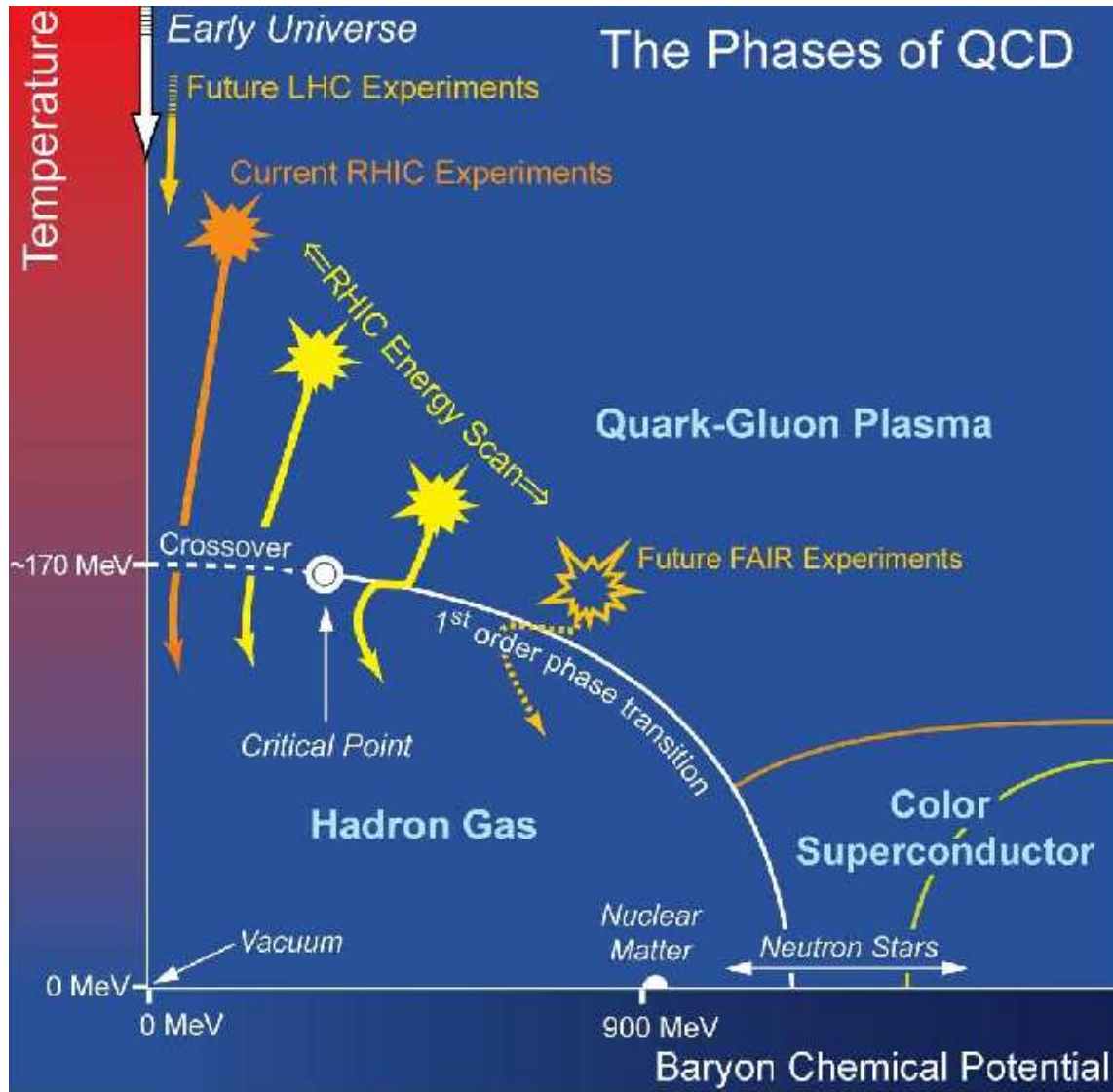
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QCD Thermodynamics with twisted mass Wilson fermions

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The tmfT Collaboration : E.-M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Müller-Preussker, M. Petschlies, O. Philipsen, and L. Zeidlewicz

- E.M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Muller-Preussker, M. Petschlies, O. Philipsen and L. Zeidlewicz, **arXiv:0905.3312** [hep-lat]
- E.M. Ilgenfritz, K. Jansen, M. P. Lombardo, M. Muller-Preussker, M. Petschlies, O. Philipsen and L. Zeidlewicz, PoS **LAT2008**
- E. M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, K. Jansen, I. Wetzorke, M. P. Lombardo and O. Philipsen, PoS **LAT2007** (2007) 238
- E. M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, K. Jansen, I. Wetzorke, M. P. Lombardo and O. Philipsen, PoS **LAT2006** (2006) 140



From NSAC Long Range Plan

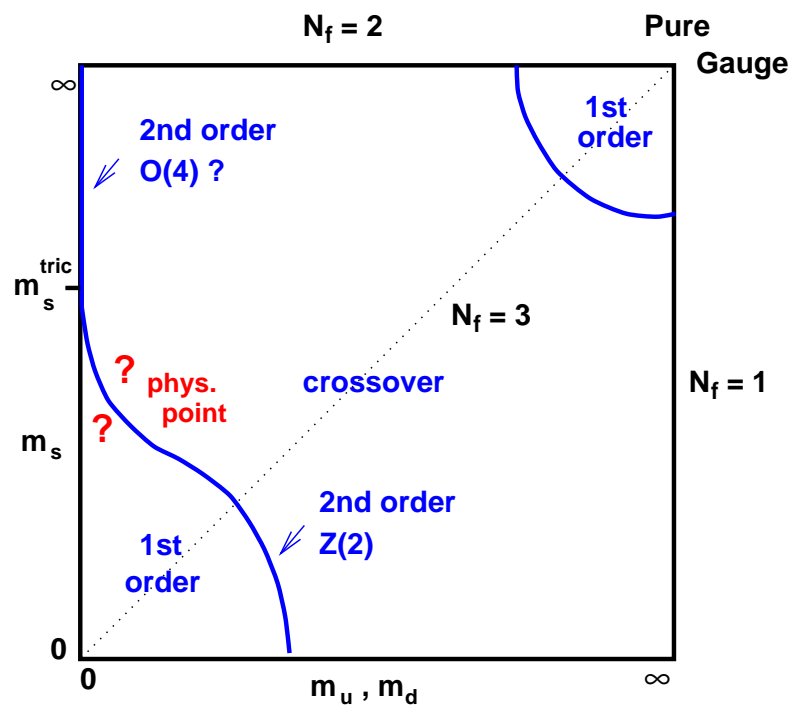
Why Twisted Mass Wilson Fermions

- LHC calls for accurate predictions at $\mu = 0$
- Important Lattice Issue : Control of the Continuum Limit
- Twisted mass allows for an automatic improvement. Extensively tested and applied at $T=0$.
- Ideally, physical quark masses (up, bottom, strange (charm))
- This work : two flavor. Important first step, and open theoretical issues.

- INTRODUCTION
- OVERVIEW OF THE RESULTS
- LOW MASS AND LATTICE ChPT
- SUMMARY
- PERSPECTIVES AND WORK IN PROGRESS

Phase transitions of QCD in the up=down, strange plane

This work : $N_f = 2$



	U(1) _A anomaly	suppressed anomaly at T_c
QCD	$SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_V$	$U(N_f)_L \otimes U(N_f)_R \rightarrow U(N_f)_V$
$N_f = 1$	crossover or first order	O(2) or first order
$N_f = 2$	O(4) or first order	$U(2)_L \otimes U(2)_R / U(2)_V$ or first order
$N_f \geq 3$	first order	first order
aQCD	$SU(2N_f) \rightarrow SO(2N_f)$	$U(2N_f) \rightarrow O(2N_f)$
$N_f = 1$	O(3) or first order	$U(2)/O(2)$ or first order
$N_f = 2$	$SU(4)/SO(4)$ or first order	first order

Universality class of QCD transition with N_f light quarks

Pisarski, Wilczek; original discussion

Basile, Pelissetto, Vicari 2005; RG analysis

The Universality Class of $N_f=2$ QCD from the Lattice:

Staggered Fermions:

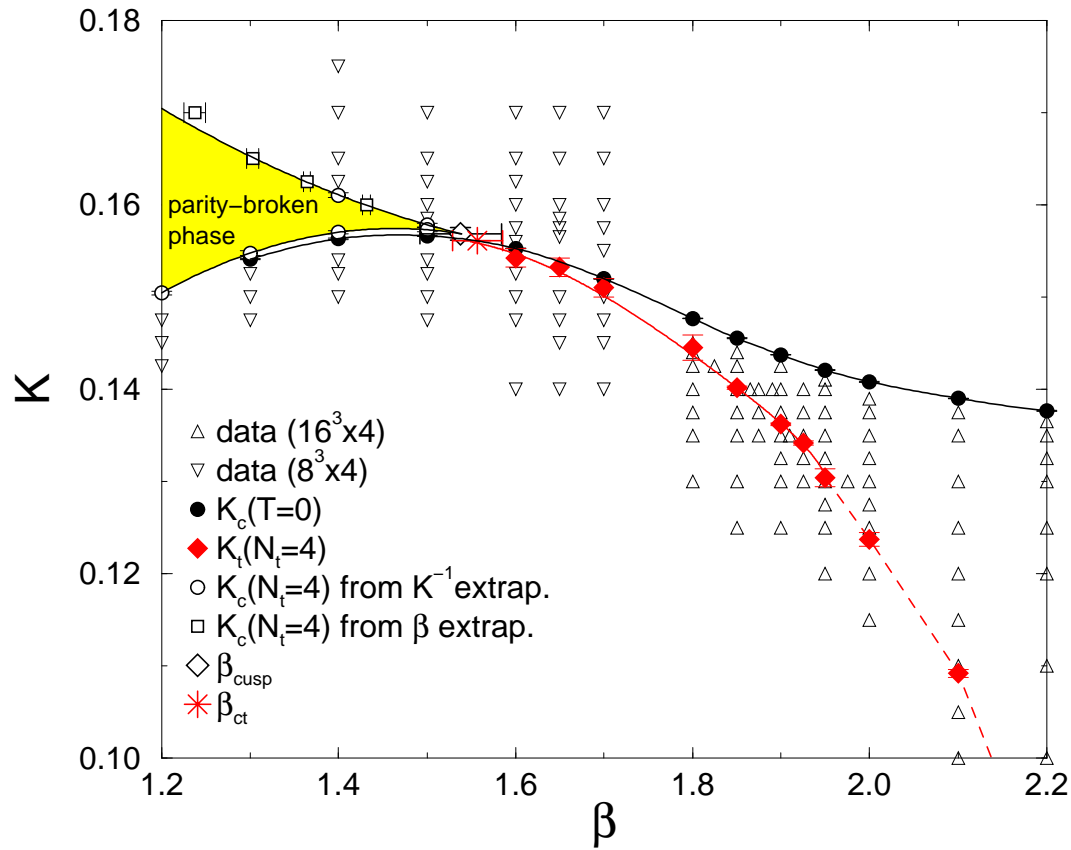
- $O(2)$ or $O(4)$ with $N_t=8$ but scaling window very narrow - other behaviours cannot be ruled out (Kogut Sinclair 2004)
- $O(2)$ at strong coupling very high precision low masses Chandrasekharan Strouthos
- First order ($O(2)$ / $O(4)$ ruled out) $N_t=4$ Pisa Group $N_t=6$ In progress
- $O(4)$ scaling visible with largish masses, T. Mendez 2005–2008

Wilson fermions:

- Apparently compatible with $O(4)$ scaling $N_t=4$ - CP-Pacs 2001

Phases of Wilson Fermions :Early Lattice results

A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D 63, 034502 (2001); Phys. Rev. D 64, 074510 (2001)



Twisted Mass QCD

- Fermion sector

$$S_q = \sum_x \left\{ (\bar{\chi}_x [\mu_\kappa + i\gamma_5 \tau_3 a\mu] \chi_x) - \frac{1}{2} \sum_{\mu=\pm 1}^{\pm 4} (\bar{\chi}_{x+\hat{\mu}} U_{x\mu} [r + \gamma_\mu] \chi_x) \right\},$$

$\mu_\kappa \equiv am_0 + 4 = 1/2\kappa$, am_0 the bare “untwisted” quark mass in lattice units and μ the twisted quark mass;

- Gauge sector: Symanzik improved

$$S_g = \beta \sum_x \left(c_0 \sum_{\mu < \nu; \mu, \nu=1}^4 \left\{ 1 - \frac{1}{3} \text{Re} U_{x\mu\nu}^{1 \times 1} \right\} + c_1 \sum_{\mu \neq \nu; \mu, \nu=1}^4 \left\{ 1 - \frac{1}{3} \text{Re} U_{x\mu\nu}^{1 \times 2} \right\} \right),$$

$U_{x\mu\nu}^{1 \times 2}$ is the planar rectangular (1×2), and we use tree-level Symanzik improved gauge action (tlSym) $c_1 = -1/12$.

Twisted Mass : A model study Mike Creutz, T=0 1996, Finite T, 2007

Effective potential for Wilson fermions

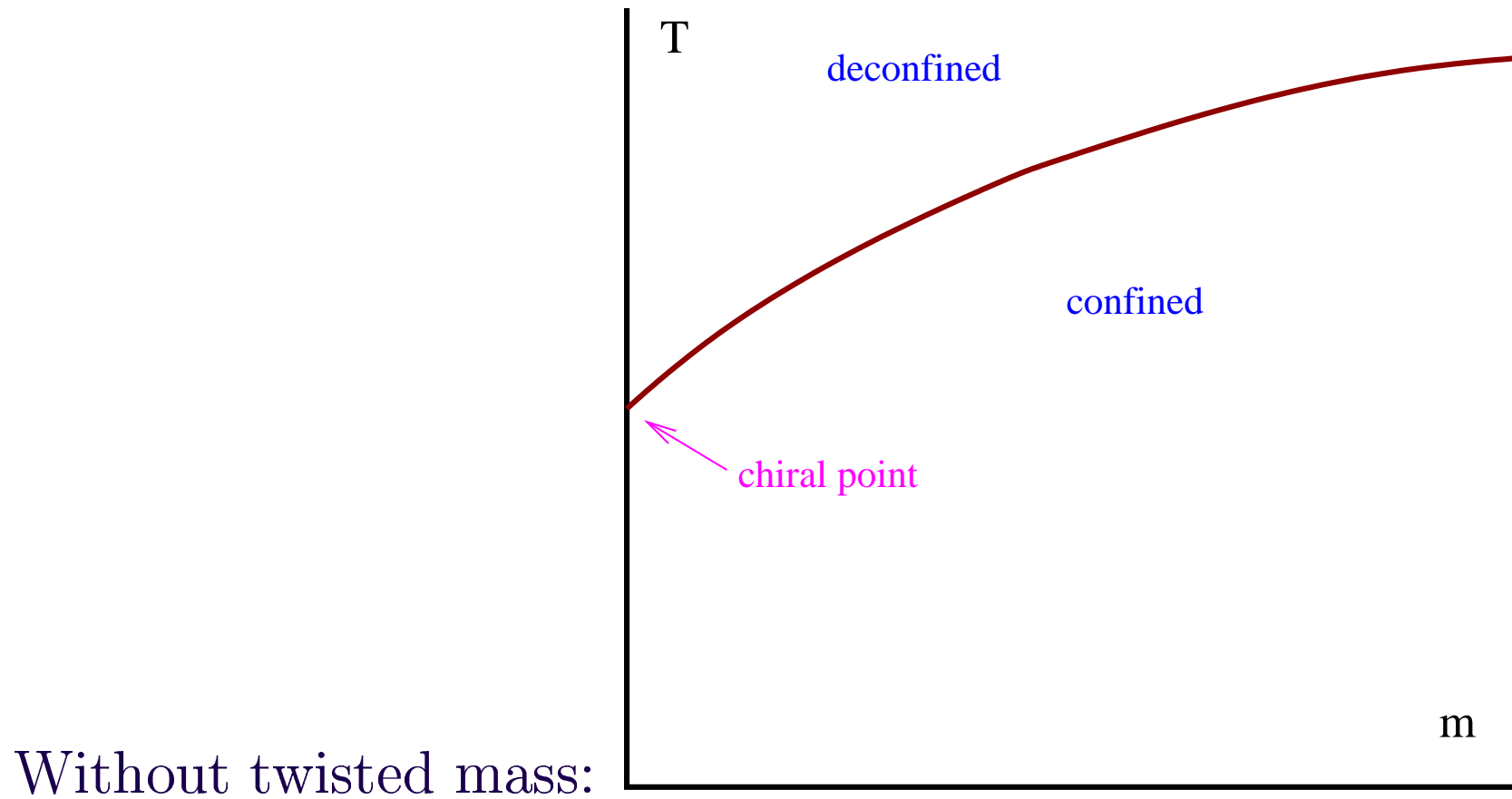
$$V(\vec{\pi}, \sigma, L) = \lambda(\sigma^2 + \vec{\pi}^2 - v^2)^2 + \alpha_0(K^2 - K_c(\beta)^2)\sigma + \alpha_1\sigma^2 + m_t\pi_3$$

- The first line is the “linear” sigma model with $O(4)$ symmetry.
- The second line is the mass term.
- The third term is a chiral symmetry breaking **lattice artifact**
- The m_t term is the twisted mass. Without the α_1 term this can be rotated into the α_0 mass term along the curve of constant $m_t^2 + \alpha_0^2(K^2 - K_c^2)^2$.
- **If $K = K_c$ lattice artifacts are reduced**

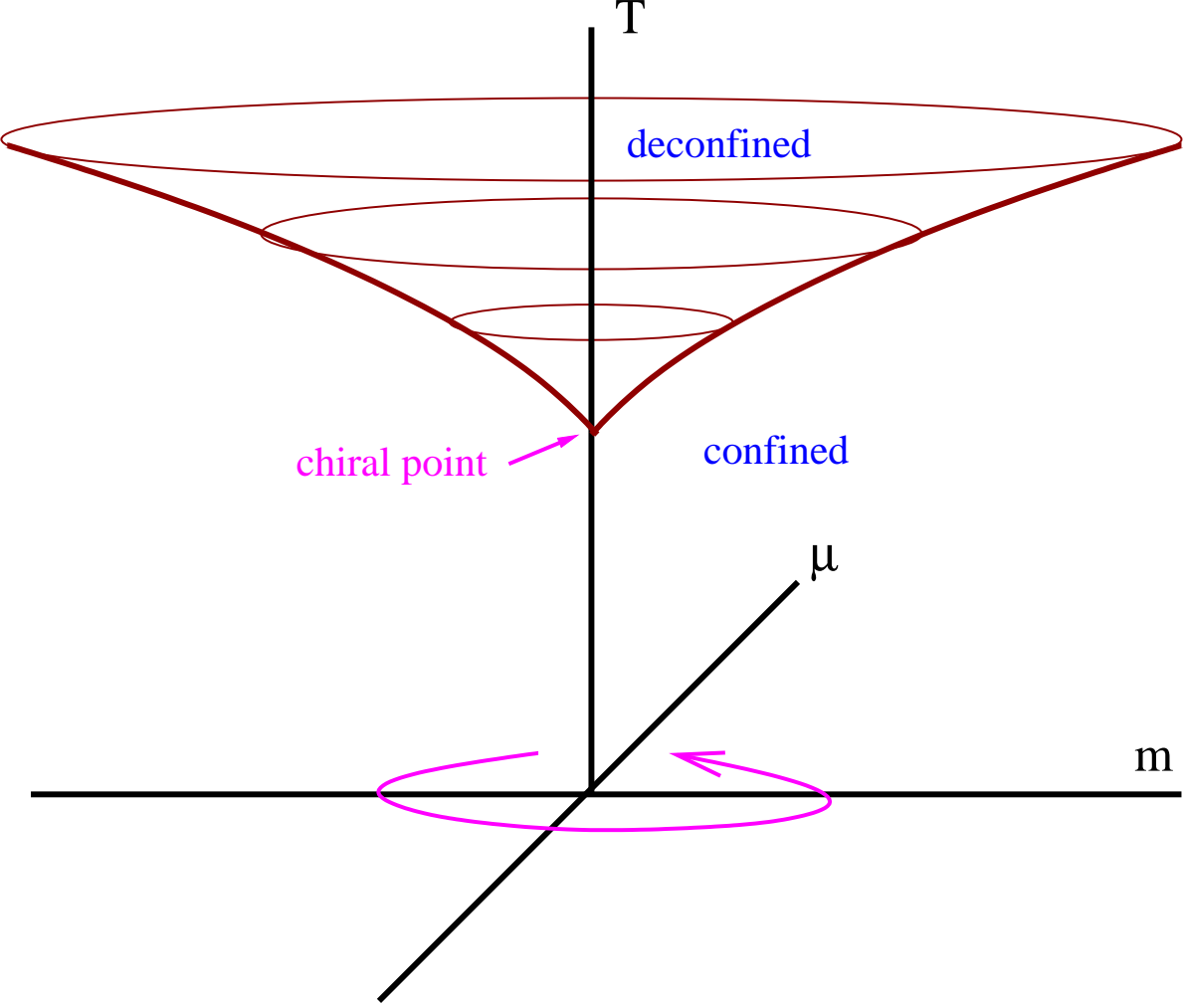
$$m_q^2 = m_t^2 + \alpha_0^2(K^2 - K_c^2)^2 + O(a)$$

.

Twisted Mass and Finite Temperature - M. Creutz 2007

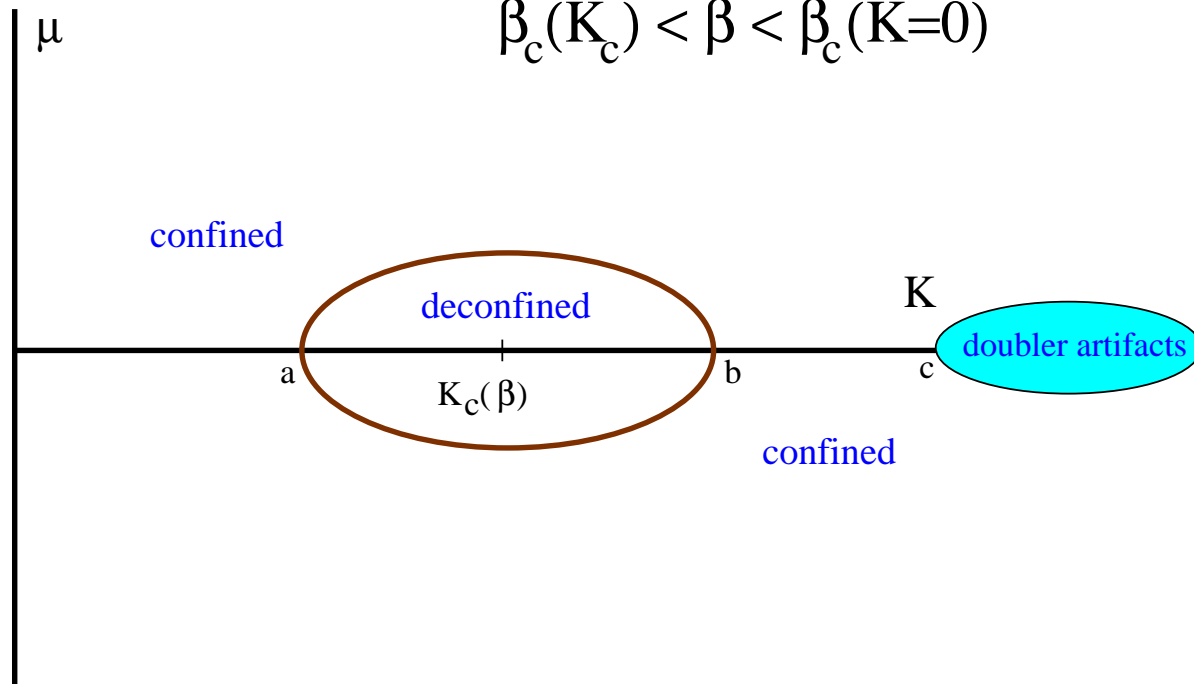


With twisted mass: the 'cone'



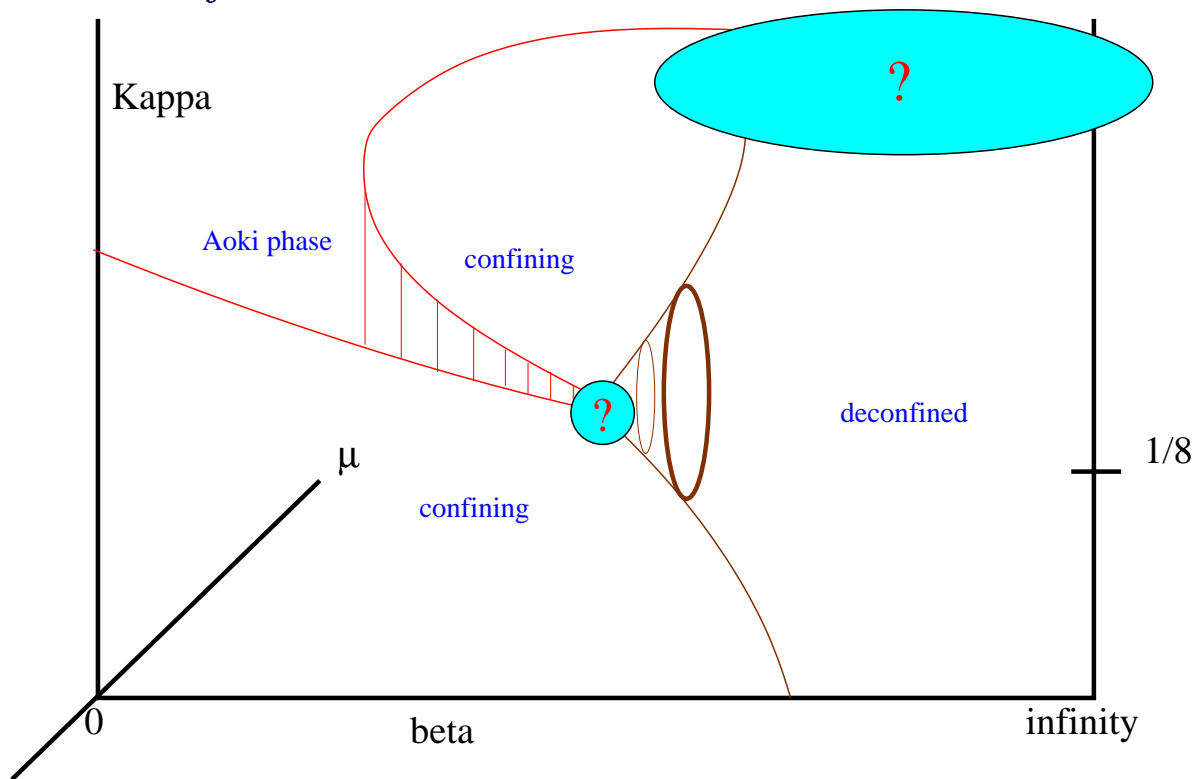
A fixed temperature $T = 1/(aN_t)$ section:

$$\beta_c(K_c) < \beta < \beta_c(K=0)$$



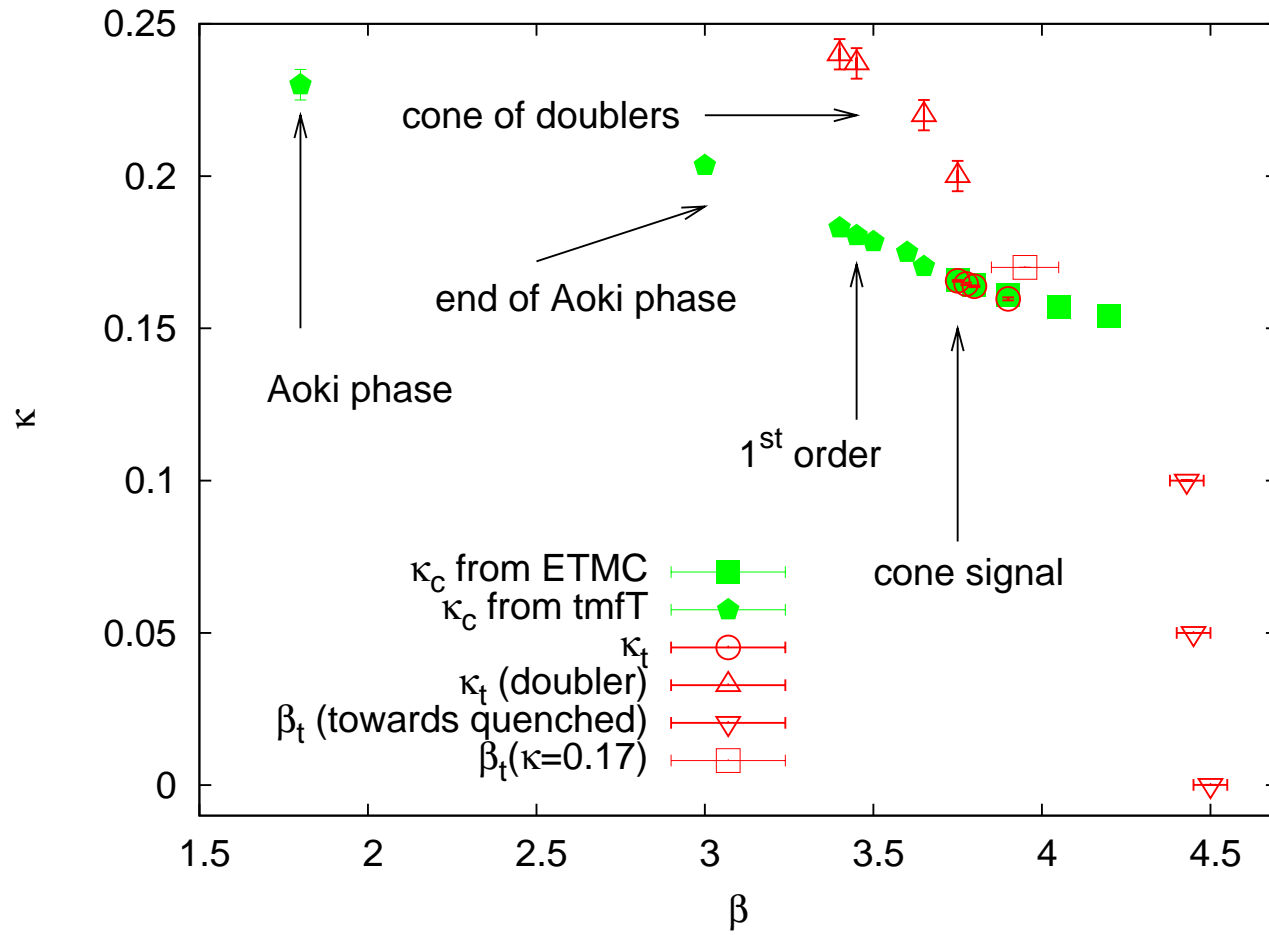
At fixed N_t and for a value of β where the infinite mass theory is confined but the massless theory is in the deconfined phase, we expect the deconfined phase to assume an approximately elliptical structure in the $K\mu$ plane.

A fixed N_t section



RESULTS

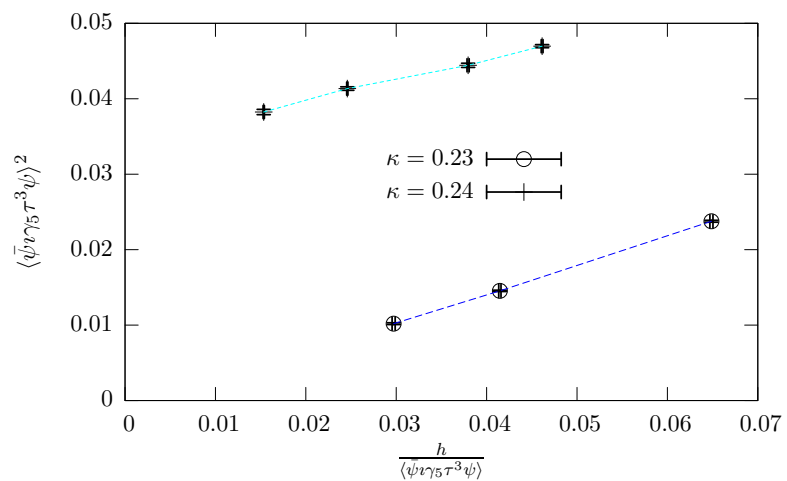
Overview



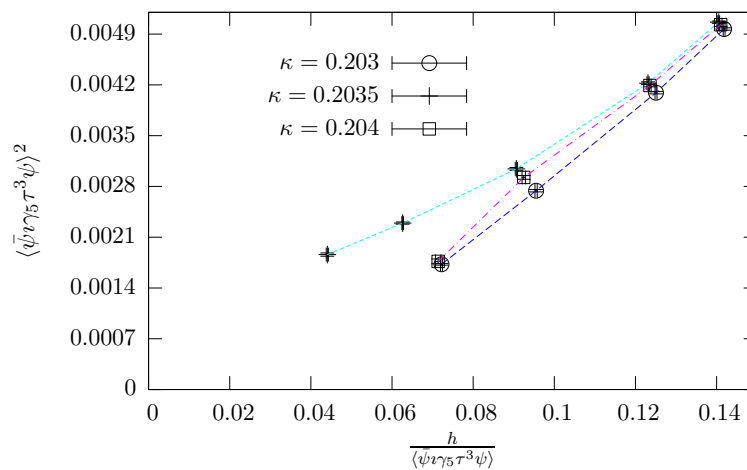
The Aoki Phase

- $\beta \in \{1.8, 3.0, 3.4\}$.
- Order parameter

$$\lim_{h \rightarrow 0} \lim_{N_\sigma \rightarrow \infty} \langle \bar{\psi} i \gamma_5 \tau^3 \psi \rangle_{N_\sigma, h} \neq 0.$$

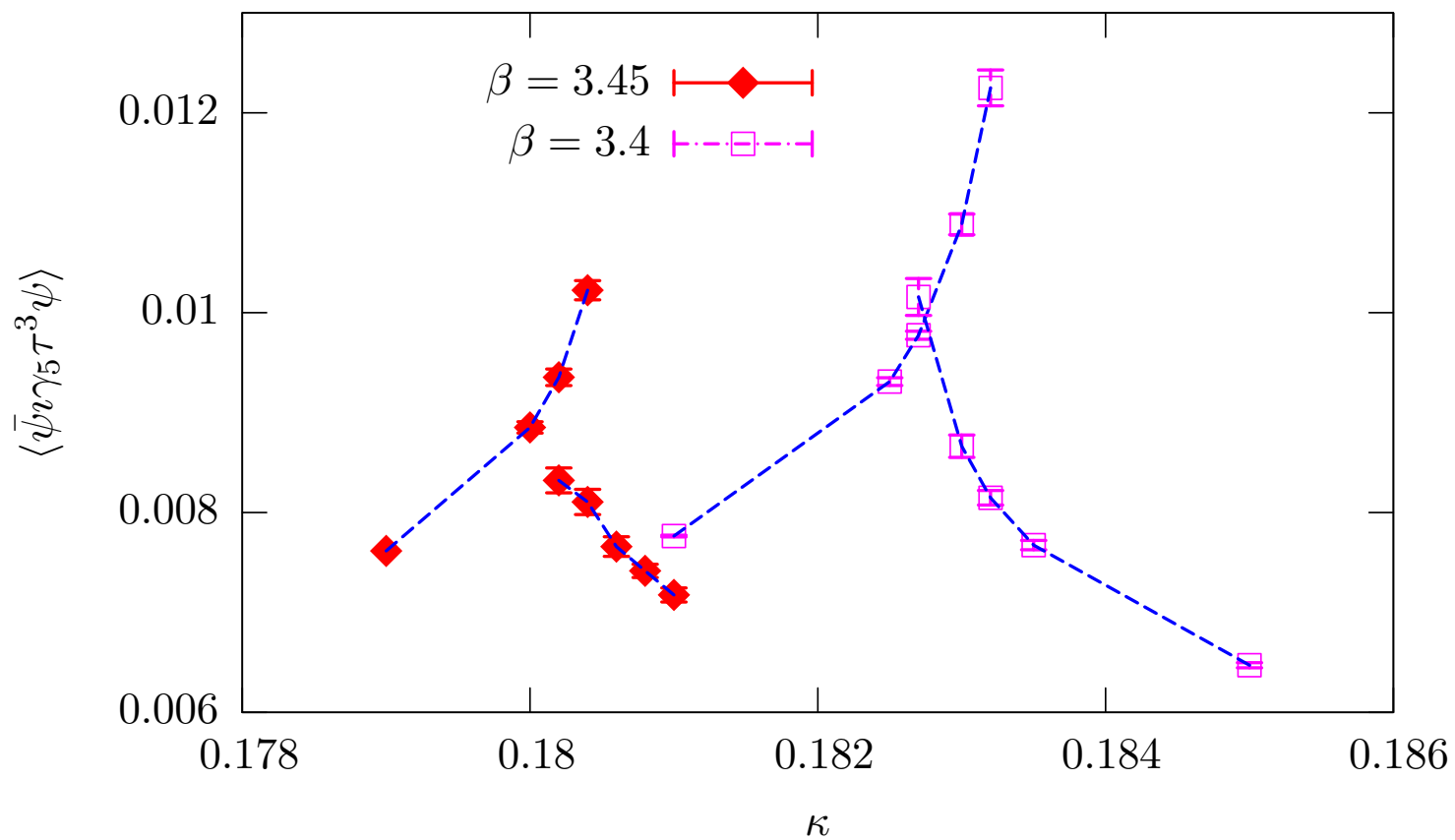


$\beta = 1.8$

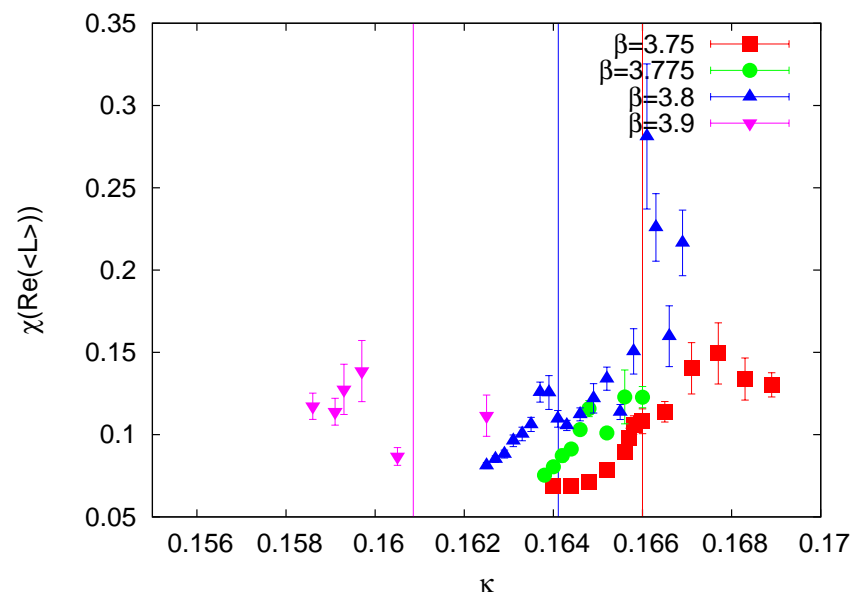
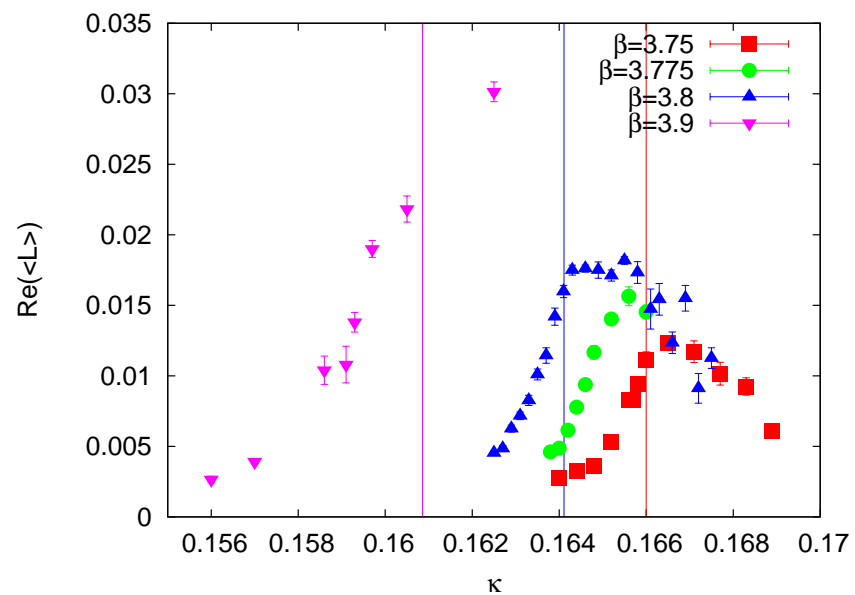


$\beta = 3.0$

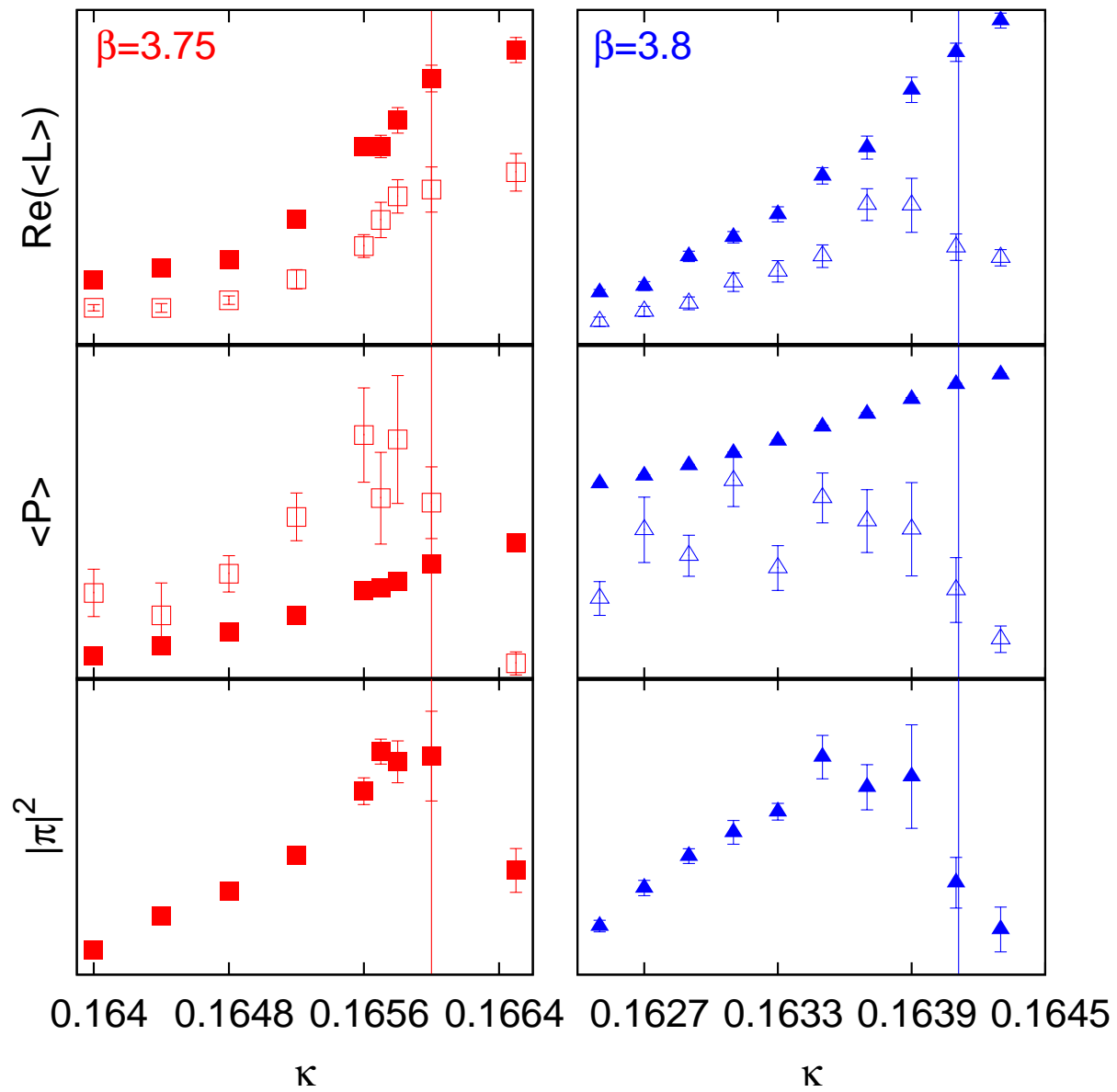
The intermediate region $\beta = 3.4, 3.45, 3.65$: a first order transition



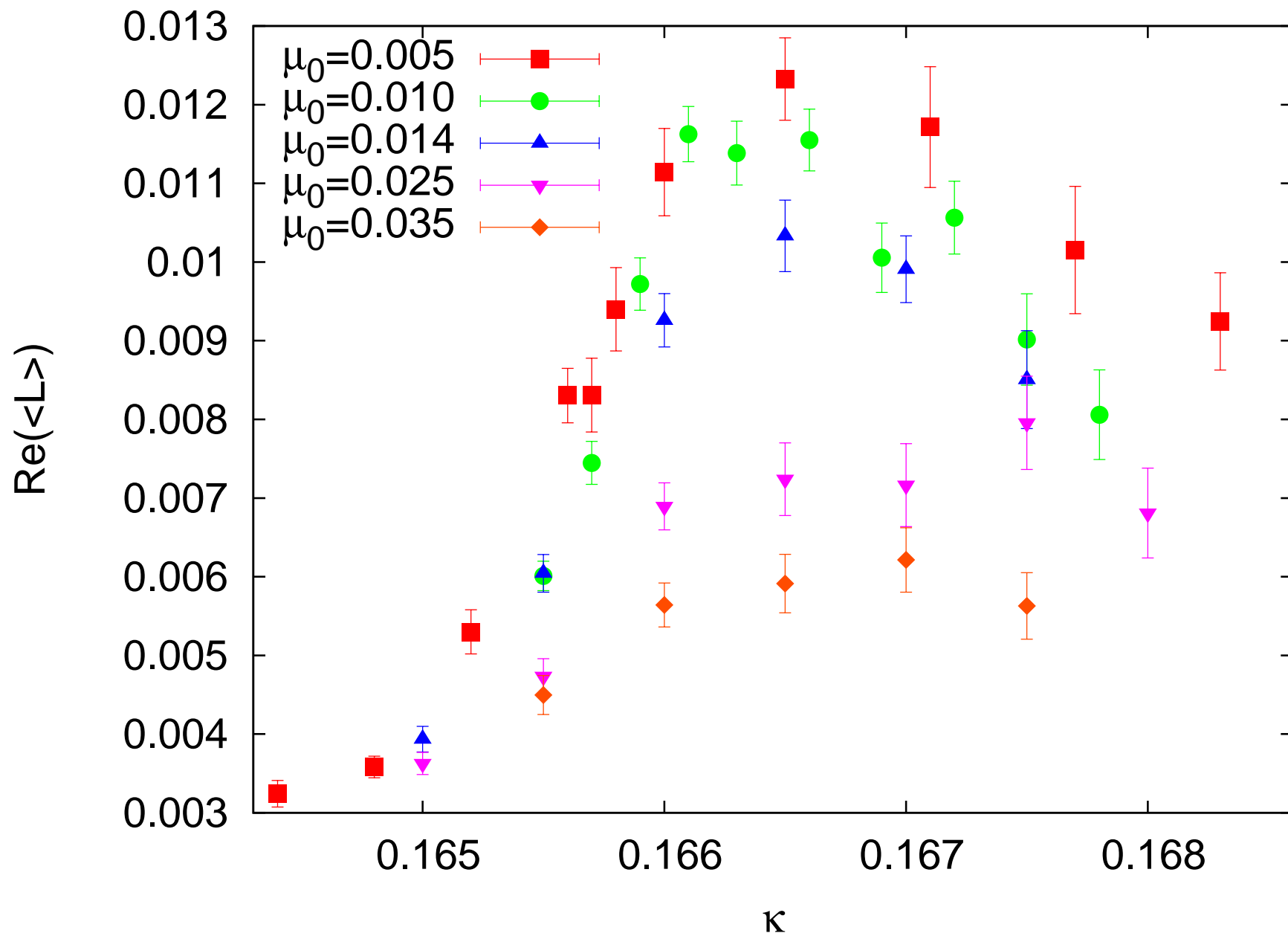
Towards the continuum limit



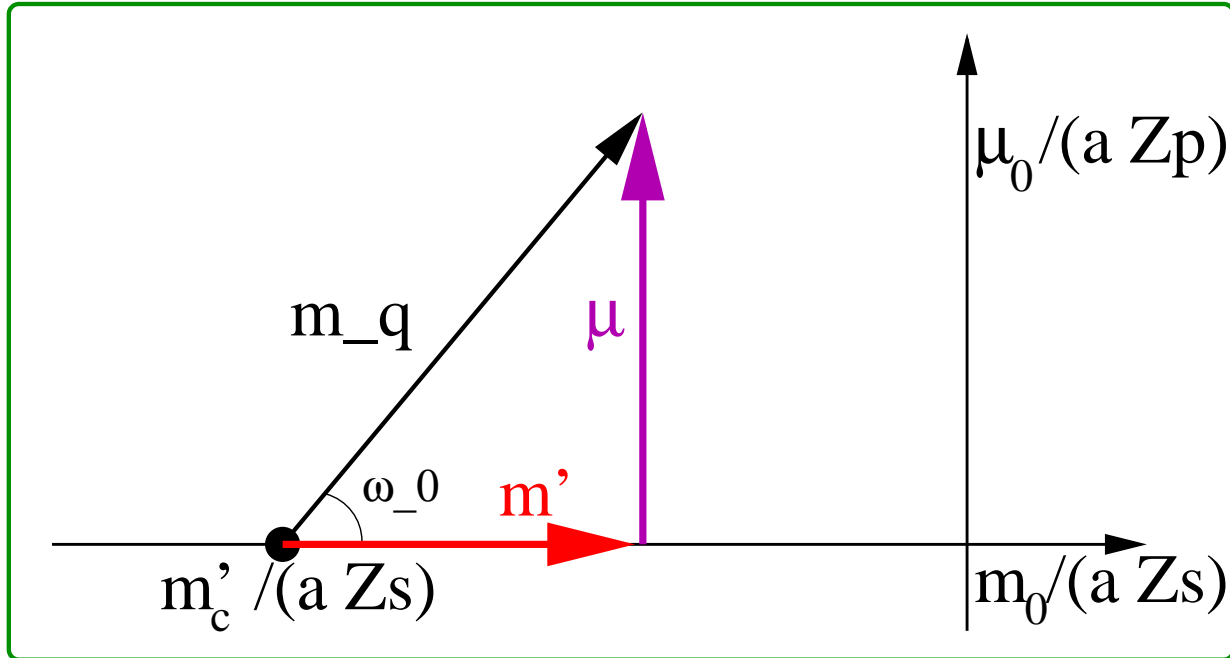
The vertical lines mark $\kappa_c(T=0, \beta)$ for $\beta = 3.9, 3.8, 3.75$ from left to right.



$\beta = 3.75$ THE LOWEST MASS POINT , and ChPT



Back to the action:
LO ($L\chi PT$)



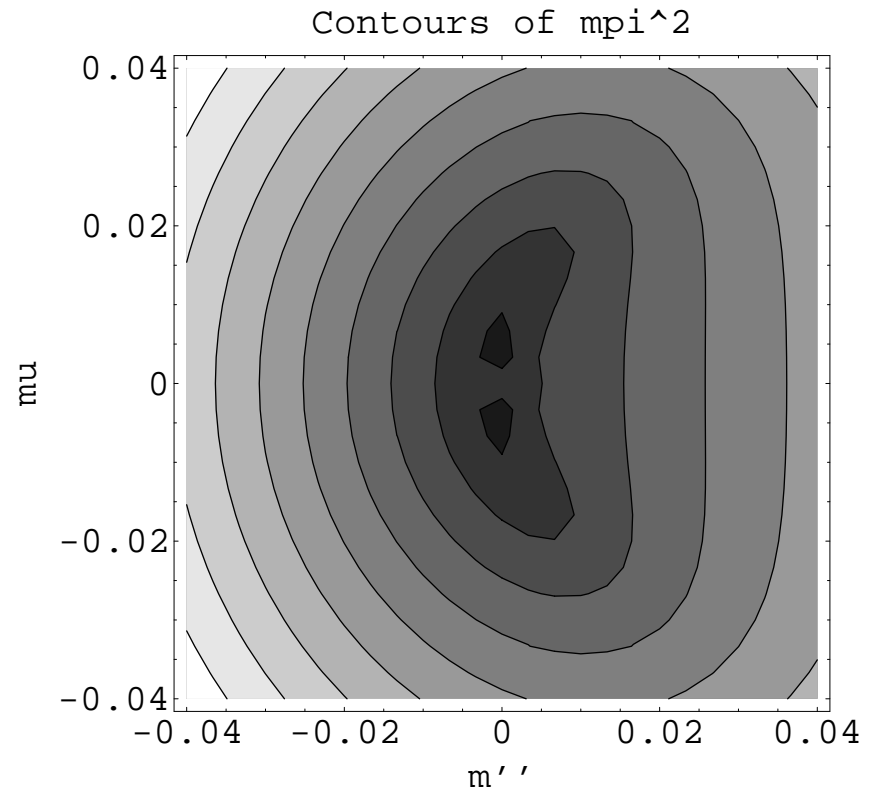
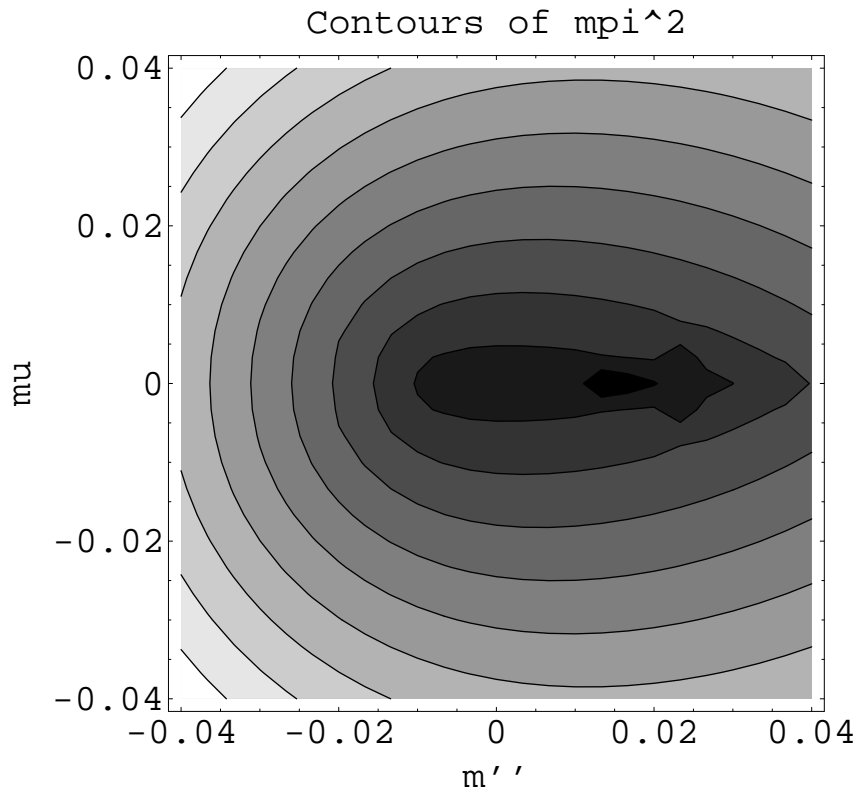
The contour plots of $m_{\pi\pm}^2$ in twisted mass plane are ellipses with axes renormalized by the Z factors. (figure after Sharpe 2006)

LO (L χ PT)

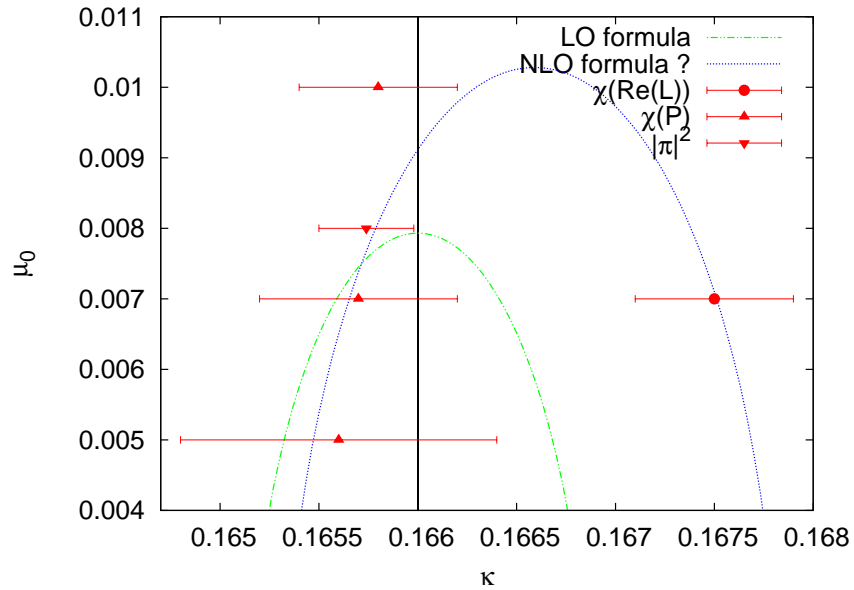
The general form of NLO results for observables is

$$m_{\pi^\pm}^2 = \widehat{m}_q \left[1 + \frac{1}{2}L_\pi + \frac{16}{f^2}\widehat{m}_q(2L_{68} - L_{45}) \right] + \widehat{m}_q \cos \omega_0 (2\delta_W - \delta_{\widetilde{W}}) + 2(\cos \omega_0)^2 w'. \quad (1)$$

This form produces contour plots of $m_{\pi^\pm}^2$ in twisted mass plane, which are distorted ellipses



Contour plots of $m_{\pi^{\pm}}^2$ in twisted mass plane, from NLO ($L\chi$ PT) for Aoki-phase (left) and first-order (right) scenarios (Sharpe 2006)

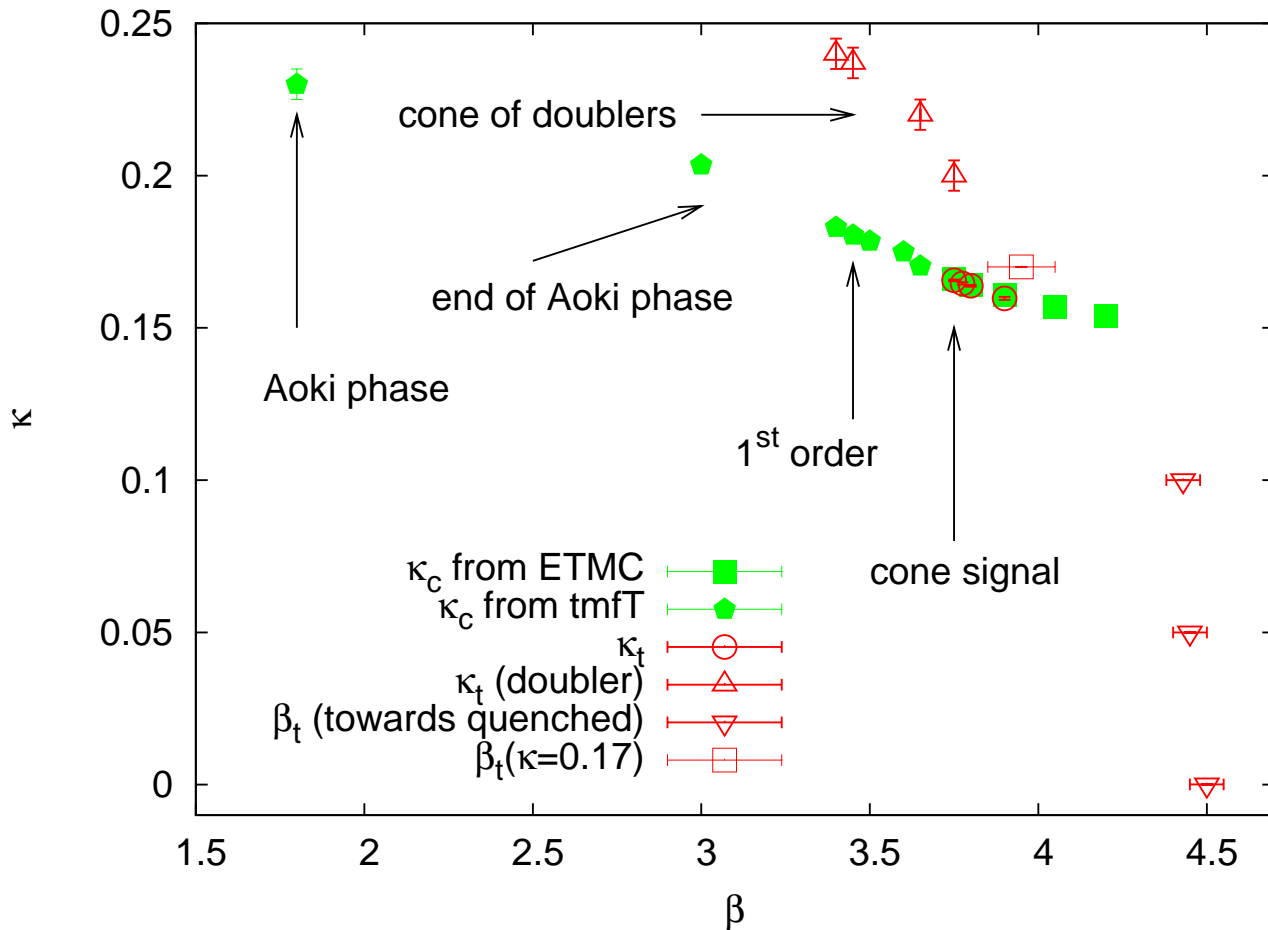


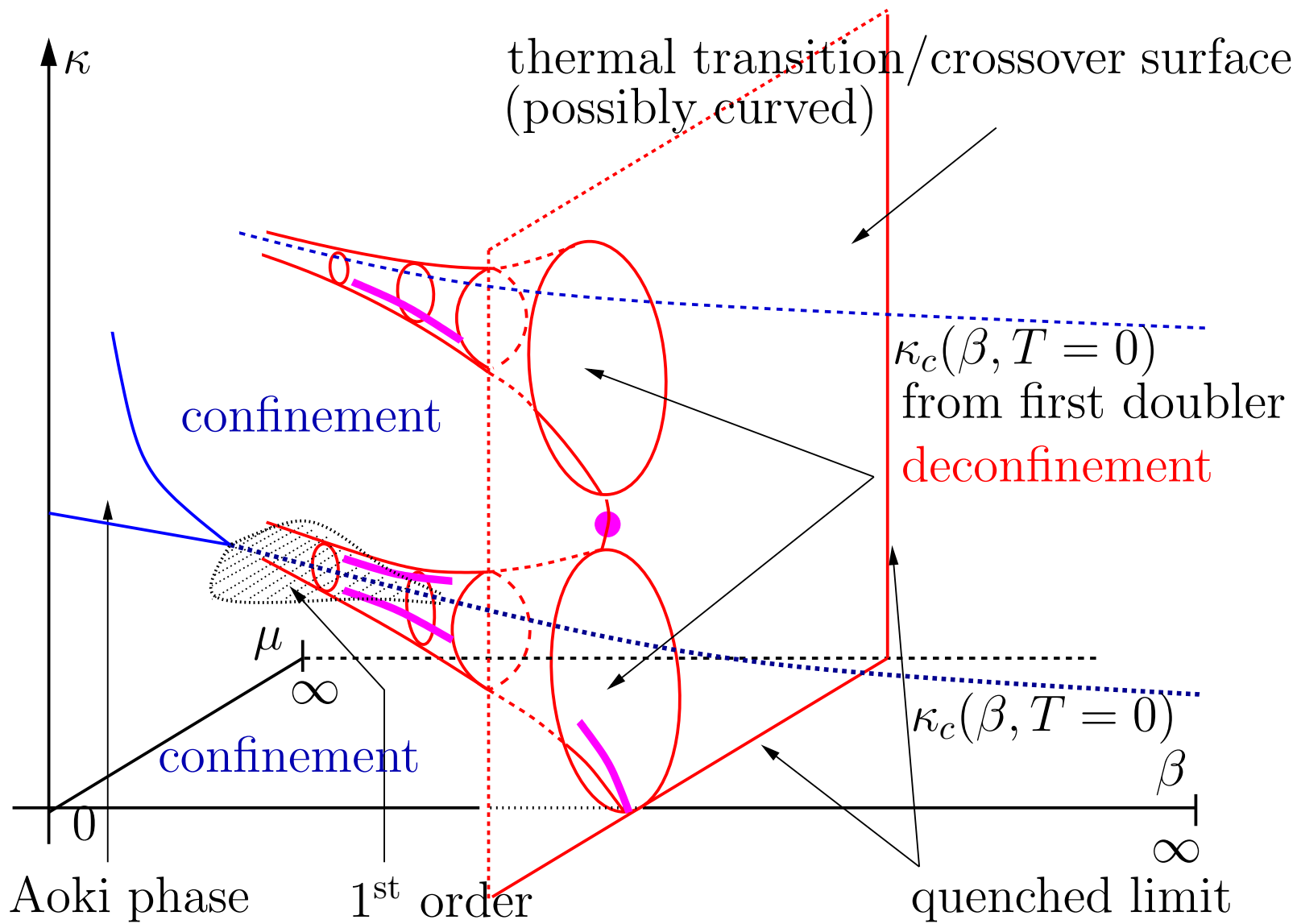
The NLO formula

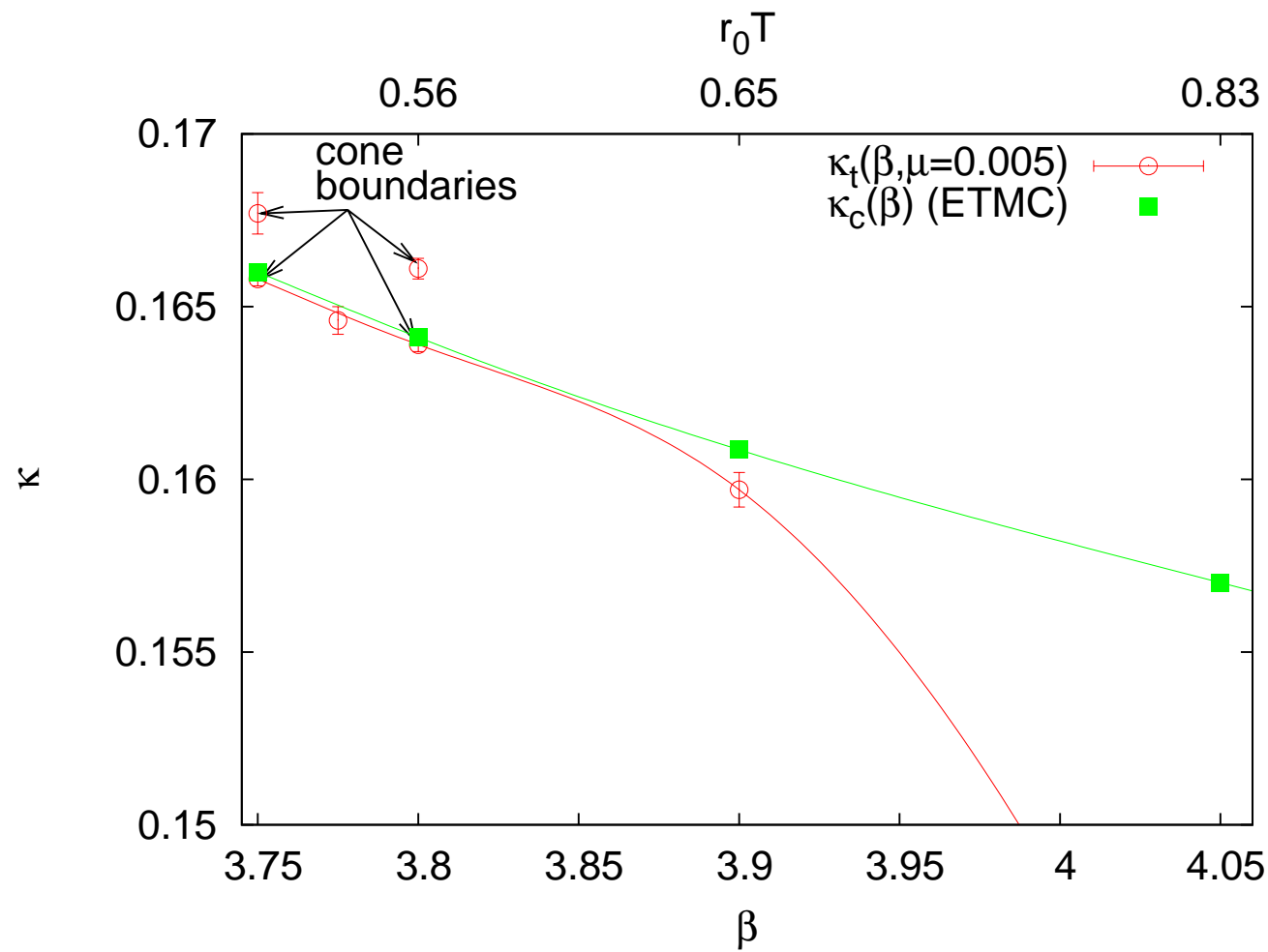
$$m_q^2 = \left(\frac{1}{Z_P^2} \mu^2 + \frac{1}{Z_S^2} \frac{1}{4} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right)^2 \right) (1 + K \cos \omega)^2 .$$

with $m_q = 0.028$, $K = 0.5$, $Z_S = 0.6$, $Z_P = 0.3$ and $\kappa_c = 0.1660$ is consistent with the data points.

SUMMARY

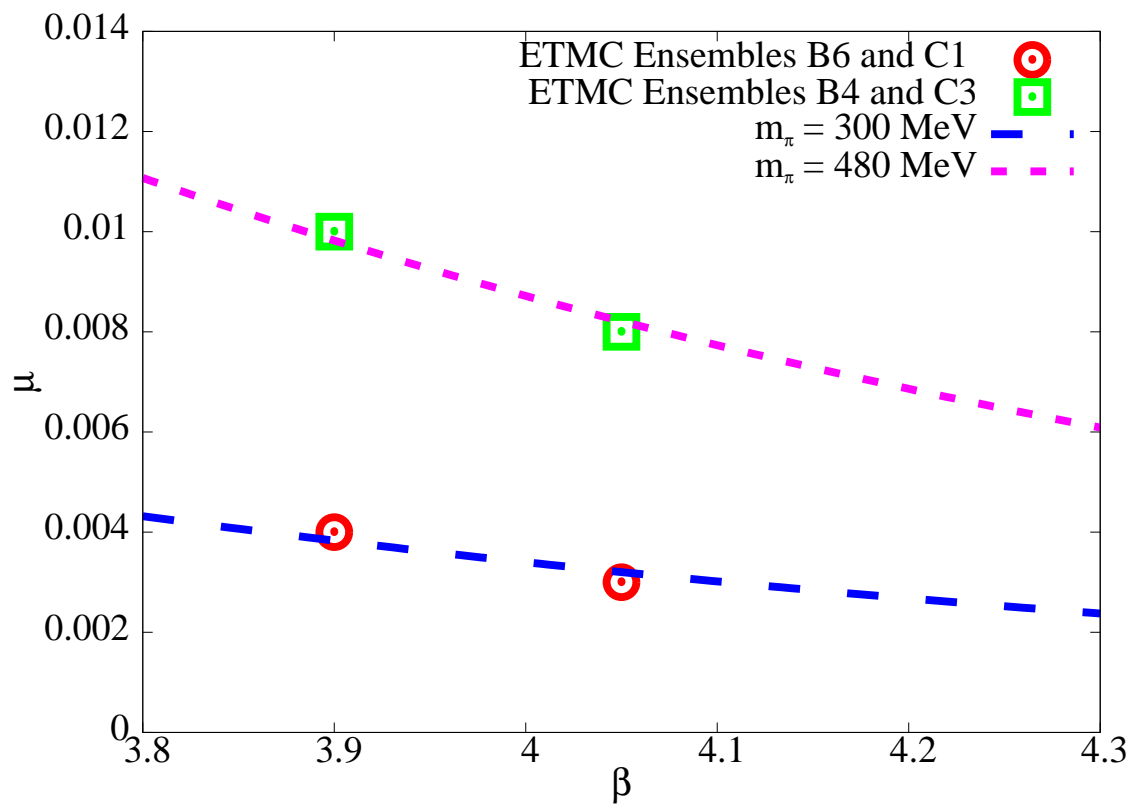




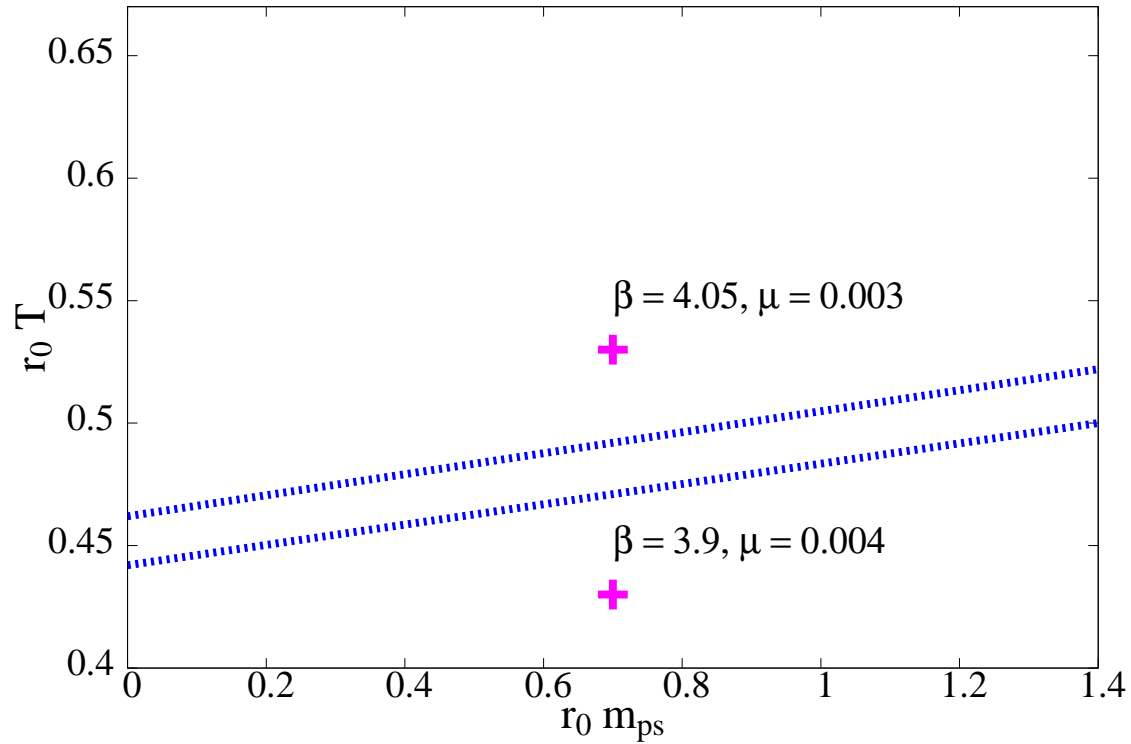


PERSPECTIVES

Trajectories of constant physics in the $\beta\mu$ plane :
 $m_\pi \simeq 300$ MeV , and $m_\pi \simeq 480$ MeV



Critical band $r_0(T - T_c) = a(r_0 m_{ps})^d$ and possibilities for twisted mass



*The vertical segment joining the two points corresponds to $m_\pi \simeq 300$ MeV,
 $N_t = 12$*

In brief

- Structure of the phase space understood.
 - Agreement with model calculations
 - Agreement with lattice NLO ChPT
- Measured thermal line at large mass $500\text{MeV} < m_\pi$, at the onset of continuum physics.
- Continuum thermodynamics with pion masses of about 300 MeV taking full advantage of the automatic improvement should be feasible with $N_t = 12$