QCD thermodynamics with colour-sextet quarks

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Introduction

Extensions of Standard Model with a strongly coupled (composite) Higgs sector.

Technicolor – QCD-like theories in which the techni-pions give mass to the W and Z through the Higgs mechanism. Must be extended to also give mass to the fermions.

Walking Technicolor – Technicolor theories where the fermion content is such that the running coupling evolves very slowly – 'walks'. This avoids some of the problems that plague other Technicolor models.

The evolution of the coupling in QCD-like theories is described by $\beta(g)$

$$\beta(g) = \mu \frac{\partial g}{\partial \mu} = -\beta_0 \frac{g^3}{(4\pi)^2} - \beta_1 \frac{g^5}{(4\pi)^4} \ldots$$

For small N_f, $\beta_0, \beta_1 > 0$ – asymptotic freedom, confinement, chiral symmetry breaking (χSB).

For large N_f, $\beta_0 > 0$ and asymptotic freedom and confinement are lost.

Between these, there is a range of N_f for which $\beta_0 > 0$, $\beta_1 < 0$. Still asymptotically free. If these 2 terms describe the physics, there is a second fixed point which is infrared attractive. The massless theory is conformally invariant.

If a chiral condensate forms before this IR fixed point is reached, this reduces the screening of the colour 'charge', and the coupling increases again – confinement. Near the would-be IR fixed point the coupling walks.

Use Lattice Gauge simulations to study candidate walking theories.

We are studying a particular candidate theory, QCD with 2 colour-sextet quarks. Staggered quarks.

For QCD with sextet quarks, asymptotic freedom is lost at $N_f = 3\frac{3}{10}$. β_1 changes sign at $N_f = 1\frac{28}{125}$.

Rainbow graph approximation predicts that a condensate forms for $N_f < 2\frac{163}{325}$. However, preliminary lattice results of DeGrand, Shamir and Svetitsky using Wilson quarks suggest $N_f = 2$ is conformal.

We are simulating thermodynamics (also studied by DeGrand, Shamir and Svetitsky). $T = 0$ to follow.

QCD with $N_f = 2$ staggered colour-sextet quarks at finite T

Wilson plaquette (triplet) action for gauge fields.

Standard staggered action for sextet quark fields, with sextet gauge fields on the links.

Staggered quarks have the advantage over Wilson quarks of having a simple chiral order parameter.

Exact RHMC algorithm used for simulations. Allows us to tune to $N_f = 2$.

Finite T allows us to study scales associated with confinement and χSB.

Changing N_t can distinguish between finite T and bulk transitions.

We are currently simulating on $8^3 \times 4$, $12^3 \times 4$ and $12^3 \times 6$ lattices. Typical runs away from transitions are 10,000 trajectories per (β, m). Close to transitions we use 50,000-100,000 trajectories per (β, m).

Simulations and Results

$N_t = 4$

For both $8^3 \times 4$ and $12^3 \times 4$ lattices we performed simulations with $m = 0.02$, $m = 0.01$ and $m = 0.005$ to attempt to access the chiral limit. We covered the range $5.0 \leq \beta = 6/g^2 \leq 7.0$.

The results on these 2 lattice sizes are consistent, so we will present results primarily from our $12^3 \times 4$ simulations.

In contrast to what was found by DeGrand, Shamir and Svetitsky, we find well separated deconfinement and chiral-symmetry restoration transitions.

Figure 1 shows the colour-triplet Wilson Line(Polyakov Loop) and the chiral condensate($\langle\bar{\psi}\psi\rangle$) as functions of $\beta = 6/g^2$, for each of the 3 quark masses on a $12^3 \times 4$ lattice.

Figure 2 shows the 'time' evolution of the triplet Wilson Line near the deconfinement transition for $m = 0.02$.

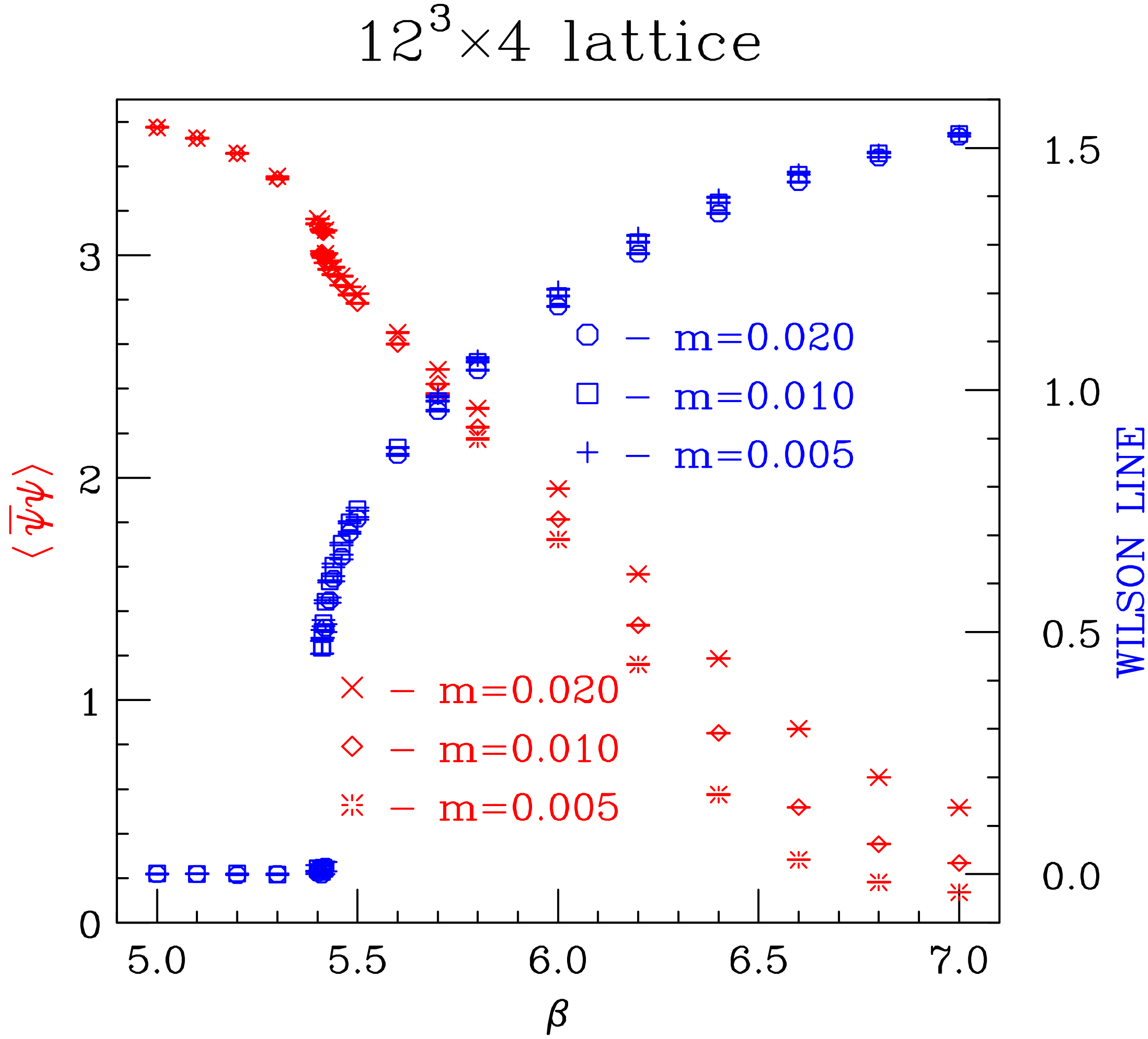

Figure 1: Wilson line and $\langle\bar{\psi}\psi\rangle$ as functions of β on a $12^3 \times 4$ lattice.

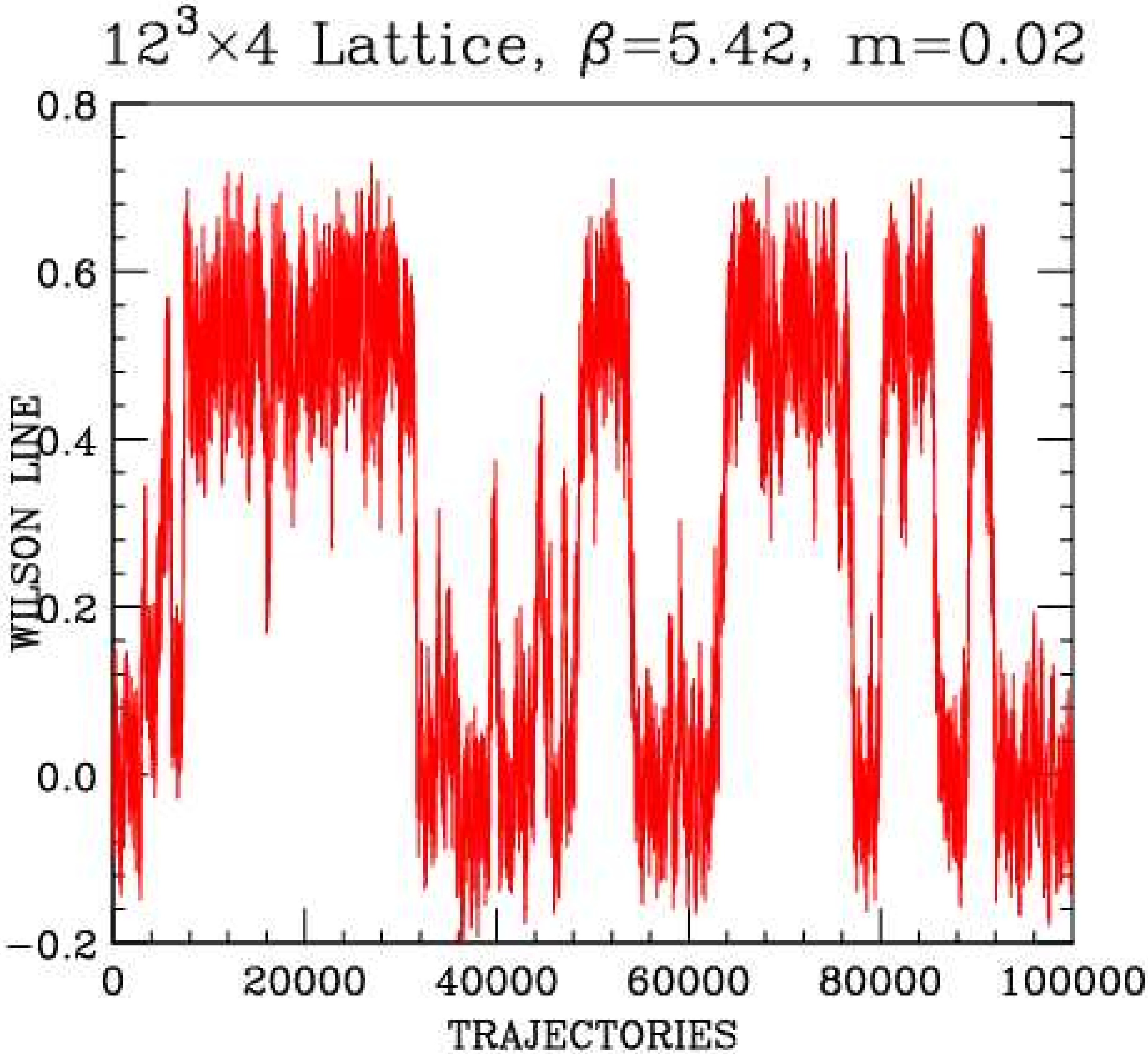

Figure 2: Evolution of the Wilson Line at $\beta = 5.42$, $m = 0.02$ on a $12^3 \times 4$ lattice.

The colour-triplet Wilson Line shows a first order(?) deconfinement transition at $\beta = 5.420(5)$ $(m = 0.02)$ and $\beta = 5.412(1)$ $(m = 0.01)$. It is close to zero below this transition and real and positive above.

The chiral symmetry restoration transition occurs at $\beta \approx 6.5$.

For $\beta < 5.9$ but significantly above the deconfinement transition we find vestiges of the broken Z_3 symmetry with a 3-state signal in the Wilson Line. However, those states with the Wilson Line oriented in the direction of one of the non-trivial cube roots of unity, while long lived, are only metastable, decaying into the state with a real positive Wilson Line. (See figures 3,4.)

At $\beta \approx 5.9$ these metastable states undergo a transition to a state with a real negative Polyakov Loop.

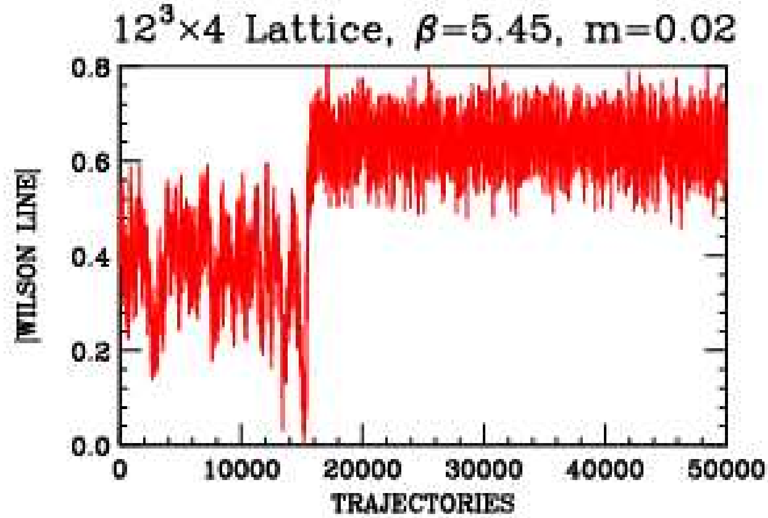

Figure 3: Time evolution of the magnitude of the triplet Wilson Line at $\beta = 5.45$ on a $12^3 \times 4$ lattice

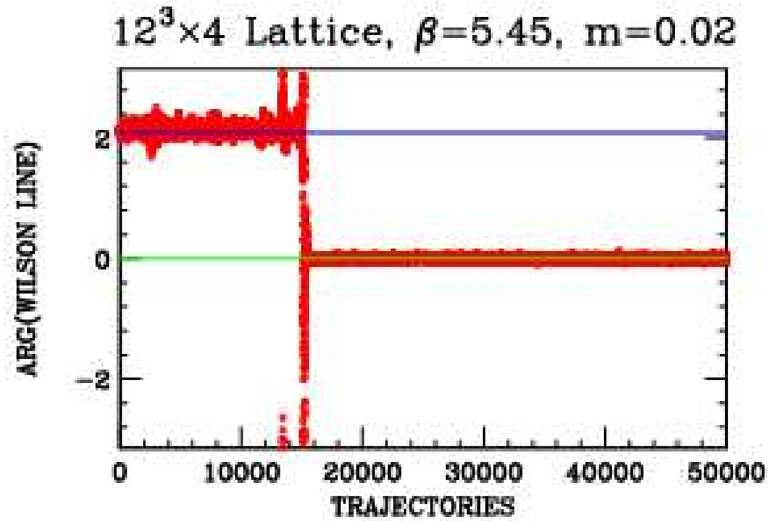

Figure 4: Time evolution of the argument of the triplet Wilson Line at $\beta = 5.45$ on a $12^3 \times 4$ lattice

$N_t = 6$

At $N_t = 6$, the remnant Z_3 symmetry is again manifest above the deconfinement transition and there is a clear 3-state signal. Tunnelings occur between these 3 states. There are tunnelings in all directions, indicating that all 3 states are stable, in contrast to $N_t = 4$.

Figure 5 shows the 3-state signal above the deconfinement transition on a $12^3 \times 6$ lattice. This graph represents 100,000 trajectories.

We separate the contribution of the state with a real Wilson Line from that from states where the Wilson line is oriented in the direction of either complex cube root of unity.

Figures 6,7 show the Wilson Lines (Polyakov Loops) and the chiral condensates. The first graph is for the states with real positive Wilson Lines. The second is for those with complex (or negative) Wilson Lines.

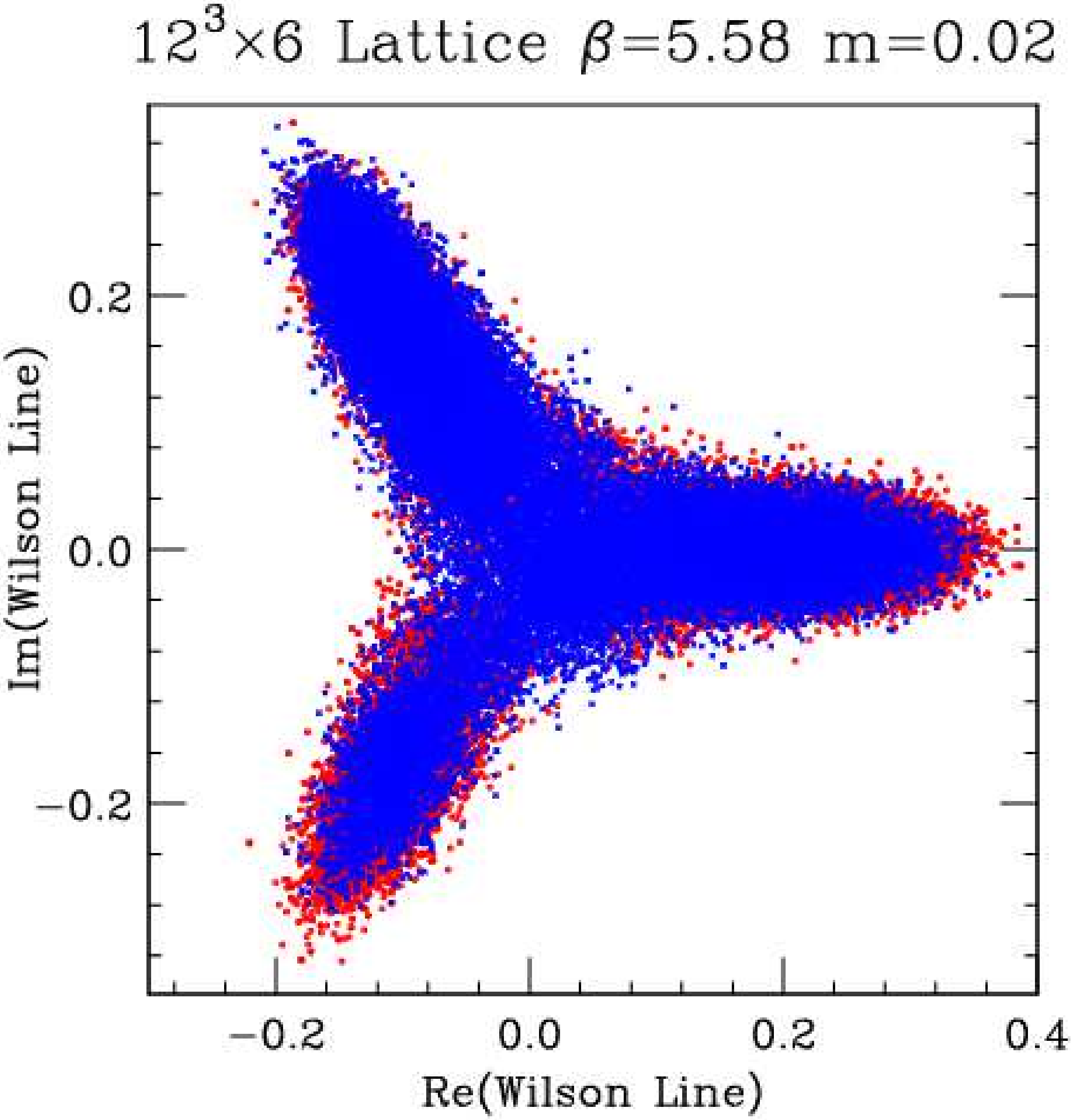

Figure 5: Scatterplot of triplet Wilson Lines at $\beta = 5.58$ in the deconfined regime on a $12^3 \times 6$ lattice.

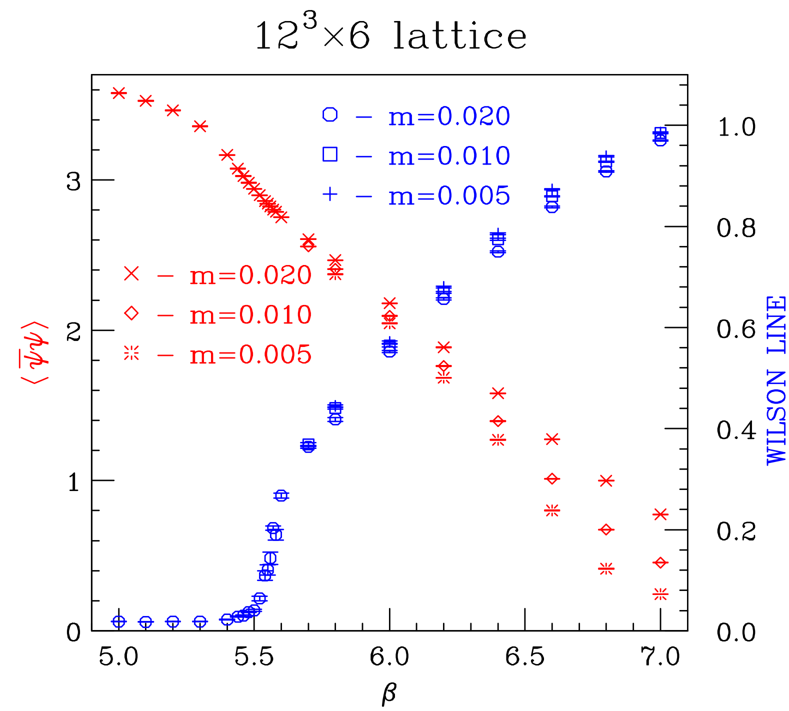

Figure 6: Triplet Wilson Line and $\langle\bar{\psi}\psi\rangle$ as functions of β for the state with a real positive Wilson Line.

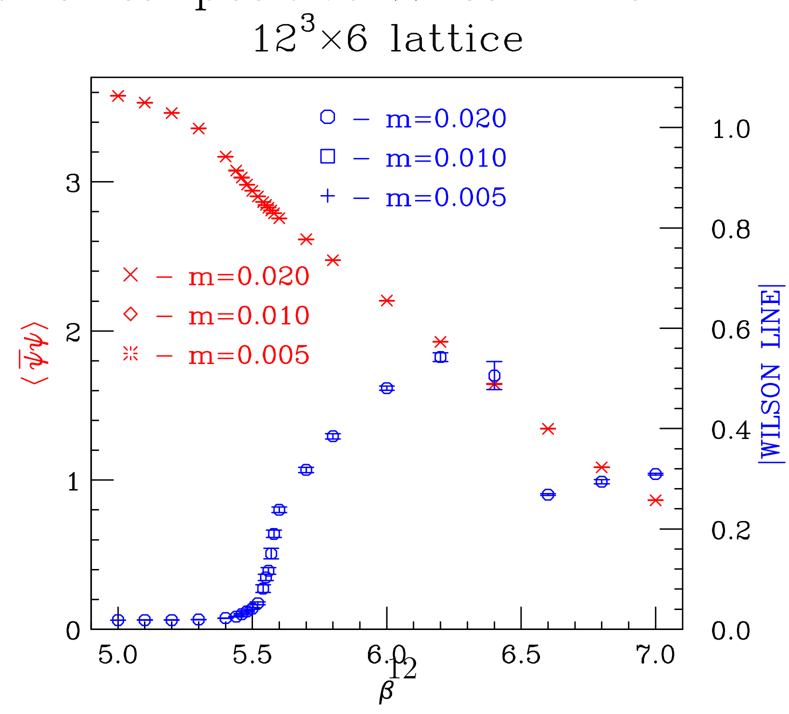

Figure 7: Triplet Wilson Line and $\langle\bar{\psi}\psi\rangle$ as functions of β for states with complex or negative Wilson Line.

The deconfinement transition is at $\beta = 5.54(4)$. The increase in β from $N_t = 4$ is expected for a finite temperature transition in an asymptotically free field theory.

The chiral-symmetry restoration transition is at $\beta \approx 6.8$. Increase over $N_t = 4$ suggests finite T transition with asymptotic freedom.

The states with complex Wilson Lines show a transition to states with real negative Wilson Lines at $\beta \approx 6.5$. (See figures 8, 9.) Large increase in β over that for $N_t = 4$ suggests that this transition could be a lattice artifact, or else could merge with the chiral transition for larger N_t.

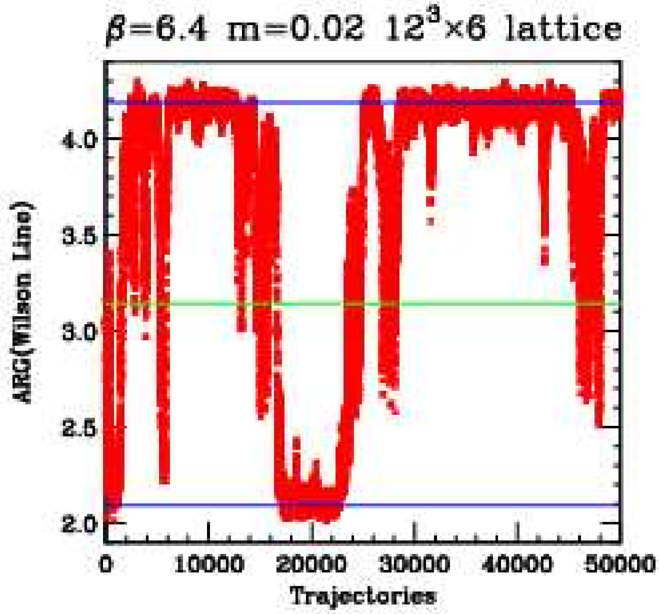

Figure 8: Time history of argument of Wilson line at $\beta = 6.4$, just below transition on $12^3 \times 6$ lattice.

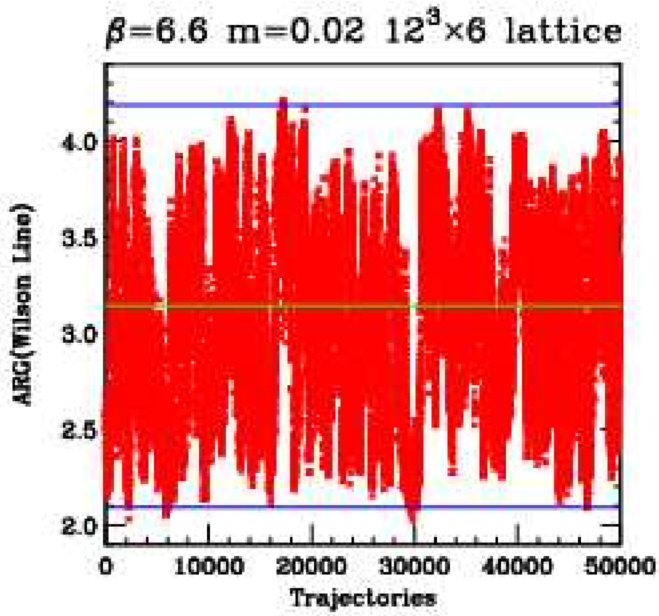

Figure 9: Time history of argument of Wilson line at $\beta = 6.6$, just above transition on $12^3 \times 6$ lattice.

Discussion and conclusions

- We are studying the thermodynamics of Lattice QCD with 2 flavours of staggered colour-sextet quarks. We find well separated deconfinement and chiral-symmetry restoration transitions. This contrasts with the case of fundamental quarks, where these 2 transitions are coincident, but is similar to the case of adjoint quarks where again these 2 transitions are separate.

- The increase in the βs for both transitions from $N_t = 4$ to $N_t = 6$ is consistent with their being finite temperature transitions for an asymptotically free theory (rather than bulk transitions).

- If there is an IR fixed point, we have yet to observe it. Our results suggest a Walking rather than a conformal behaviour – very preliminary.

- Why is this phase diagram so different from that for Wilson quarks (DeGrand, Shamir & Svetitsky)? Is it because there is an infrared fixed point, and we are on the strong-coupling side of it? Are our quark masses too large to see the chiral limit?

- At $N_t = 4$, $\beta_d(m = 0.02) = 5.420(5)$, $\beta_d(m = 0.01) = 5.412(1)$; deconfinement appears first order. $\beta_\chi \approx 6.5$.
- At $N_t = 6$, $\beta_d(m = 0.02) = 5.54(4)$. $\beta_\chi \approx 6.8$.
- For the deconfined phase there is a 3-state signal, the remnant of now-broken Z_3 symmetry. For $N_t = 4$ the states with complex Polyakov Loops appear metastable. For $N_t = 6$ all 3 states appear stable. Breaking of Z_3 symmetry is seen in the magnitudes of the Polyakov Loops for the real versus complex states.
- The existence of the 3-state signal is presumably because the formation of the chiral condensates at short distances suppresses the contribution of the quarks at the confinement scale, so the deconfined but chirally broken phase is more similar to the quenched theory. (Should compare to deconfined phase with heavy colour-triplet quarks).
- Between the deconfinement and chiral transitions, we find a third transition where the Wilson Lines in the directions of the 2 non-trivial roots of unity change to real negative Wilson Lines. This transition occurs for $\beta \approx 5.9$ ($N_t = 4$) and $\beta \approx 6.5$ ($N_t = 6$). This rapid increase suggests that the transition is a lattice artifact.

- If this third transition is real, the fact that the magnitude of the negative Polyakov Loop is roughly one third of that for the positive Polyakov Loop, suggests that this transition is associated with colour symmetry breaking $SU(3) \longrightarrow SU(2) \times U(1)$.
- 2-state, charge conjugation violating phase just below this third transition.
- Need larger lattices – $18^3 \times 6$ and $16^3 \times 8$. Smaller quark masses.
- To understand this theory more fully, we need to study its zero temperature behaviour, measuring its spectrum, string tension, potential, f_π... Measurement of the running of the coupling constant for weak coupling is needed.
- Apply lattice simulations to other candidate theories. Staggered fermions or overlap/domain-wall fermions for good chiral properties.
- These simulations were performed on the Cray XT4 (Franklin) at NERSC, and on the Linux Cluster (Abe) at NCSA under an LRAC grant.

Appendix

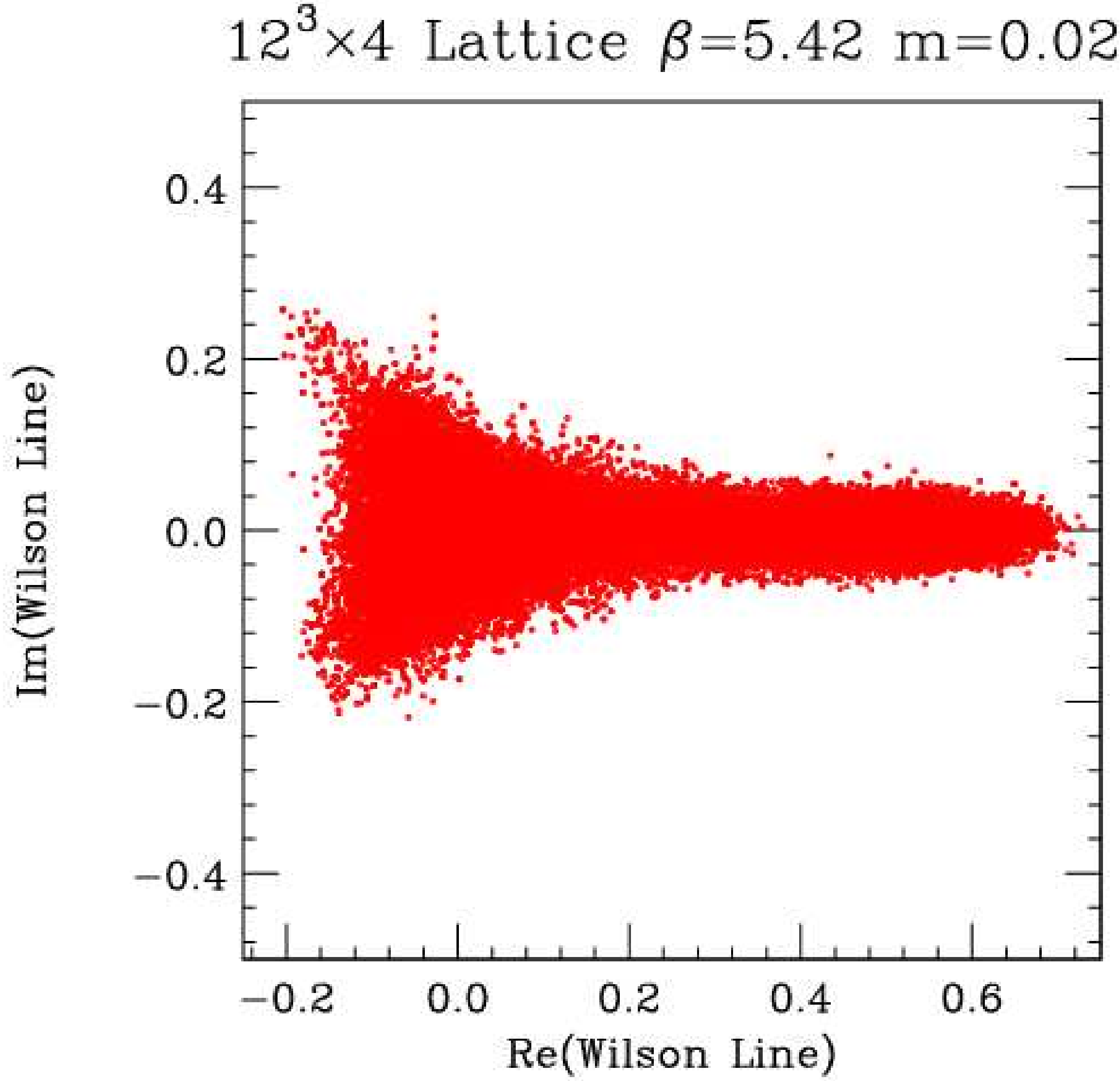

Figure 10: Scatterplot of values of Wilson Line (Polyakov Loop) for each trajectory at $\beta = 5.42$ and $m = 0.02$ on a $12^3 \times 4$ lattice

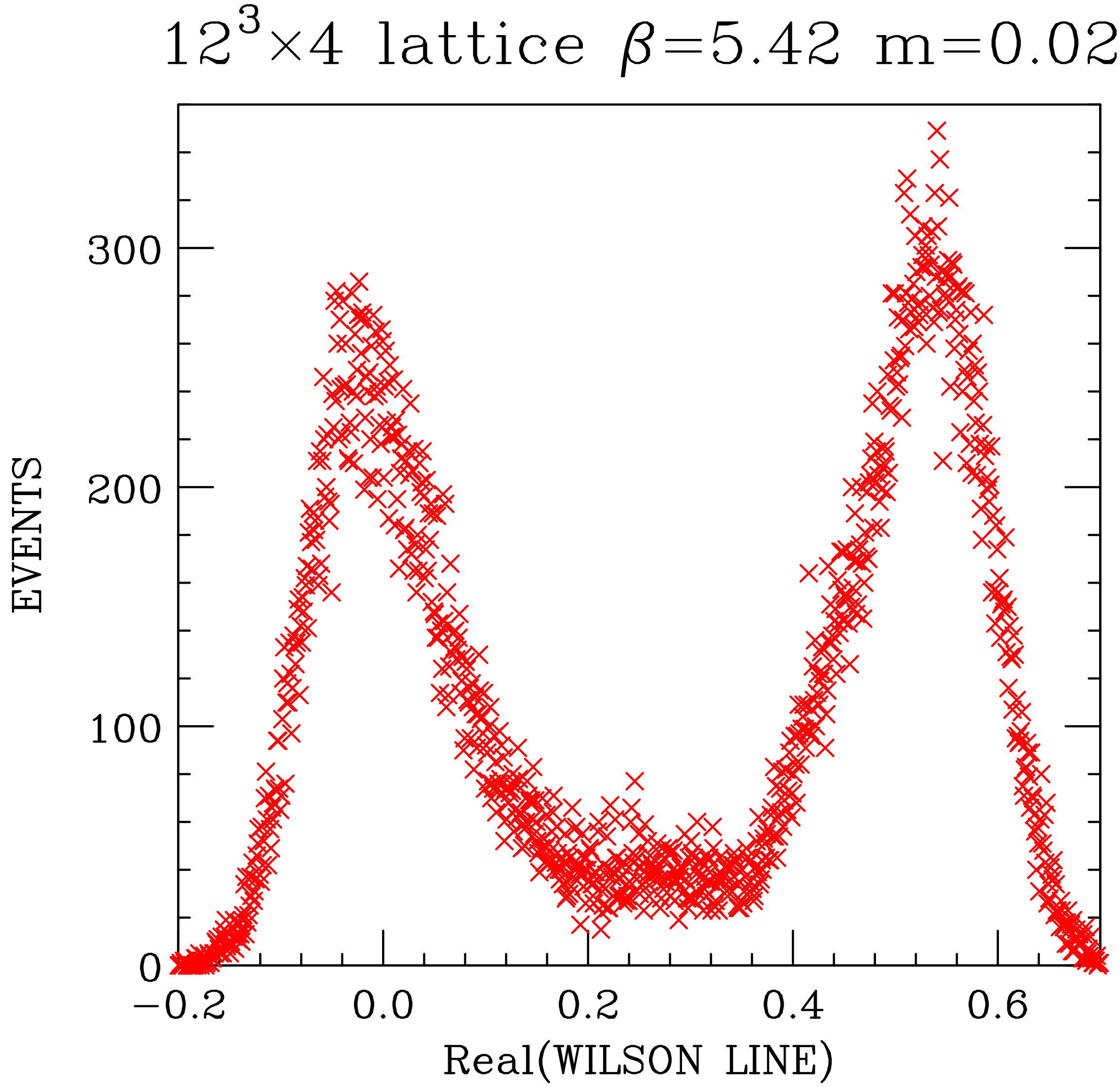

Figure 11: Histogram of the real part of the Wilson Line (Polyakov Loop) for each trajectory at $\beta = 5.42$ and $m = 0.02$ on a $12^3 \times 4$ lattice

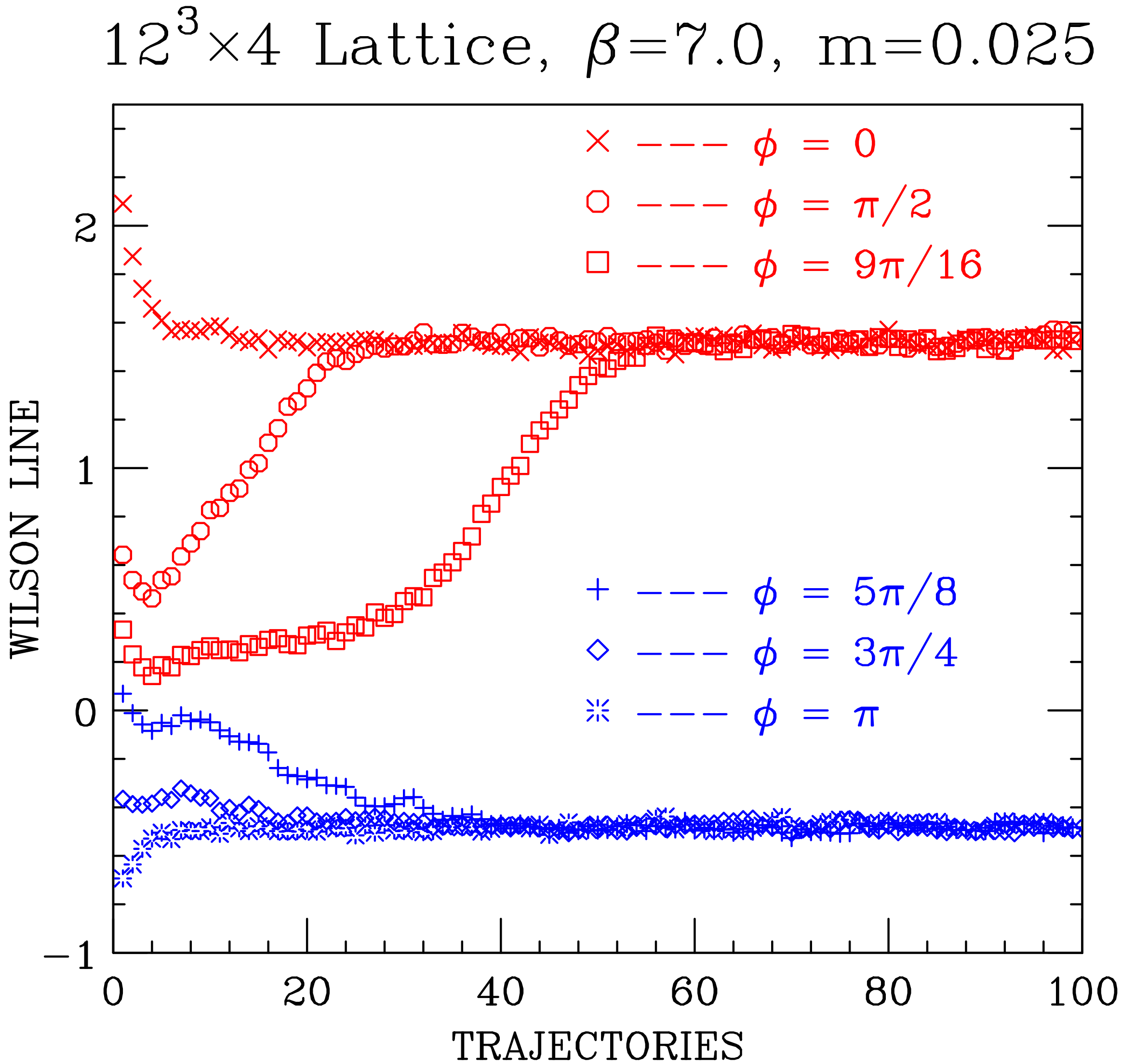

Figure 12: Evolution of Wilson Line from ordered start. Start with all $U_\mu = 1$, except for timelike links on a single timeslice where $U_4 = \mathrm{diag}\left(1, e^{i\phi}, e^{-i\phi}\right)$.