Nonperturbative Vacuum Structure of QED

Sang Pyo Kim

Kunsan National University & APCTP

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What is quantum vacuum structure in strong background fields?
(gauge fields or curvature of spacetime)

Gauge Theory (QED, QCD) and Quantum Field Theory in Curved Spacetime

What do we know in physics?

- Quotation from Mattuck, "A Guide to Feynman Diagrams in the Many-Body Problems" ('76)
- In 18th century Newtonian mechanics, the three-body problem was insoluble.
- With the birth of general relativity around 1910 and quantum electrodynamics in 1930, the two- and one-body problems became insoluble.
- Within modern quantum field theory, the problem of zero bodies (vacuum) is insoluble.

Outline

- Motivation
- Effective Action
- Strong QED
- Conclusion

Strong Background Field Theory

QED/QCD

- External gauge fields
- QCD/Yang-Mills theory
  cf. Maxwell theory
  (linear & superposition)
- Renormalizable
- Schwinger mechanism
  (pair production by electric fields)
- Constant E-field
- Constant B-field
- E-B duality

QG

- Spacetime curvature
- Einstein gravity
  (nonlinear & no-superposition)
- Non-renormalizable
- Particle production
  (Hawking radiation or cosmic particles)
- de Sitter(dS) space
- Anti-de Sitter(AdS) space
- dS-AdS duality?

Can we test strong field theory?

QED

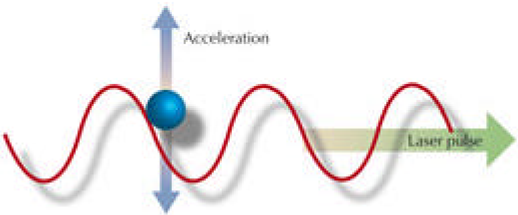
- Compact stars
  - neutron stars
  - magnetars
  - strange quark stars
- ELI(Extreme Light Infrastructure by EU consortium)

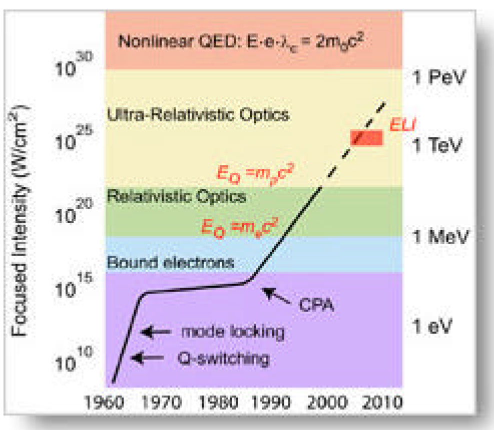
QG

- Primordial black holes
- Planck scale structure of the universe
- Gravitational wave detectors for information of the black holes
- Mini-black holes from LHC?

Yes, the answer is affirmative!

Extreme Light Infrastructure





- X-ray free electron lasers, Extreme light
- Schwinger limit (critical strength)

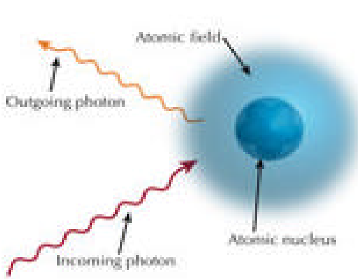
$$E_c = \frac{mc^2}{q\,l_C} = \frac{m^2c^3}{q\hbar}$$

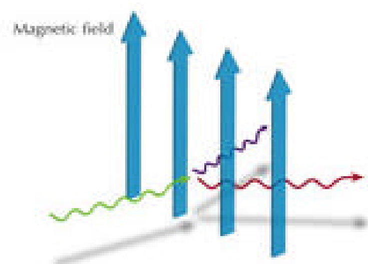
- Critical strength for e-e+ pair production

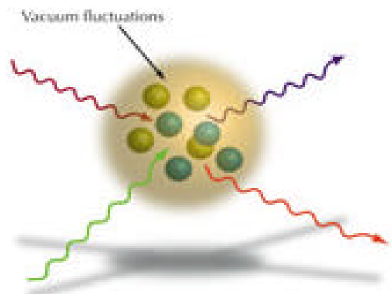
$$E_c = 2.2\times10^{15}(V/cm)$$

Extreme Light Infrastructure

- Can test strong QED

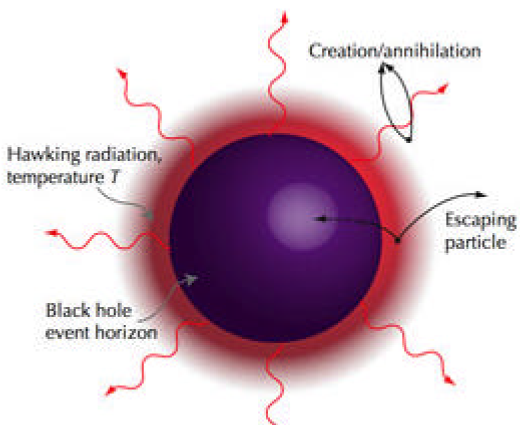


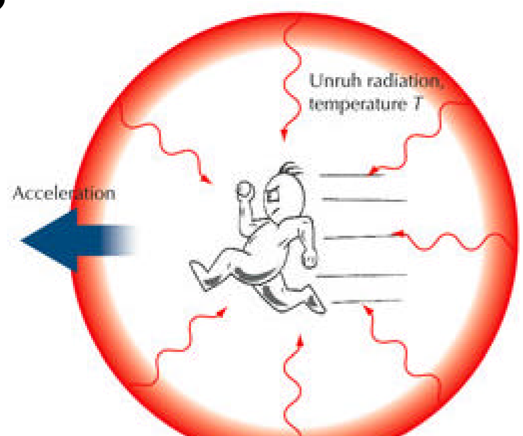




(Delbruck scattering) (Photon splitting) (Pair production)

- Can test the Hawking-Unruh radiation($a > 10^{24} g$)





One-Loop Effective Action of QED

- One-loop effective action of spinor QED

$$S_{eff} = -i \ln \det(i\not{D} - m) = -\frac{i}{2} \ln \det(\not{D}^2 + m^2)$$

- One-loop effective action of scalar QED

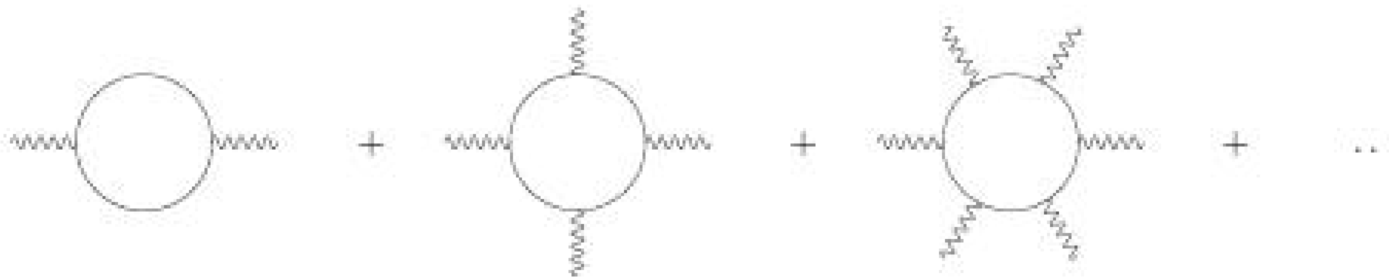
$$S_{eff} = +\frac{i}{2} \ln \det(D^2 + m^2)$$

- Schwinger's proper-time integral method

$$S_{eff} = -\frac{i}{2} \mathrm{Tr} \ln(\not{D}^2 + m^2) = -\frac{i}{2} \int_0^\infty \frac{ds}{s} \mathrm{Tr} \exp\left[-(\not{D}^2 + m^2)s\right]$$

Diagrammatic Representation of Exact One-Loop Effective Action

- Diagrammatic perturbative expansion of one-loop effective action



- External legs: EM fields or gravitons
- Internal loops: bosons or fermions

One-Loop Effective Action in Constant EM-Fields

- **Gauge invariant forms**

$$F = \frac{1}{4} F^{mn} F_{mn} = \frac{1}{2}\left(\mathrm{B}^2 - \mathrm{E}^2\right), \quad G = \frac{1}{4} F^{mn} F^{*}_{mn} = \mathrm{B} \cdot \mathrm{E}$$

$$X = \sqrt{2(F + iG)} = X_r + iX_i$$

- **Heisenberg-Euler and Schwinger effective action/volume and time**

$$L_{eff} = -F - \frac{1}{8p^2} \int_0^\infty ds \frac{e^{-m^2 s}}{s^3} \left[(qs)^2 G \frac{\mathrm{Re}\cosh(qXs)}{\mathrm{Im}\cosh(qXs)} - 1 - \frac{2}{3}(qs)^2 F \right]$$

Effective Action of Pure B-Field

- Spinor QED in pure magnetic field

$$L_{eff}^{(1)}(B) = -\frac{(qB)^2}{8p^2}\int_0^\infty ds\,\frac{e^{-m^2 s/qB}}{s^2}\left[\coth(s) - \frac{1}{s} - \frac{s}{3}\right]$$

- Asymptotic expansion

$$L_{eff}^{(1)}(B) = -\frac{m^4}{8p^2}\sum_{n=0}^{\infty}\frac{\mathrm{B}_{2n+4}}{(2n+4)(2n+3)(2n+2)}\left(\frac{2qB}{m^2}\right)^{2n+4}$$

Effective Action of Pure E-Field

- Spinor QED in pure electric field

$$L_{eff}^{(1)}(E) = -\frac{(qE)^2}{8p^2}\int_0^\infty ds\, \frac{e^{-m^2 s/qE}}{s^2}\left[\cot(s) - \frac{1}{s} + \frac{s}{3}\right]$$

- Asymptotic expansion is NOT Borel summable

$$L_{eff}^{(1)}(E) = -\frac{m^4}{8p^2}\sum_{n=0}^{\infty}\frac{(-1)^n \mathrm{B}_{2n+4}}{(2n+4)(2n+3)(2n+2)}\left(\frac{2qE}{m^2}\right)^{2n+4}$$

- Imaginary part: $\mathrm{Im}\, L_{eff}^{(1)}(E) = \frac{(qE)^2}{8p^3}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-pm^2 n/qE}$

Electro - Magnetic Duality

- Spinor QED in pure electric field

$$L_{eff}^{(1)}(E) = -\frac{(qE)^2}{8p^2}\int_0^\infty ds\,\frac{e^{-m^2 s/qE}}{s^2}\left[\cot(s) - \frac{1}{s} + \frac{s}{3}\right]$$

- Analytic continuation

$$E \Leftrightarrow iB, \qquad s \Leftrightarrow it$$

- Spinor QED in pure magnetic field

$$L_{eff}^{(1)}(B) = -\frac{(qB)^2}{8p^2}\int_0^\infty dt\,\frac{e^{-m^2 t/qB}}{t^2}\left[\coth(t) - \frac{1}{t} - \frac{t}{3}\right]$$

Schwinger-DeWitt Proper-Time Formalism for Effective Action(1)

- **Scalar QED or minimal scalar in dS&AdS**
  - Wave operator

$$H(x)\Phi = 0, \quad H(x) = -D^m D_m + m^2, \quad D_m = \partial_m - iqA_m(x)$$

  - Green function

$$HG = 1, \quad H(x)G(x,x') = \frac{d(x-x')}{\sqrt{-g(x)}}$$

$$G(x,x') = \int_0^\infty d(is) \langle x | e^{-isH} | x' \rangle$$

Schwinger-DeWitt Proper-Time Formalism for Effective Action(2)

– **Proper-time state and Schrodinger equation**

$$\left| x, s \right\rangle = e^{isH} \left| x \right\rangle, \quad \lim_{s \to 0} \left\langle x, s \,|\, x', 0 \right\rangle = d(x - x') / \sqrt{-g}$$

$$i\partial / \partial s \left\langle x, s \,|\, x', 0 \right\rangle = H(x) \left\langle x, s \,|\, x', 0 \right\rangle$$

– **Propagator in a general coordinate system**

$$\left\langle x, s \,|\, x', 0 \right\rangle = \frac{i}{(4pis)^{d}} e^{-im^2 s} \exp\left[i \frac{t^2}{4s} \right] \Delta^{1/2}(x, x') F(x, x'; is)$$

$$t = \int_0^s ds' \sqrt{g_{mn} (dx^m / ds')(dx^n / ds')}$$

$$\Delta(x, x') = -[-g(x)]^{-1/2} \det[-\partial_m \partial_n (t^2 / 2)][-g(x')]^{-1/2}$$

$$F(x, x'; is) = 1 + (is) f_1(x, x') + (is)^2 f_2(x, x') + \cdots$$

Schwinger-DeWitt Proper-Time Formalism for Effective Action(3)

– **Effective action**

$$e^{iW} = \int d[f] e^{iS[f]} = \frac{1}{\sqrt{\det(iH/2pm^2)}}, \quad W = \frac{i}{2}\mathrm{Tr}\,\ln\left(\frac{iH}{2pm^2}\right)$$

– $dW = (i/2)\mathrm{Tr}(dH/H)$ **leads to the effective action**

$$W = -\frac{i}{2}\int d^d x\sqrt{-g}\int_0^\infty d(is)\frac{1}{(is)}\left\langle x\,|\,e^{-isH}\,|\,x'\right\rangle$$

$$= \frac{1}{2}\int d^d x\sqrt{-g}\int_0^\infty d(is)\frac{e^{-im^2 s}}{(is)(4ps)^{d/2}}F(x,x';is)$$

$$f_1 = R,\ \ f_2 = \frac{1}{30}R_{;m}{}^{;m} + \frac{1}{12}R^2 + \frac{1}{180}R_{mnab}R^{mnab} - \frac{1}{180}R_{mn}R^{mn}$$

Evolution Operator Method for Effective Action(1)

- In-and out-state formalism [Schwinger ('51), Nikishov ('70), DeWitt ('75), Ambjorn et al ('83)]

$$e^{iW} = e^{i\int dt d^3x L_{eff}} = \langle 0, \mathrm{out} \mid 0, \mathrm{in} \rangle$$

- Bogoliubov transformation

$$a_{\mathrm{k,out}} = a_{\mathrm{k,in}} a_{\mathrm{k,in}} + b^*_{\mathrm{k,in}} b^+_{\mathrm{k,in}} = U_{\mathrm{k}} a_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

$$b_{\mathrm{k,out}} = a_{\mathrm{k,in}} b_{\mathrm{k,in}} + b^*_{\mathrm{k,in}} a^+_{\mathrm{k,in}} = U_{\mathrm{k}} b_{\mathrm{k,in}} U^+_{\mathrm{k}}$$

Evolution Operator Method for Effective Action(2)

– Evolution operator in terms of two-mode squeeze operator [Caves, Schumaker ('85)]

$$U_{\mathrm{k}} = S_{\mathrm{k}} P_{\mathrm{k}}, \quad \begin{cases} P_{\mathrm{k}} = \exp[iq_{\mathrm{k}}(a^{+}_{\mathrm{k,in}} a_{\mathrm{k,in}} + b^{+}_{\mathrm{k,in}} b_{\mathrm{k,in}} + 1)] \\ S_{\mathrm{k}} = \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2ij_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2ij_{\mathrm{k}}}) \end{cases}$$

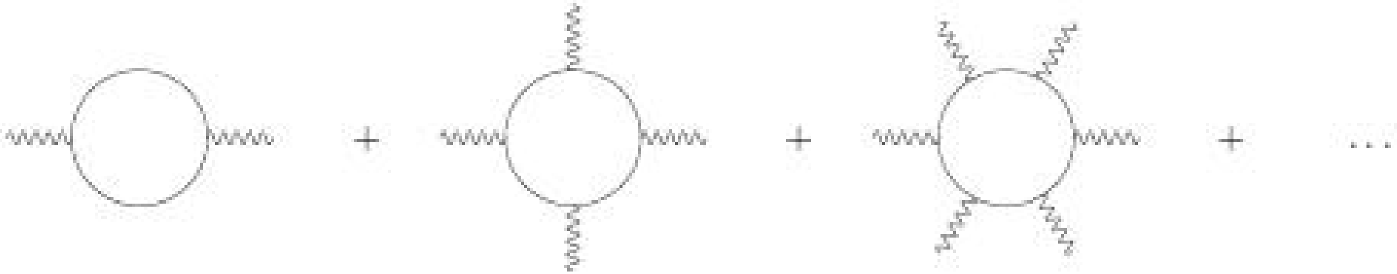
$$a_{\mathrm{k}} = e^{-iq_{\mathrm{k}}} \cosh r_{\mathrm{k}}, \quad b^{*}_{\mathrm{k}} = -e^{-iq_{\mathrm{k}}} (e^{2ij_{\mathrm{k}}} \cosh r_{\mathrm{k}})$$

Evolution Operator Method for Effective Action(3)

– The outgoing vacuum is the two-mode squeezed state of the in-going vacuum

$$\left|0,\mathrm{out}\right\rangle = \prod_{\mathrm{k}} U_{\mathrm{k}} \left|0,\mathrm{in}\right\rangle = \prod_{\mathrm{k}} e^{iq_{\mathrm{k}}} S_{\mathrm{k}} \left|0,\mathrm{in}\right\rangle$$

– Diagrammatic representation for pair production $S_{\mathrm{k}} = \exp[r_{\mathrm{k}}(a_{\mathrm{k,in}} b_{\mathrm{k,in}} e^{-2ij_{\mathrm{k}}} - a^{+}_{\mathrm{k,in}} b^{+}_{\mathrm{k,in}} e^{-2ij_{\mathrm{k}}})]$



Evolution Operator Method for Effective Action(4)

[SPK, H.K. Lee, Y. Yoon, PRD 78 ('08)]

– Zero-temperature effective action for scalar

$$W = -i \ln\langle 0, \mathrm{out} \,|\, 0, \mathrm{in} \rangle = i \sum_{\mathrm{k}} \ln a_{\mathrm{k}}^{*}$$

– Zero-temperature effective action for spinor

$$W = -i \ln\langle 0, \mathrm{out} \,|\, 0, \mathrm{in} \rangle = -i \sum_{\mathrm{k}} \ln a_{\mathrm{k}}^{*}$$

Evolution Operator Method for Effective Action(5)

[SPK, H.K. Lee, Y. Yoon, PRD 78 ('08)]

– Vacuum persistence for scalar QED

$$\left|\langle 0,\mathrm{out}\,|\,0,\mathrm{in}\rangle\right|^2 = e^{-2\,\mathrm{Im}\,W} = e^{-V\sum_{\mathrm{k}}\ln(1+|b_{\mathrm{k}}|^2)}$$

– Vacuum persistence for spinor QED

$$\left|\langle 0,\mathrm{out}\,|\,0,\mathrm{in}\rangle\right|^2 = e^{-2\,\mathrm{Im}\,W} = e^{+V\sum_{\mathrm{k}}\ln(1-|b_{\mathrm{k}}|^2)}$$

How to find the Bogoliubov coefficients?

Vacuum Persistence and Spin-Statistics Inversion

[W-Y. P. Hwang, SPK, arXiV:0906.3813, PRD]

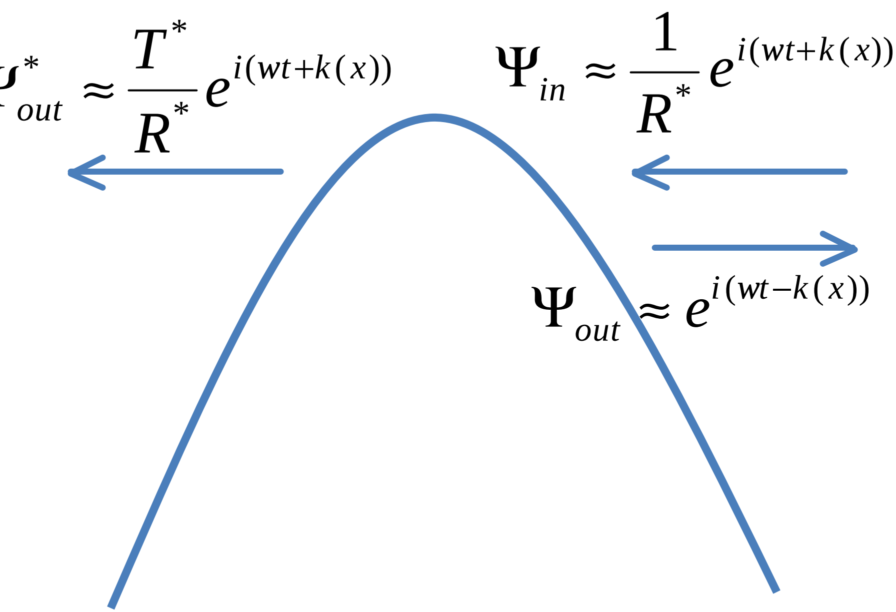
- The vacuum persistence in terms of the transverse energy $e = k_\perp^2 / m$

$$2\,\mathrm{Im}(L_{eff}) = (-1)^{2s+1} \frac{(2|s|+1)qEm}{8p} \int_0^\infty de \ln[1 + (-1)^{2s+1} N(e)]$$

- The vacuum persistence in terms of the instanton action $N(e) = e^{-S(e)}$

$$2\,\mathrm{Im}(L_{eff}) = \frac{m}{4p} \sum_s \int_{-\infty}^{\infty} \frac{dk_z}{2p} \int_0^\infty de \, \frac{e dS(e)/de}{e^{S(e)} + (-1)^{2s}}$$

Boundary Condition for Static Systems (B-field, AdS)



Tunneling under the Barrier

- Flux conservation

$$\left|\frac{1}{R}\right|^2 - \left|\frac{T}{R}\right|^2 = 1$$

- Bogoliubov coefficients for scalar QED

$$a = \frac{1}{R^*}, \quad b = \frac{T^*}{R^*}$$

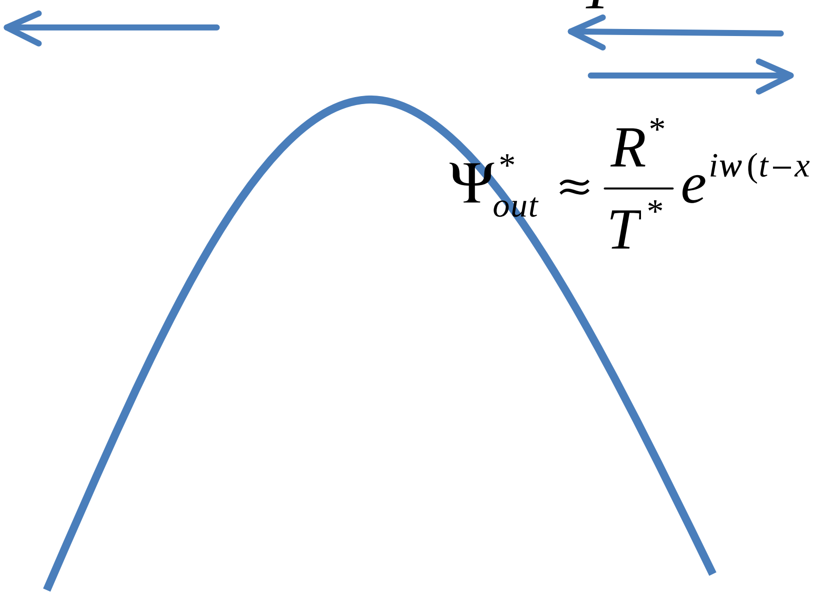
- Brezin, Itzykson, PRD 2 ('70)
- SPK, D. N. Page, PRD 65 ('02); 73 ('06)

Boundary Condition for Nonstatic Systems (E-field, dS)

$$\Psi_{in} \approx e^{i(w(t)+kx)}$$

$$\Psi_{out} \approx \frac{1}{T^*} e^{i(w(t)+kx)}$$

$$\Psi^*_{out} \approx \frac{R^*}{T^*} e^{iw(t-x)}$$



Scattering over the Barrier

- Flux conservation

$$\left|\frac{1}{T}\right|^2 - \left|\frac{R}{T}\right|^2 = 1$$

- Bogoliubov coefficients for scalar QED

$$a = \frac{1}{T^*}, \quad b = \frac{R^*}{T^*}$$

Renormalized Exact One-Loop Effective Action(1)

[SPK, H.K. Lee, Y. Yoon, PRD78 ('08)]

- Effective action/volume and time for a constant E-field
  - Bogoliubov coefficient for scalar and spinor

$$a_{\mathrm{k}} = \frac{\sqrt{2p}}{\Gamma(-p)} e^{-i(p+1)p/2}, \qquad p = -\frac{1}{2} - i\frac{m^2 + \mathrm{k}^2 + 2isqE}{2(qE)}$$

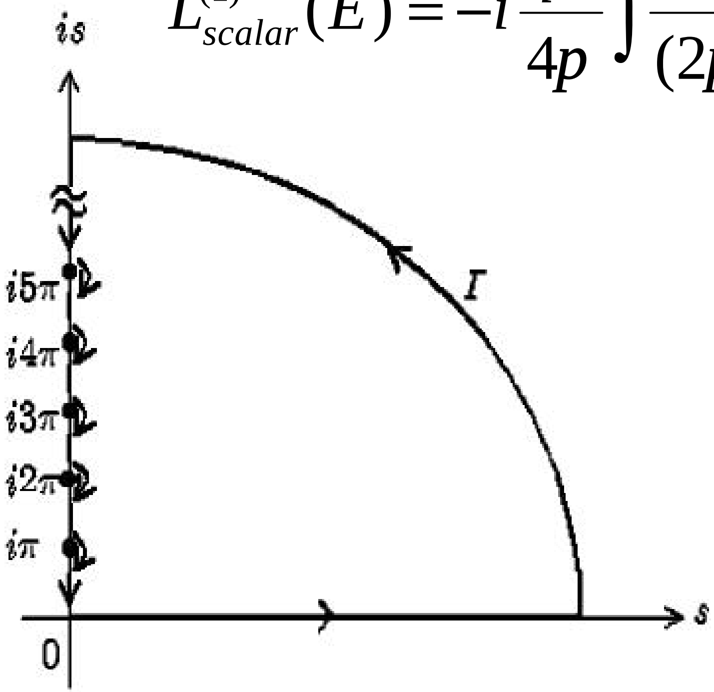
  - Gamma function regularization

$$\ln\Gamma(z) = \int_0^\infty \frac{ds}{s}\left[\frac{e^{-zs}}{1-e^{-s}} - \frac{e^{-s}}{1-e^{-s}} + (z-1)e^{-s}\right]$$

Renormalized Exact One-Loop Effective Action(2)

- **Scalar QED effective action**

$$L^{(1)}_{scalar}(E) = -i\frac{qE}{4p}\int\frac{d^2\mathrm{k}_\perp}{(2p)^2}\int_0^\infty ds\,\frac{e^{(p^*+1/2)s}}{s}\left[\frac{1}{\sinh(s/2)}-\frac{2}{s}+\frac{s}{12}\right]$$



- **Contour integral and gamma function regularization**

Renormalized Exact One-Loop Effective Action(3)

- **Scalar QED: Renormalized Effective action/volume and time for a constant E-field**

$$L^{(1)}_{scalar}(E) = -\frac{(qE)^2}{16p^2} P\int_0^\infty ds \frac{e^{-m^2 s/qE}}{s^2}\left[\frac{1}{\sin s} - \frac{1}{s} - \frac{s}{6}\right]$$

- **Spinor QED: Renormalized Effective action/ volume and time for a constant E-field**

$$L^{(1)}_{spinor}(E) = -\frac{(qE)^2}{8p^2} P\int_0^\infty ds \frac{e^{-m^2 s/qE}}{s^2}\left[\cot(s) - \frac{1}{s} + \frac{s}{3}\right]$$

Renormalized Exact One-Loop Effective Action(4)

- **Scalar QED: Schwinger Pair Production in a constant E-field**

$$2\,\mathrm{Im}(L^{(1)}_{scalar}) = \frac{(qE)^2}{8p^3}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\exp\left[-\frac{pm^2 n}{qE}\right]$$

$$= \frac{qE}{2p}\int\frac{dk_\perp^2}{(2p)^2}\ln\left(1+N_{\mathrm{k}}\right), \qquad \left(N_{\mathrm{k}} = e^{-\frac{p\,(m^2+\mathrm{k}_\perp^2)}{qE}}\right)$$

- **Spinor QED: Schwinger Pair Production**

$$2\,\mathrm{Im}(L^{(1)}_{spinor}) = \frac{(qE)^2}{4p^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left[-\frac{pm^2 n}{qE}\right]$$

$$= -\frac{qE}{p}\int\frac{dk_\perp^2}{(2p)^2}\ln\left(1-N_k\right)$$

Renormalized Exact One-Loop Effective Action(5)

- Renormalized effective action in scalar and spinor QED in a Pulsed E-Field with/without B

$$E(t) = E_0 \mathrm{sech}^2\left(\frac{t}{t}\right)$$

- For details, SPK, H.K. Lee, Y. Yoon, PRD 78 ('08)
- Renormalized effective action in a spatially localized E-field (SPK, in preparation)

$$E(z) = E_0 \mathrm{sech}^2\left(\frac{z}{L}\right)$$

Conclusion

- The renormalized effective action of scalar and spinor QED.
- The in-and out-state formalism for the exact renormalized effective action in constant E-B fields and localized electric fields.
- Schwinger pair production or vacuum persistence at zero or finite temperature. [SPK, H.K. Lee, Y. Yoon, PRD79 ('09)]

Possible Application to QCD

- The abelian sectors of QCD may have the vacuum structure similar to QED.
- Quark and gluon pair production in color electric fields in connection with quark-gluon plasma.
- Vacuum polarization of QCD in analogy with QED.