Finite Temperature/Density Simulation of Two-Color QCD

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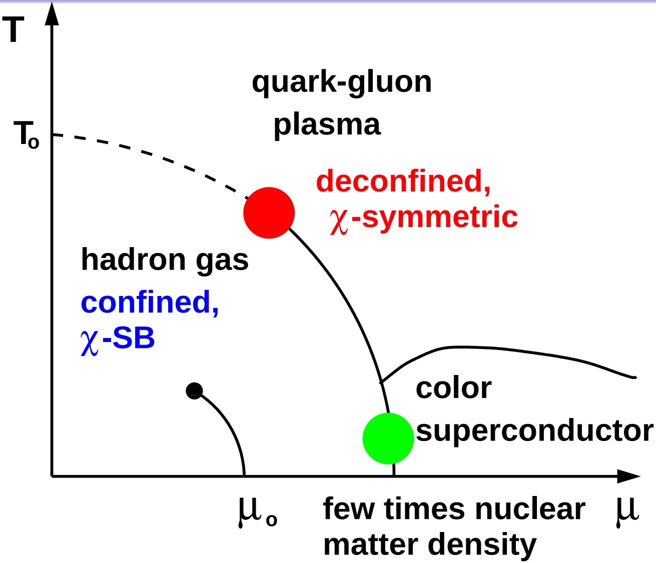
QCD phase diagram



QCD phase diagram

- QCD is asymptotically free theory
- high T $\longrightarrow$ weak coupling:

  deconfinement and restored chiral symmetry
- high μ $\longrightarrow$ weak coupling:

  weak attraction
- Existence of the critical point at finite T and $\mu \neq 0$
- Existence of new phase at low T and $\mu \neq 0$

QCD phase diagram

the strong interaction is strong!!!

non-perturbative study method is needed

$\longrightarrow$ Lattice QCD

Complex Action

- Lattice QCD is essential

- But with finite chemical potential QCD lagrangin is complex

$$M(\mu) = \gamma_\mu D_\mu + m + \mu\gamma_0 \tag{1}$$

Λ such that $M^\dagger(\mu) = \Lambda M(\mu)\Lambda^{-1}$ can't be found

- Monte Carlo simulation is difficult (importance sampling can be accomplished only after large cancellation)

Complex Action

- consider an Gaussian integral analytically

$$\langle\sigma^2\rangle(z) = \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}\sigma^2+iz\sigma}}{\int d\sigma\, e^{-\frac{1}{2}x^2+iz\sigma}} \tag{1}$$

$$= \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}(\sigma-iz)^2-\frac{1}{2}z^2}}{\int d\sigma\, e^{-\frac{1}{2}(\sigma-iz)^2-\frac{1}{2}z^2}} \tag{2}$$

$$= \frac{\int d\sigma'\, (\sigma'+iz)^2\, e^{-\frac{1}{2}\sigma'^2-\frac{1}{2}z^2}}{\int d\sigma'\, e^{-\frac{1}{2}\sigma'^2-\frac{1}{2}z^2}} \tag{3}$$

$$= \frac{\int d\sigma'\, \sigma'^2\, e^{-\frac{1}{2}\sigma'^2-\frac{1}{2}z^2}}{\int d\sigma'\, e^{-\frac{1}{2}\sigma'^2-\frac{1}{2}z^2}} - z^2 \tag{4}$$

$$= 1 - z^2 \tag{5}$$

Complex Action

- consider the same Gaussian integral in Monte Carlo

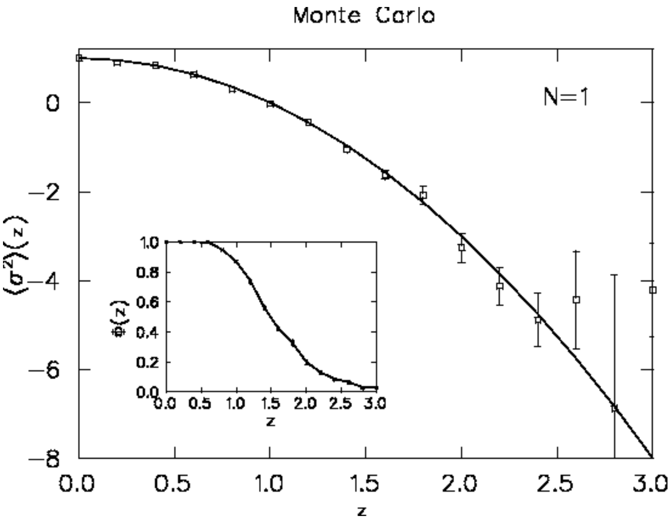
$$\langle\sigma^2\rangle(z) = \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}\sigma^2+iz\sigma}}{\int d\sigma\, e^{-\frac{1}{2}x^2+iz\sigma}} \tag{1}$$

$$= \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}\sigma^2}\cos(z\sigma)}{\int d\sigma\, e^{-\frac{1}{2}\sigma^2}\cos(z\sigma)} \tag{2}$$

$$= \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}\sigma^2+\log(\cos(z\sigma))}}{\int d\sigma\, e^{-\frac{1}{2}\sigma^2+\log\cos(z\sigma)}} \tag{3}$$

(4)

Complex Action



Complex Action

- consider the same Gaussian integral in complex Langevin (S.Koonin and C. Adami, PRC63, 034319(2001))

$$\langle \sigma^2 \rangle (z) = \frac{\int d\sigma\, \sigma^2\, e^{-\frac{1}{2}\sigma^2 + iz\sigma}}{\int d\sigma\, e^{-\frac{1}{2}x^2 + iz\sigma}} \tag{1}$$

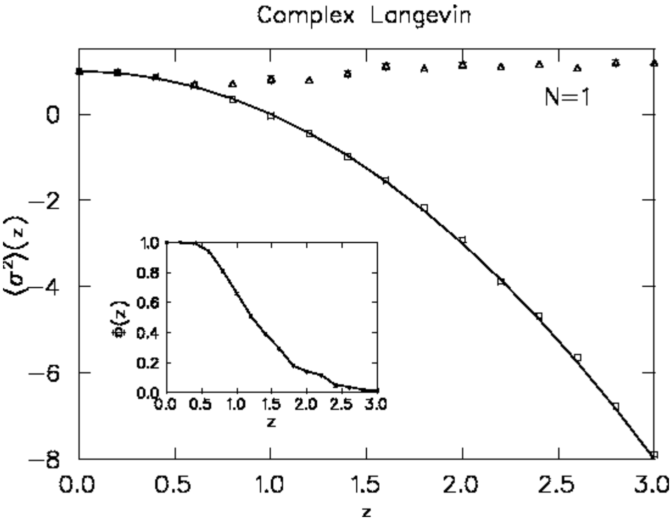
$$S = -\frac{1}{2}\sigma^2 + iz\sigma \tag{2}$$

$$\frac{\partial \sigma_R(t)}{\partial t} = -\frac{1}{2}\mathrm{Re}\left(\frac{\partial S}{\partial \sigma}\right) + \eta(t) \tag{3}$$

$$\frac{\partial \sigma_I(t)}{\partial t} = -\frac{1}{2}\mathrm{Im}\left(\frac{\partial S}{\partial \sigma}\right) \tag{4}$$

(5)

Complex Action



Complex Action

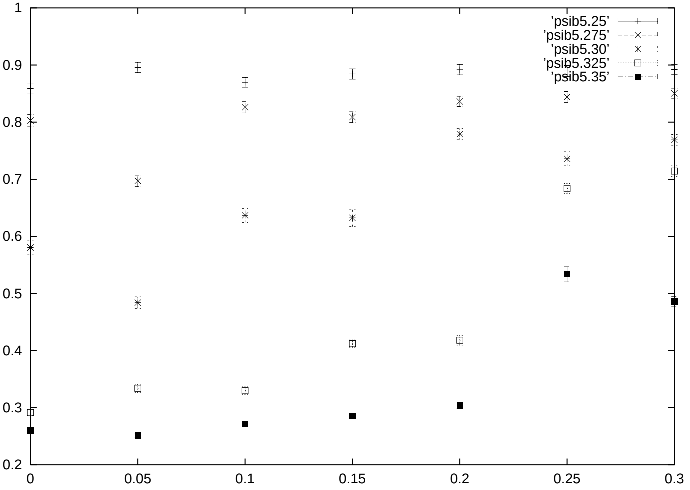
- 4-dimensional Thirring term and quantum chromodynamics with finite chemical potential,

  S.K., Prog. Theor. Phys. Suppl. **153** (2004) 349

Complex Action

$$
\begin{aligned}
\mathcal{L} &= \overline{\psi}(\partial\!\!\!/ + m)\psi + \mu\overline{\psi}\gamma_4\psi && (1)\\
&\to \overline{\psi}(\partial\!\!\!/ + m)\psi + \mu\overline{\psi}\gamma_4\psi + \frac{G}{2}(\overline{\psi}\gamma_\mu\psi)^2 && (2)\\
&\to \exp[-\int d^4x\{\mu\overline{\psi}\gamma_4\psi + \frac{G}{2}(\overline{\psi}\gamma_\mu\psi)^2\}] && \\
&\to \int[d\tilde{B}_4][d\tilde{B}_i]\exp\Big[-\int d^4x\{\frac{1}{2G}\tilde{B}_4^2 && \\
&+ i\tilde{B}_4(\overline{\psi}\gamma_4\psi + \frac{\mu}{G}) + \frac{1}{2G}\tilde{B}_i^2 + i\tilde{B}_i\overline{\psi}\gamma^i\psi - \frac{\mu^2}{2G}\}\Big], && (3)\\
\mathcal{L}_{\mathrm{HLS}} &= \overline{\psi}(D\!\!\!\!/ + m)\psi + \frac{1}{2G}(\partial_i\theta + B_i)^2 + \frac{1}{2G}(\partial_4\theta + B_4 + i\mu)^2 && (4)
\end{aligned}
$$

Complex Action



2-Color QCD

- Two color QCD or SU(2) gauge theory in large chemical potential is different from QCD

- For SU(2),

$$(C\gamma_5)\tau_2 M(\mu)(C\gamma_5)^{-1}\tau_2 = M^*(\mu) \quad (1)$$

- $\det M(\mu)$ is real (but does not mean that it is positive)

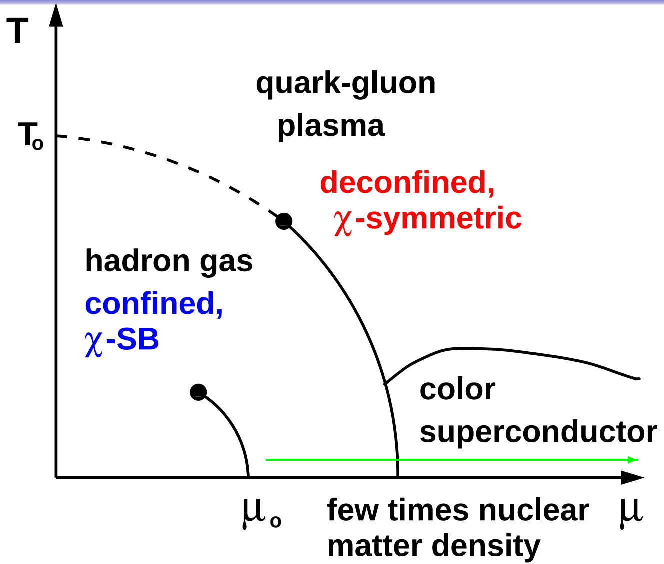
- There is spontaneous chiral symmetry breaking
$\longrightarrow$ pion is light

- But qq is a color singlet $\longrightarrow$ diquark condensate does not break color symmetry

2-Color QCD

- Model for QCD but
- gluon sector is similar to QCD
- large chemical potential region can be studied

Low T, μ


T
quark-gluon
plasma
T o
deconfined,
χ-symmetric
hadron gas
confined,
χ-SB
color
superconductor
μ o
few times nuclear
matter density
μ

Low T, μ

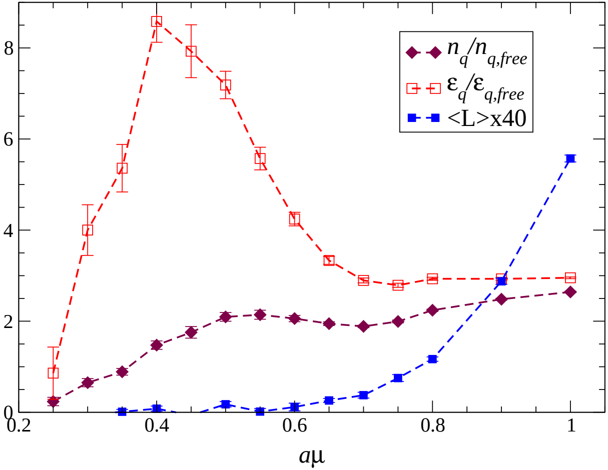
- Two color QCD with heavy quark:

  S. Hands, S. K., J.-I. Skullerud, Eur.Phys.J.C48:193,2006

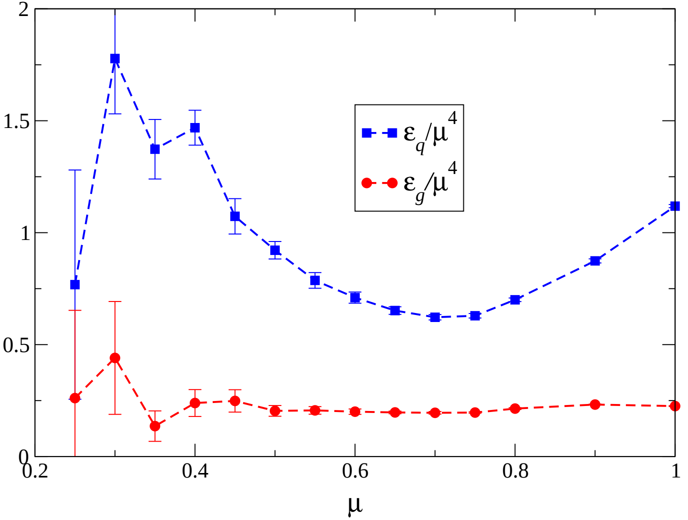
  $8^3 \times 16, \beta = 1.7$ Wilson quark

  $\kappa = 0.168, m_\pi a \sim 0.8$

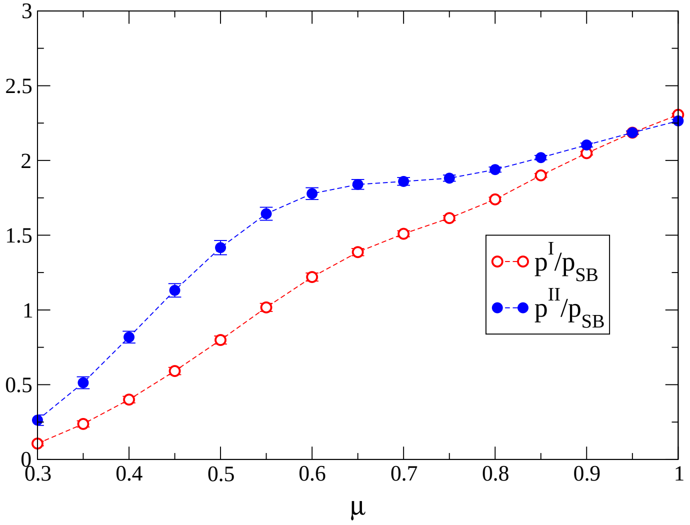
Low T, μ



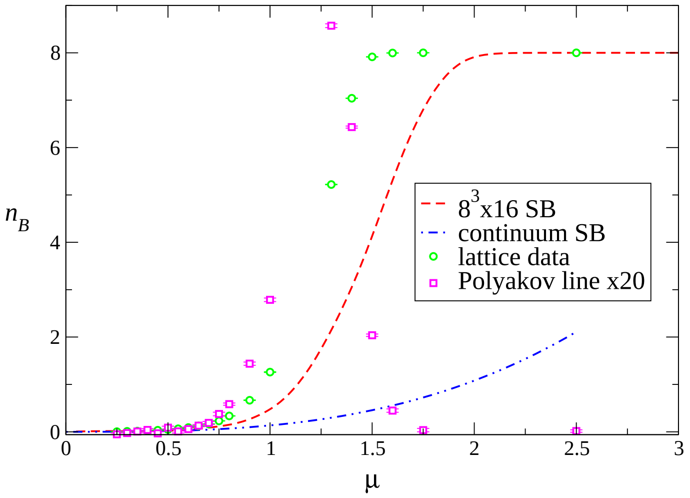
Low T, μ



Low T, μ



Low T, μ



8
6
4
2
0
n_B
0
0.5
1
1.5
2
2.5
3
μ
8^3x16 SB
continuum SB
lattice data
Polyakov line x20

Low T, μ

- Hint of three different phases

  Hadronic phase

  Bose-Einstein Condensed(BEC) phase

  Bardeen-Cooper-Schrieffer(BCS) phase

- BEC-BCS transition is not a sharp transition

  strong attraction $\longrightarrow$ tightly bound boson

  $\longrightarrow$ boson condesation

  weaker attraction $\longrightarrow$ loosely bound Cooper pair

  $\longrightarrow$ superconducting phase

Low T, μ

- P. Nozieres, S.Schmitt-Rink, J. of Low Temp. Phys. 59, 195(1985)

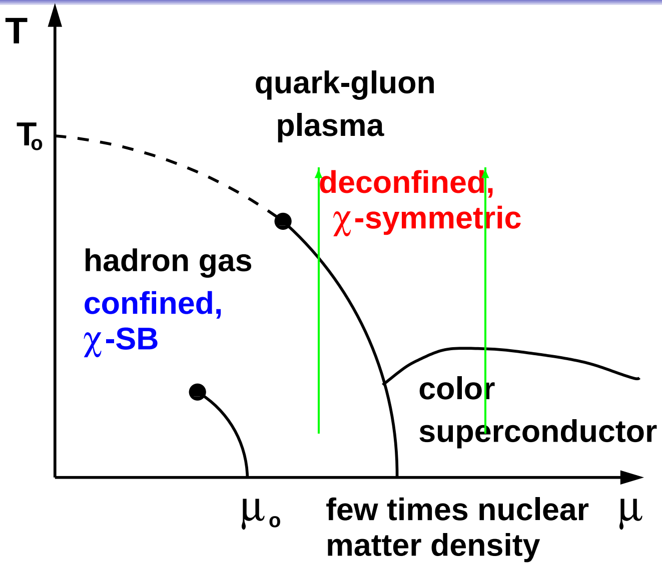
finite temperature transition of BEC phase is different

$$T_c = (2\pi/M)(N_p/2.612)^{2/3}$$

from that of BCS phase

$$T_c = (e^{\gamma}/\pi)\Delta_F$$

Finite T, μ


T
T_o
quark-gluon
plasma
deconfined,
χ-symmetric
hadron gas
confined,
χ-SB
color
superconductor
μ_o
few times nuclear
matter density
μ

Finite T, μ

- temperature is defined as $T = \frac{1}{N_\tau a}$

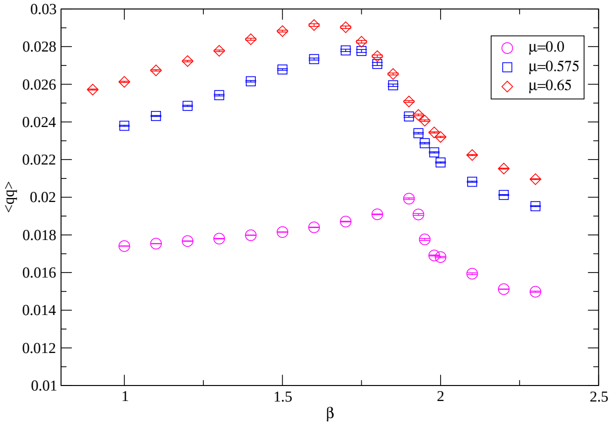
  $\leftarrow$ a is controlled by gauge coupling constant
  $\leftarrow$ different temperature means a different gauge coupling constant

- periodic boundary condition on bosonic field, anti-periodic boundary condition on fermionic field

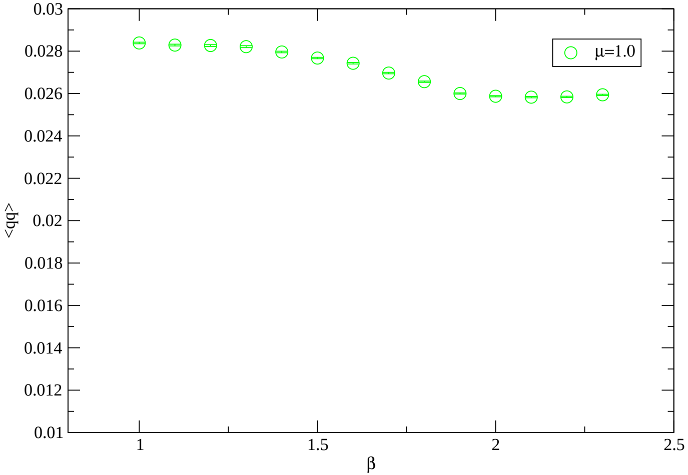
- no. of spatial lattice sites (N_s) should be bigger than N_τ

- phases are distinguished by order parameters

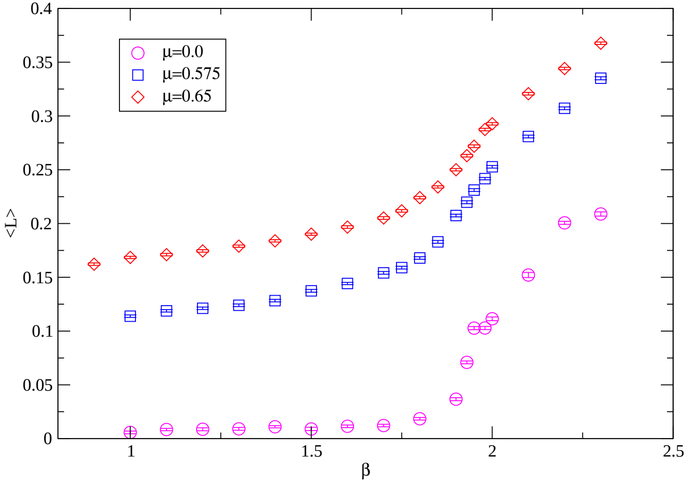
Finite T, μ


0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
<qq>
1
1.5
2
2.5
β
μ=0.0
μ=0.575
μ=0.65

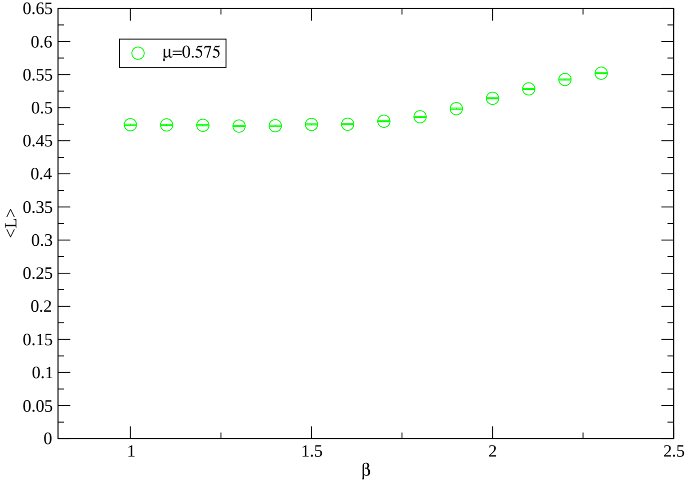
Finite T, μ


0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
<qq>
1
1.5
2
2.5
β
μ=1.0

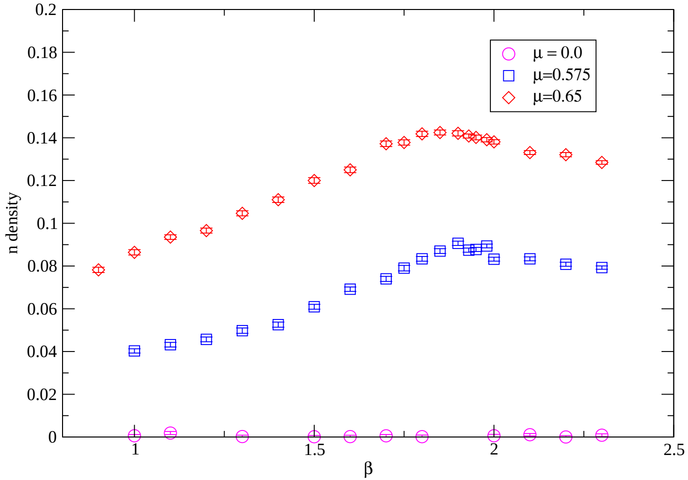
Finite T, μ



Finite T, μ

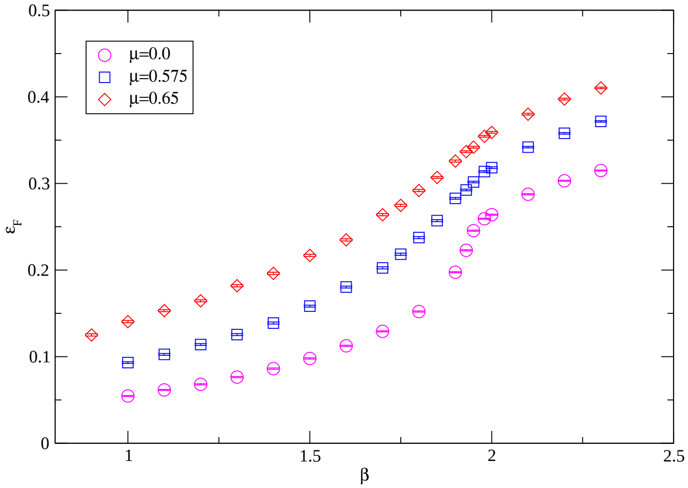


Finite T, μ

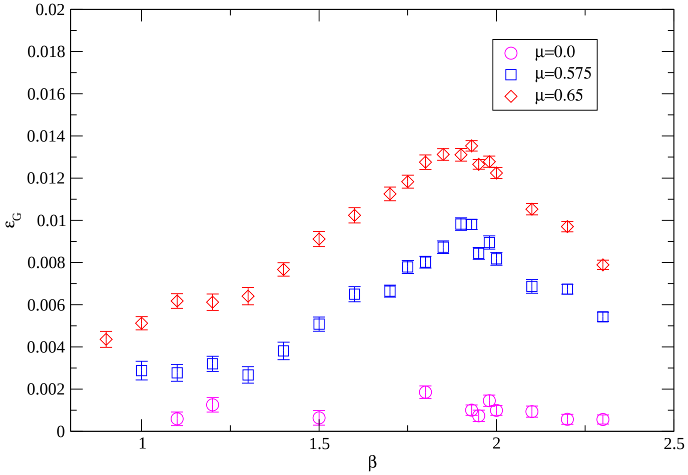


0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
n density
1
1.5
2
2.5
β
μ = 0.0
μ=0.575
μ=0.65

Finite T, μ



Finite T, μ


0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
ε_G
1
1.5
2
2.5
β
μ=0.0
μ=0.575
μ=0.65

Discussion

- diquark condensate behavior shows interesting behavior

  for $\mu = 0.575, 0.65$, the diquark condensate doesn't change much for $1.75 < \beta < 2.0$

- further study is needed

- many parameters to scan $\rightarrow$ Grid computing