[APCTP Focus Program] Recent Developments in Neutrino ...(In memory of Prof. Benjamin W. Lee), APCTP, Pohang, 23 June 2009

The Horava Gravity : The Theory of 97 % ?

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Based on arXiv:0905.4480v3, and work to appear

Excuse

- **There are several points that I don’t understand, still !**
- **But, what I can say is what I can do !**

0. Outline

1. Horava gravity and its IR modification

2. FRW cosmology in IR modified Horava gravity

3. Comparison with observational data

4. Open problems

1. Motivation of IR modification of Horava gravity

Horava gravity ~ Einstein gravity (with a deformation parameter λ)

+ non-covariant deformations with higher spatial derivatives (up to 6 orders)

+ "detailed balance" in the coefficients (5 constant parameters: $\kappa, \lambda, \nu, \mu, \Lambda_W$)

Cf. In Einstein gravity, we have 3 fundamental constants $c,\ G,\ \Lambda$

Detailed balance condition:

- We need (foliation preserving Diff invariant) potential term having 6^{th} order spatial derivatives at most (power-counting renormalizable with z=3) :

$$S_V = \int dt\, d^D x \sqrt{g} N\, V[g_{ij}]$$

- There are large numbers of possible terms, which are invariant by themselves, like …

- …, like

$$\nabla_k R_{ij} \nabla^k R^{ij}, \qquad \nabla_k R_{ij} \nabla^i R^{jk}, \qquad R \Delta R, \qquad R^{ij} \Delta R_{ij};$$

$$R^3, \qquad R^i_j R^j_k R^k_i, \qquad R R_{ij} R^{ij},$$

- But there are too many couplings for explicit computations, though some of them may be constrained by the stability and unitarity. We need some pragmatic way of reducing in a reliable manner.

- Horava required the potential to be of

$$S_V = \frac{\kappa^2}{8}\int dt\, d^D x \sqrt{g} N\, E^{ij} \mathcal{G}_{ijk\ell} E^{k\ell},$$

by demanding

$$\sqrt{g}E^{ij} = \frac{\delta W[g_{k\ell}]}{\delta g_{ij}}$$

for some action W, and $\mathcal{G}_{ijk\ell}$, the inverse of De Witt metric

$$G^{ijk\ell} = \frac{1}{2}\left(g^{ik}g^{j\ell} + g^{i\ell}g^{jk}\right) - \lambda g^{ij} g^{k\ell}$$

- There is a similar method in non-equilibrium critical phenomena.

- W is 3-dimensional Euclidean action.
- First, we may consider Einstein-Hilbert action,

$$W = \frac{1}{\kappa_W^2} \int d^D\mathrm{x} \sqrt{g}(R - 2\Lambda_W).$$

then, this gives 4'th-derivative order potential

$$S_V = \frac{\kappa^2}{8\kappa_W^4} \int dt\, d^D\mathrm{x} \sqrt{g} N \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \mathcal{G}_{ijk\ell} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right).$$

- So, this is not enough to get 6'th order !!

- **In 3-dim, we also have a peculiar, 3'rd-derivative order action, called (gravitational) Chern-Simons action.**

$$W = \frac{1}{w^2}\int_\Sigma \omega_3(\Gamma).$$

$$\omega_3(\Gamma) = \mathrm{Tr}\left(\Gamma \wedge d\Gamma + \frac{2}{3}\Gamma \wedge \Gamma \wedge \Gamma\right) \equiv \varepsilon^{ijk}\left(\Gamma^m_{i\ell}\partial_j\Gamma^\ell_{km} + \frac{2}{3}\Gamma^n_{i\ell}\Gamma^\ell_{jm}\Gamma^m_{kn}\right) d^3\mathrm{x}$$

- **This produce the potential**

$$-\frac{\kappa^2}{2w^4}C_{ij}C^{ij}$$

with the Cotton tensor $C^{ij} = \varepsilon^{ik\ell}\nabla_k\left(R^j_\ell - \frac{1}{4}R\delta^j_\ell\right)$

- Then, in total, he got the 6’th order

$$S = \int dt\, d^3x \sqrt{g}\, N \left\{ \frac{2}{\kappa^2} K_{ij} G^{ijk\ell} K_{k\ell} - \frac{\kappa^2}{2} \left[\frac{1}{w^2} C^{ij} - \frac{\mu}{2} \left(R^{ij} - \frac{1}{2} R g^{ij} + \Lambda_W g^{ij} \right) \right] \times \mathcal{G}_{ijk\ell} \left[\frac{1}{w^2} C^{k\ell} - \frac{\mu}{2} \left(R^{k\ell} - \frac{1}{2} R g^{k\ell} + \Lambda_W g^{k\ell} \right) \right] \right\}.$$

from

$$W = \frac{1}{w^2} \int \omega_3(\Gamma) + \mu \int d^3x \sqrt{g}(R - 2\Lambda_W).$$

So, we have 5 constant parameters, which seems to be minimum, from the detailed balancing.

- Some improved UV behaviors are expected, i.e., renormalizability

⇒ Predictable Quantum Gravity !!(?)

- But, it seems that the detailed balance condition is too strong to get general spacetimes with an arbitrary cosmological constant.

- For example, there is no Minkowski , i.e., vanishing c.c. vacuum solution ! (Lu, Mei, Pope)

- A "soft" breaking of the detailed balance is given by the action :

$$S_g = \int dt d^3x \sqrt{g} N \left[\frac{2}{\kappa^2}\left(K_{ij}K^{ij} - \lambda K^2\right) - \frac{\kappa^2}{2\nu^4} C_{ij}C^{ij} + \frac{\kappa^2 \mu}{2\nu^2} \epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)\ell}{}_k \right.$$
$$\left. - \frac{\kappa^2\mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{4\lambda-1}{4}(R^{(3)})^2 - \Lambda_W R^{(3)} + 3\Lambda_W^2\right) + \frac{\kappa^2\mu^2\omega}{8(3\lambda-1)} R^{(3)} \right]$$

IR modification term

- It is found that there does exit the black hole which converges to the usual Schwarzschild solution in Minkowski limit, i.e., $\Lambda_W \to 0$ for $\lambda = 1$ (s.t. Einstein-Hilbert in IR) (Kehagias, Stetsos) .

- **Black hole solution for $\Lambda_W \to 0$ limit ($\lambda = 1$):**

$$ds^2 = -N(r)^2 c^2 dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right)$$

$$N^2 = f = 1 + \omega r^2 - \sqrt{r[\omega^2 r^3 + 4\omega M]}$$

$$\approx 1 - \frac{2M}{r} + \mathcal{O}(r^{-4})$$

~ Schwarzshild Solution

: Independently of ω **!!**

$$(G = c \equiv 1)$$

General Remarks

KS considered $\omega = 8\mu^2(3\lambda - 1)/\kappa^2$ but it can be considered as an independent parameter: One more parameter than the Horava gravity with the detailed balance, i.e., we have 6 constant parameters

$$\kappa, \lambda, \nu, \mu, \Lambda_W, \omega$$

IR modification parameter

- Cosmological constant ~ Λ_W <0, i.e., AdS, for consistency ! (Horava)

- **dS , i.e., positive c.c., can be obtained by the continuation (Lu,Mei,Pope):**

$$\mu \longrightarrow i\mu, \;\; \nu^2 \longrightarrow -i\nu^2, \;\; \omega \longrightarrow -\omega$$

$$\Lambda_W > 0$$

- **Cf: KS:** $\omega = +16\mu^2/\kappa^2 \rightarrow \omega = -16\mu^2/\kappa^2$

2. FRW cosmology in IR modified Horava gravity

- Homogeneous, isotropic cosmological solution of FRW form :

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

$$k = +1, 0, -1$$

- For a perfect fluid with energy density ρ and pressure p, the IR modified Horava action gives …

Friedman equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6(3\lambda-1)}\left[\rho \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{-k^2}{R_0^4 a^4} + \frac{2k(\Lambda_W - \omega)}{R_0^2 a^2} - \Lambda_W^2\right)\right]$$

$$\frac{\ddot{a}}{a} = \frac{\kappa^2}{6(3\lambda-1)}\left[-\frac{1}{2}(\rho+3p) \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)}\left(\frac{k^2}{R_0^4 a^4} - \Lambda_W^2\right)\right].$$

R_0 is the current (a=1) radius of curvature of universe

[Upper (Lower) sign for AdS (dS)]

Remarks

- The $1/a^4$ term, which is the contribution from the higher-derivative terms in Horava gravity, exists only for, $k \neq 0$ i.e., non-flat universe and becomes dominant for small a : The cosmological solutions for GR are recovered at large scales.
- There is no contribution from the soft IR modification to the second Friedman Eq.: Identical to that of Lu,Mei,Pope.

What is the implication of the Horava gravity to our universe ?

Is there any critical test of the theory if we are live in Horava gravity ?

It seems to be "yes" ! How ?

How to test the theory ?

- If we are live in the Horava gravity (with some IR modifications), the additional contributions to the Friedman Eq. from the higher-(spatial) derivative terms may not be distinguishable from the dark energy with (including C.C. term)

$$\rho_{\mathrm{D.E.}} = \pm \frac{3\kappa^2\mu^2}{8(3\lambda-1)} \left(\frac{-k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - \Lambda_W^2 \right),$$

$$p_{\mathrm{D.E.}} = \mp \frac{\kappa^2\mu^2}{8(3\lambda-1)} \left(\frac{k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - 3\Lambda_W^2 \right),$$

- **We would see the Friedman Eq. as**

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\tilde{G}}{3c^2}(\rho_{\rm matter} + \rho_{\rm D.E.}) - \frac{c^2 k}{a^2},$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi\tilde{G}}{3c^2}[(\rho_{\rm matter} + \rho_{\rm D.E.}) + 3(p_{\rm matter} + p_{\rm D.E.})],$$

where

$$c^2 \equiv \mp\frac{\kappa^4\mu^2\Lambda_W}{8(3\lambda-1)^2}, \quad \tilde{G} = \frac{\kappa^2 c^2}{16\pi(3\lambda-1)} \quad \Lambda = \frac{3}{2}\Lambda_W c^2;$$

- **The Eq. of state parameter is given by**

$$w_{\rm D.E.} = \frac{p_{\rm D.E.}}{\rho_{\rm D.E.}} = \left(\frac{k^2 - 2k\bar{\omega}a^2 - 3\bar{\Lambda}_W^2 a^4}{3k^2 + 6k\bar{\omega}a^2 + 3\bar{\Lambda}_W^2 a^4} \right)$$

$$\bar{\omega} \equiv \omega R_0^2, \;\; \bar{\Lambda}_W = \Lambda_W R_0^2$$

- **And it depends on the constant parameters** k, ω, Λ_W **...**

Remarks

- c^2 is non-negative always !
- The Newton's constant G can be negative, i.e., anti-gravity, for $\lambda < 1/3$:

➡ $\lambda_c = 1/3$ is the upper bound for the consistency with our universe, i.e., no anti-gravity: Physical bound

Remarks Cont'd

- **The definition of speed of light seems to have some ambiguity: One might consider include ω term in c^2 ,**

$$c^2 \equiv \mp \frac{\kappa^4 \mu^2 (\Lambda_W - \omega)}{8(3\lambda - 1)^2}$$

??

, rather than including in $\rho_{\mathrm{D.E.}}$ as

$$\rho_{\mathrm{D.E.}} = \pm \frac{3\kappa^2 \mu^2}{8(3\lambda - 1)} \left(\frac{-k^2}{R_0^4 a^4} - \frac{2k\omega}{R_0^2 a^2} - \Lambda_W^2 \right)$$

- **But c^2 can be negative when $|\omega| > |\Lambda_W|$.**

- **Actually there are infinitely many definitions of c^2, depending on how much the ω term contributes to c^2. Here, I do not consider all these possibilities but consider only the simplest choice which can be matched with the experiments: In experiments, I need at least two parameters to fit to data and I have two parameters Λ_W, ω in my choice also.**

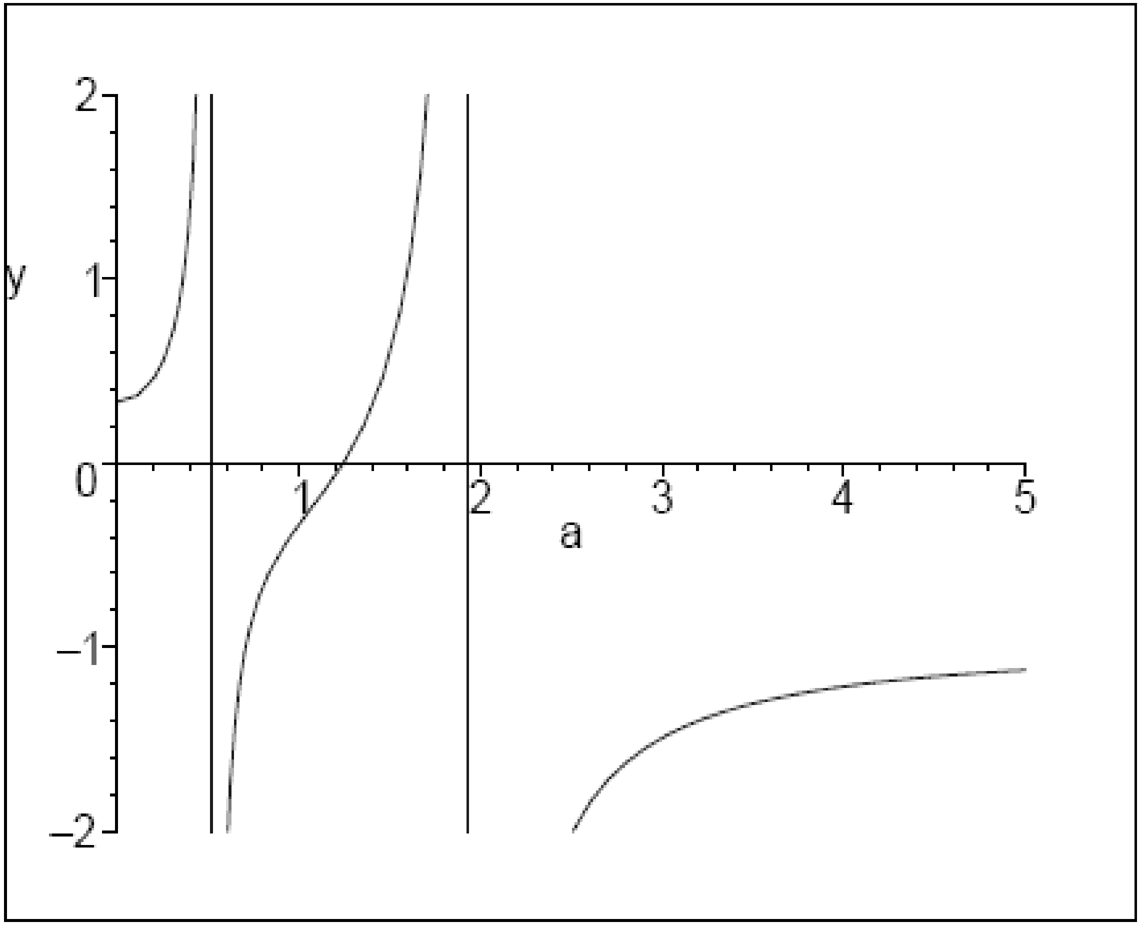


FIG. 4: Plot of equation of state parameter $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $\omega^2 > \Lambda_W^2$, $k\omega < 0$. There are two infinite discontinuities of $w_{\rm D.E.}$ at $\tilde{a}^{\pm} = \sqrt{-k\omega \pm |k|\sqrt{\omega^2 - \Lambda_W^2}}/|\Lambda_W|R_0$ where $\rho_{\rm D.E.}$ vanishes. Here, I considered $|\omega|R_0^2 = 2, |\Lambda_W|R_0^2 = 1$ case ($\omega R_0^2 = -2, k = +1$ or $\omega R_0^2 = +2, k = -1$).

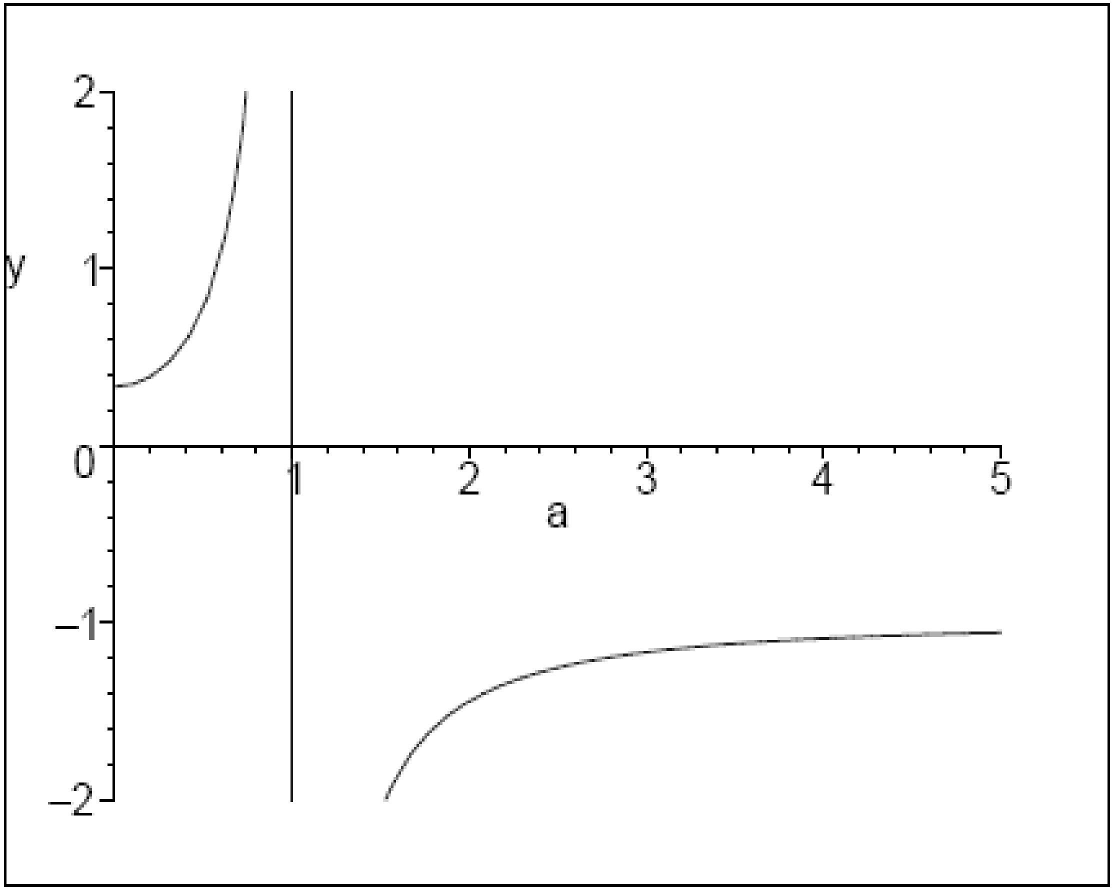


FIG. 5: Plot of equation of state parameter $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $\omega^2 = \Lambda_W^2,\ k\omega < 0$. The two points of infinite discontinuities $\tilde{a}^{\pm}$ in Fig.4 merge as $|\omega|$ approaches to $|\Lambda_W|$ and they meet at $\tilde{a}^{\pm} = \sqrt{|k|}/|\Lambda_W|R_0$ when $\omega^2 = \Lambda_W^2$. In this plot, I considered $|\omega|R_0^2 = |\Lambda_W|R_0^2 = 1$ ($\omega R_0^2 = -1, k = +1$ or $\omega R_0^2 = +1, k = -1$).

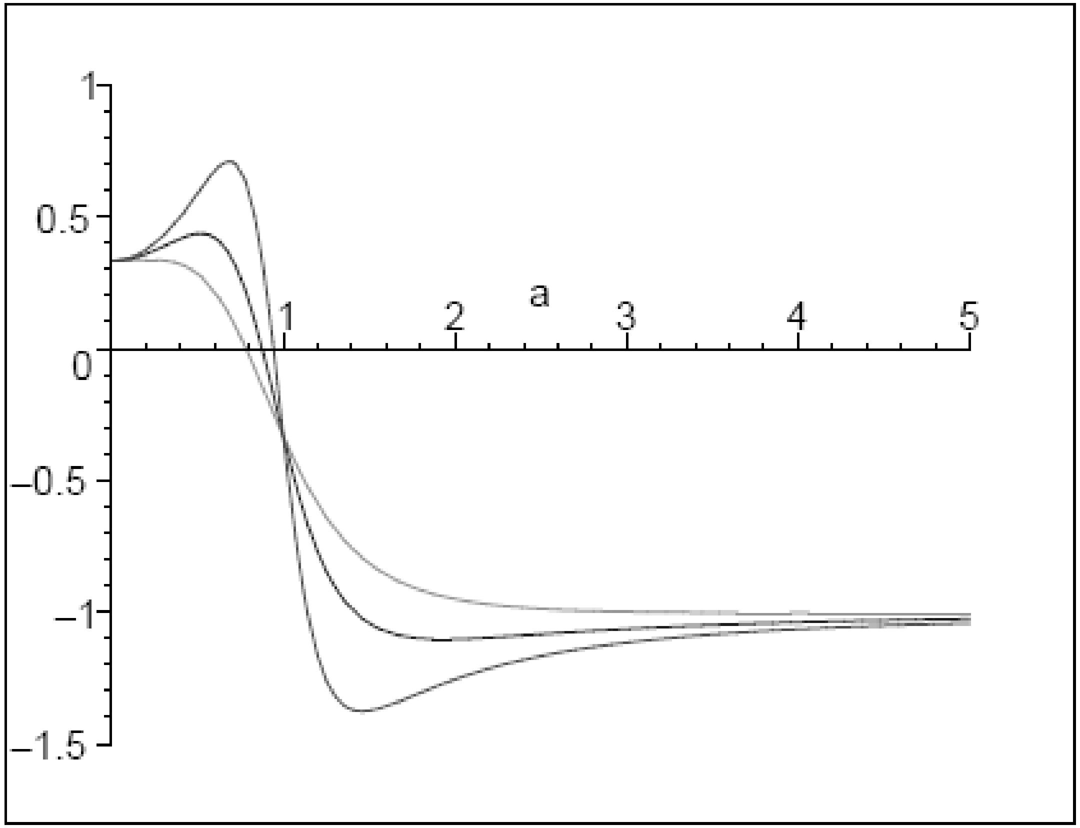


FIG. 6: Plots of equation of state parameters $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $\omega^2 < \Lambda_W^2$, $k\omega < 0$ ($\omega R_0^2 = +1/1.3, +1/2, +1/10$, $k = -1$ or $\omega R_0^2 = -1/1.3, -1/2, -1/10$, $k = +1$ with $\Lambda_W R_0^2 = 1$ (top to bottom in the left region)). When $|\omega|$ is not far from $|\Lambda_W|$, there is a region where $w_{\rm D.E.}$ is fluctuating beyond the UV and IR limits and this can be understood as a smooth deformation of the plot of Fig.5. When $|\omega|$ is small enough, $w_{\rm D.E.}$ is monotonically decreasing from $1/3$ in the UV limit to -1 in the IR limit.

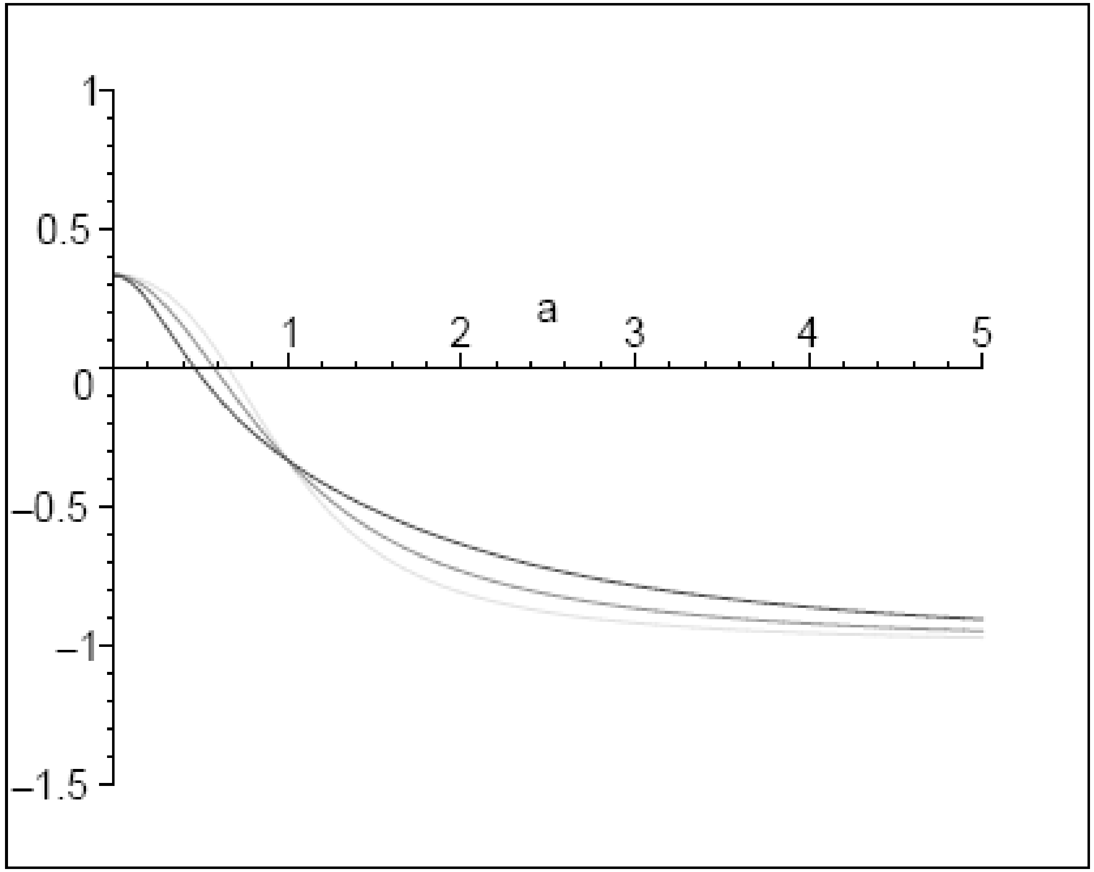


FIG. 7: Plots of equation of state parameters $w_{\rm D.E.}$ vs. scale factor $a(t)$ for $k\omega > 0$ ($\omega R_0^2 = +2, +1, +1/2$, $k = +1$ or $\omega R_0^2 = -2, -1, -1/2$, $k = -1$ with $|\Lambda_W| R_0^2 = 1$ (top to bottom in the left region). In this case $w_{\rm D.E.}$ is "always" monotonically decreasing from $1/3$ in the UV limit to -1 in the IR limit.

3a. Comparison with observational data I

(1)Deceleration to Acceleration transition

Astrophys. J. **607**, 665 (2004)

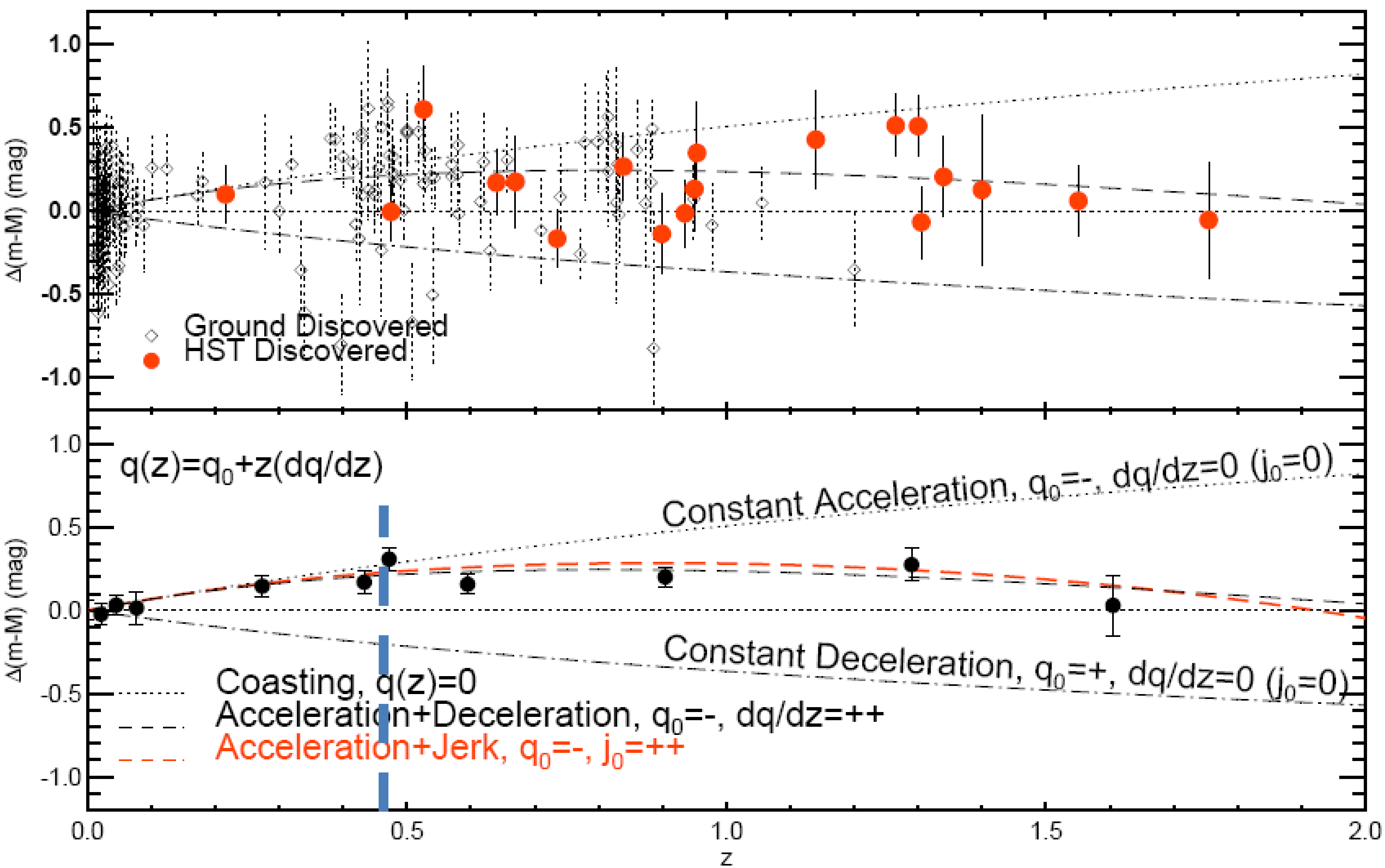
Type Ia Supernova Discoveries at $z > 1$ From the *Hubble Space Telescope:* Evidence for Past Deceleration and Constraints on Dark Energy Evolution[1]

To Appear in the Astrophysical Journal, June 2004

Adam G. Riess[2], Louis-Gregory Strolger[2], John Tonry[3], Stefano Casertano[2], Henry C. Ferguson[2], Bahram Mobasher[2], Peter Challis[4], Alexei V. Filippenko[5], Saurabh Jha[5], Weidong Li[5], Ryan Chornock[5], Robert P. Kirshner[4], Bruno Leibundgut[6], Mark Dickinson[2], Mario Livio[2], Mauro Giavalisco[2], Charles C. Steidel[7], Narciso Benitez[8] and Zlatan Tsvetanov[8]

ABSTRACT

We have discovered 16 Type Ia supernovae (SNe Ia) with the *Hubble Space Telescope (HST)* and have used them to provide the first conclusive evidence for cosmic deceleration that preceded the current epoch of cosmic acceleration.



Ground Discovered
HST Discovered
Δ(m-M) (mag)
q(z)=q0+z(dq/dz)
Constant Acceleration, q0=-, dq/dz=0 (j0=0)
Constant Deceleration, q0=+, dq/dz=0 (j0=0)
Coasting, q(z)=0
Acceleration+Deceleration, q0=-, dq/dz=++
Acceleration+Jerk, q0=-, j0=++
Δ(m-M) (mag)
z

Y. Gong, astro-ph/0405446: $z_{\rm T} = 0.30.$

Class. Quant. Grav. **22**, 2121 (2005)

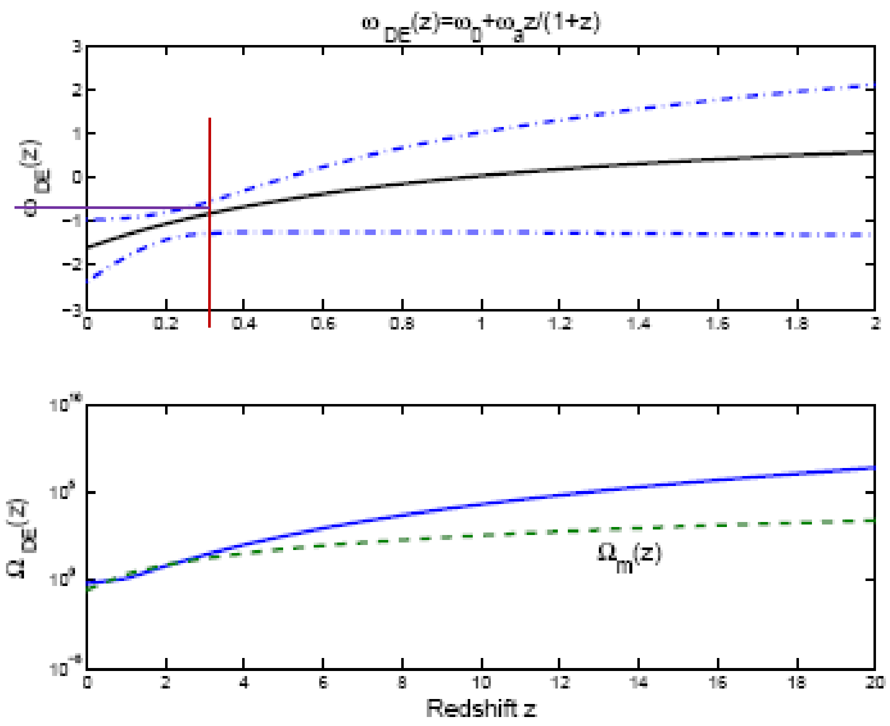


Figure 5. The best fit to the 157 gold sample SNe Ia with the prior $\Omega_{\rm m0} = 0.3 \pm 0.04$. The upper panel shows $\omega_{\rm DE}(z)$, the dotted dash lines are the 1σ regions. The lower panel shows $\Omega_{\rm m}(z)$ and $\Omega_{\rm DE}(z)$

- Actually, in our Horava gravity (the second Friedman Eq.), there is the transition point from deceleration to acceleration phase, neglecting matter contributions, at

$$a_T = \sqrt{|k|/|\Lambda_W|}$$

- If I use $a_T \sim 1/1.03 \approx 0.9709$ or $z_T \sim 0.30$ ($z = 1/a - 1$), I get $|\Lambda_W| \sim (1.03)^2 R_0^{-2} \approx 1.0609 R_0^{-2}$ for the non-flat universe with $|k| = 1$.

Remarks

- **At the transition point, the theory predicts $w_{\mathrm{D.E.}} = -1/3$, independently of the parameters k, ω, Λ_W !**

(2) Non-flatness Ω_k : $\Omega_{\text{matter}} + \Omega_{\text{D.E.}} + \Omega_k = 1$

ApJ, in press, January 5, 2007

Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Observations: Implications for Cosmology

D. N. Spergel[1,2], R. Bean[1,3], O. Doré[1,4], M. R. Nolta [4,5], C. L. Bennett[6,7], J. Dunkley[1,5], G. Hinshaw[6], N. Jarosik [5], E. Komatsu [1,8], L. Page[5], H. V. Peiris [1,9,10], L. Verde [1,11], M. Halpern [12], R. S. Hill[6,15], A. Kogut [6], M. Limon [6], S. S. Meyer [9], N. Odegard [6,15], G. S. Tucker [13], J. L. Weiland[6,15], E. Wollack [6], E. L. Wright [14]

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ABSTRACT

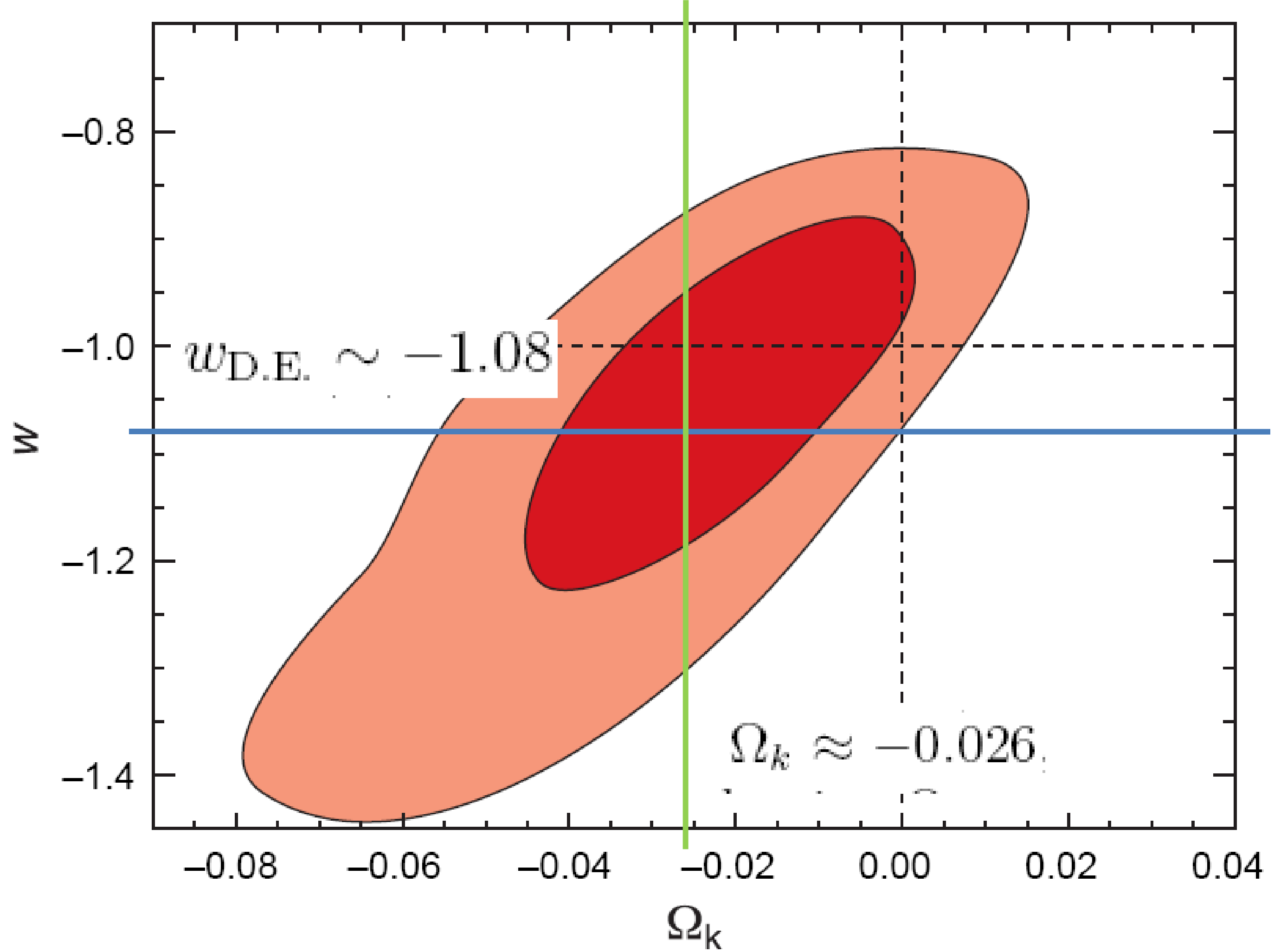


Fig. 17.— Constraints on a non-flat universe with quintessence-like dark energy with constant w (Model M10 in Table 3). The contours show the 2-d marginalized contours for w and Ω_k based on the the CMB+2dFGRS+SDSS+supernova data sets. This figure shows that with the full combination of data sets, there are already strong limits on w without the need to assume a flat universe prior. The marginalized best fit values for the equation of state and curvature are $w = -1.08 \pm 0.12$ and $\Omega_k = -0.026^{+0.016}_{-0.015}$ at the 68% confidence level.

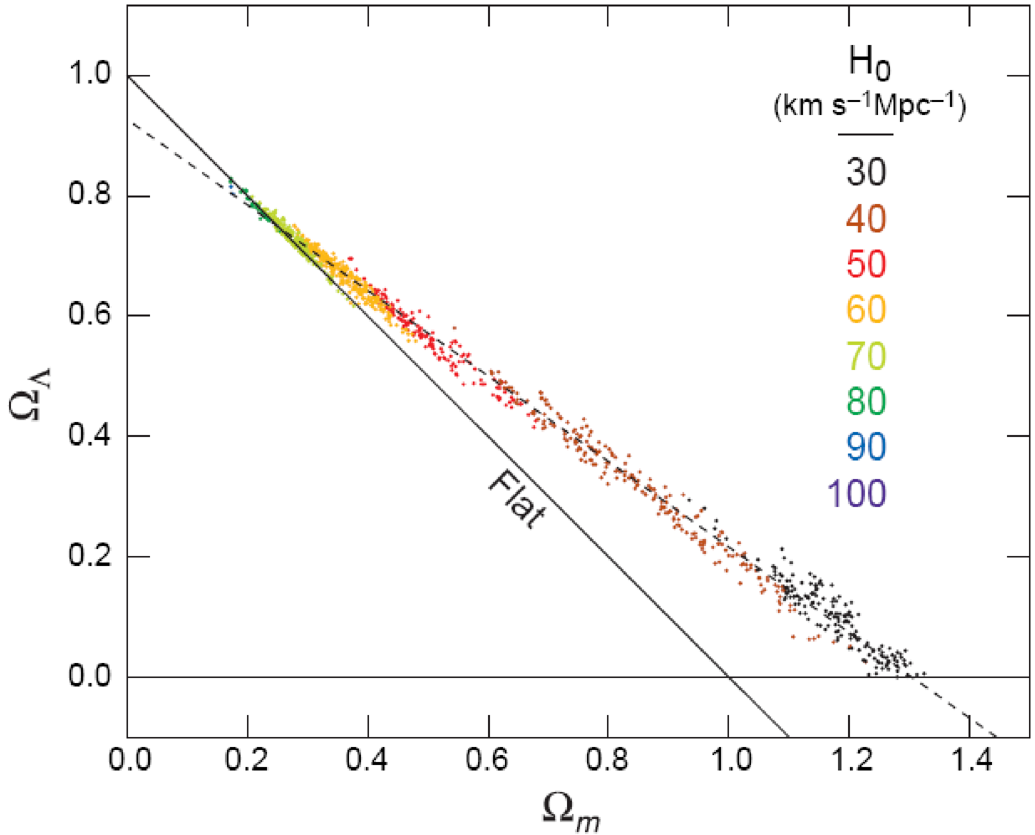


Fig. 20.— Range of non-flat cosmological models consistent with the WMAP data only. The models in the figure are all power-law CDM models with dark energy and dark matter, but without the constraint that $\Omega_m + \Omega_\Lambda = 1$ (model M10 in Table 3). The different colors correspond to values of the Hubble constant as indicated in the figure. While models with $\Omega_\Lambda = 0$ are not disfavored by the WMAP data only ($\Delta\chi^2_{eff} = 0$; Model M4 in Table 3), the combination of WMAP data plus measurements of the Hubble constant strongly constrain the geometry and composition of the universe within the framework of these models. The dashed line shows an approximation to the degeneracy track: $\Omega_K = -0.3040 + 0.4067\Omega_\Lambda$. Note that for these open universe models, we assume a flat prior on Ω_Λ.

- If I use $\Omega_k \sim -0.026$ in the current epoch $(a=1)$ and

$$\Omega_k = \mu^2 k|\Lambda_W|L_{\rm P}^2/2a^2H^2R_0^2M_{\rm P}^2$$

for the Hubble parameter $H \equiv \dot{a}/a$,

$M_{\rm P}/L_{\rm P} \equiv 2(3\lambda-1)/\kappa^2$ and $k=-1$, I get

$$\mu \sim 0.2214 H_0 R M_{\rm P}/L_{\rm P}$$

- If I use $w_{\rm D.E.} \sim -1.08$ with $k=-1$

$|\Lambda_W| \sim (1.03)^2 R_0^{-2} \approx 1.0609 R_0^{-2}$, I get

$$\omega \sim 1.0067 R_0^{-2}$$

To summarize,

- For $k = -1$, I get the constant parameters with

$$|\Lambda_W| \sim (1.03)^2 R_0^{-2} \approx 1.0609 R_0^{-2}$$

$$\omega \sim 1.0067 R_0^{-2}$$

$$\mu \sim 0.2214 H_0 R M_\mathrm{P} / L_\mathrm{P}$$

which predicts the evolution of $w_{\mathrm{D.E.}}$ as one of the curves of $\omega < |\Lambda_W|$.

- **If I use** $R_0 \sim 6.2017\ c/H_0$ **from** $\Omega_k = kc^2/H_0^2 R_0^2 \sim -0.026$ **and** $H_0 \sim 70\text{km s}^{-1}\text{Mpc}^{-1}$ **, I get**

$$\Lambda_W \sim 1.5018 \times 10^{-9}\text{Mpc}^{-2},$$

$$\omega \sim 1.4251 \times 10^{-9}\text{Mpc}^{-2}$$

$$\mu \sim 5.6636 \times 10^{35}\text{kg s}^{-1}$$

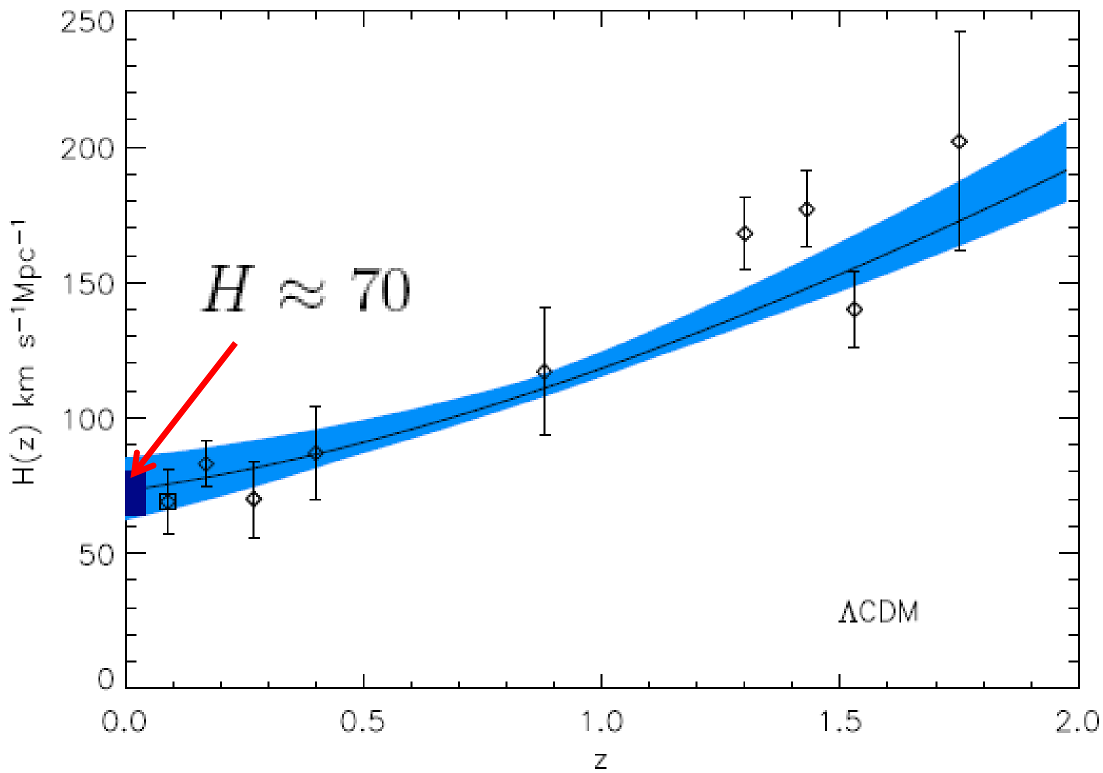
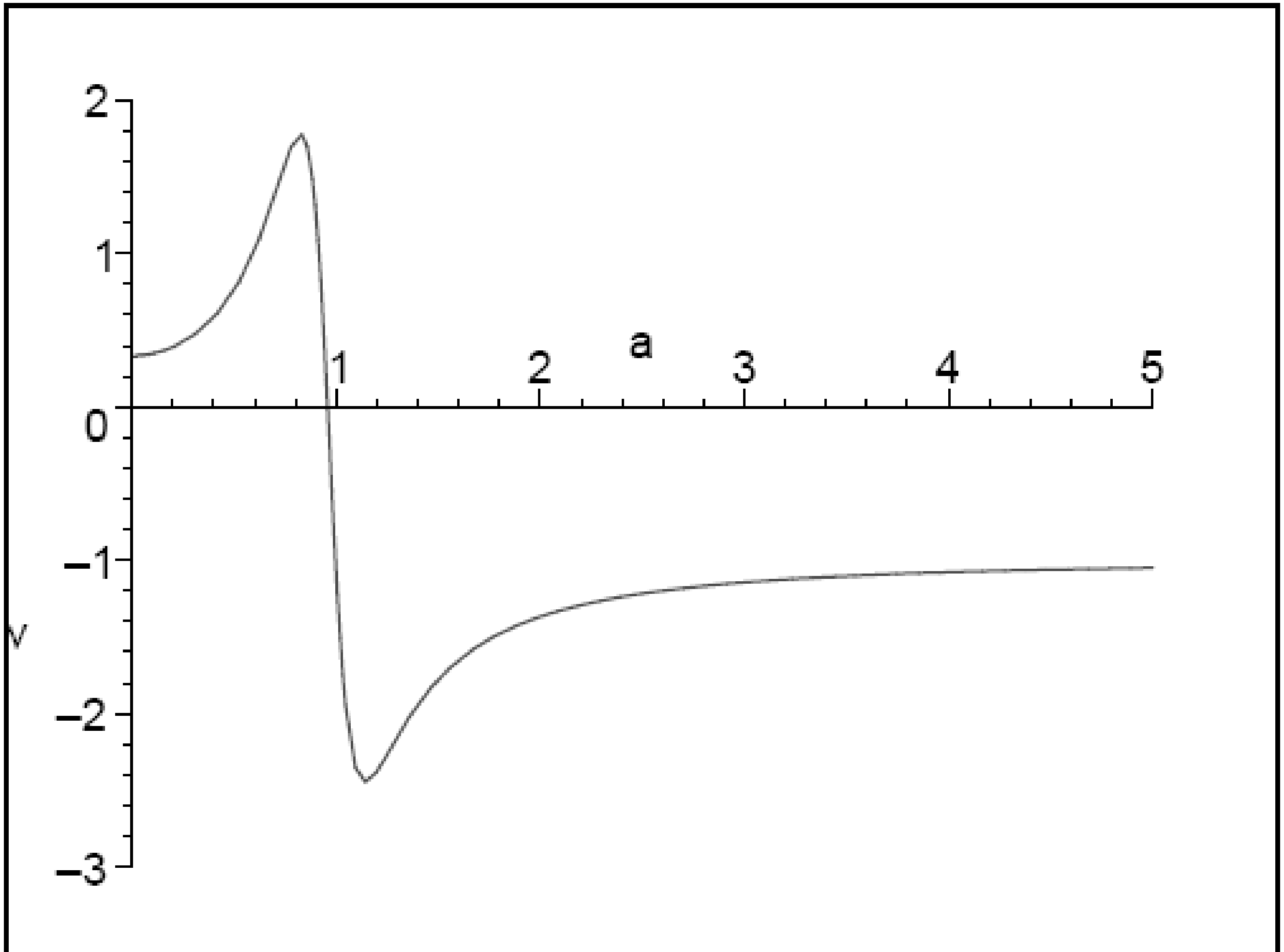
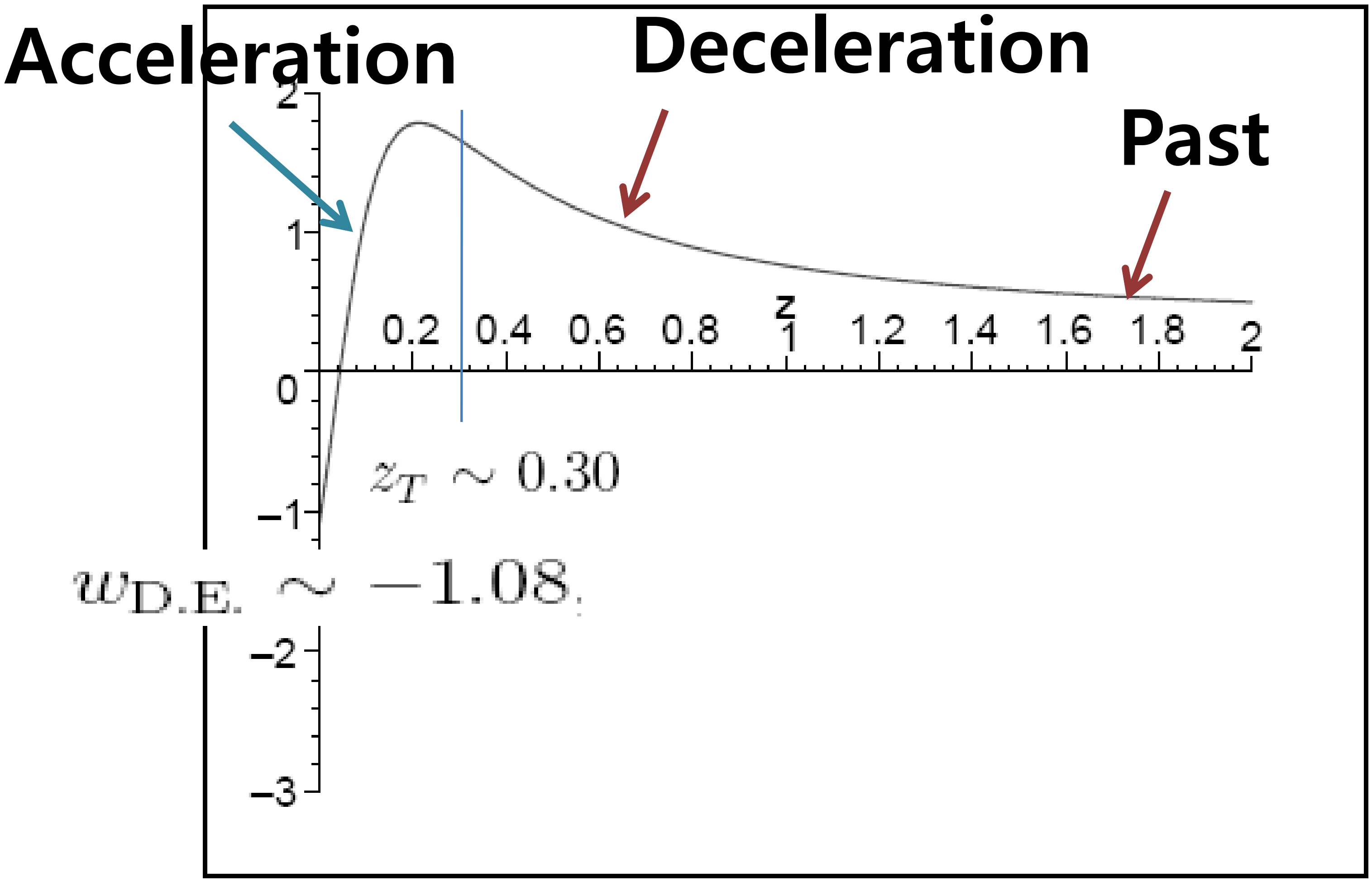


Fig. 4.— The ΛCDM model fit to the WMAP data predicts the Hubble parameter redshift relation. The blue band shows the 68% confidence interval for the Hubble parameter, H. The dark blue rectangle shows the HST key project estimate for H_0 and its uncertainties (Freedman et al. 2001). The other points are from measurements of the differential ages of galaxies, based on fits of synthetic stellar population models to galaxy spectroscopy. The squares show values from Jimenez et al. (2003) analyses of SDSS galaxies. The diamonds show values from Simon et al. (2005) analysis of a high redshift sample of red galaxies.

So, our theory predicts



- **Or, in the astronomer's convention**



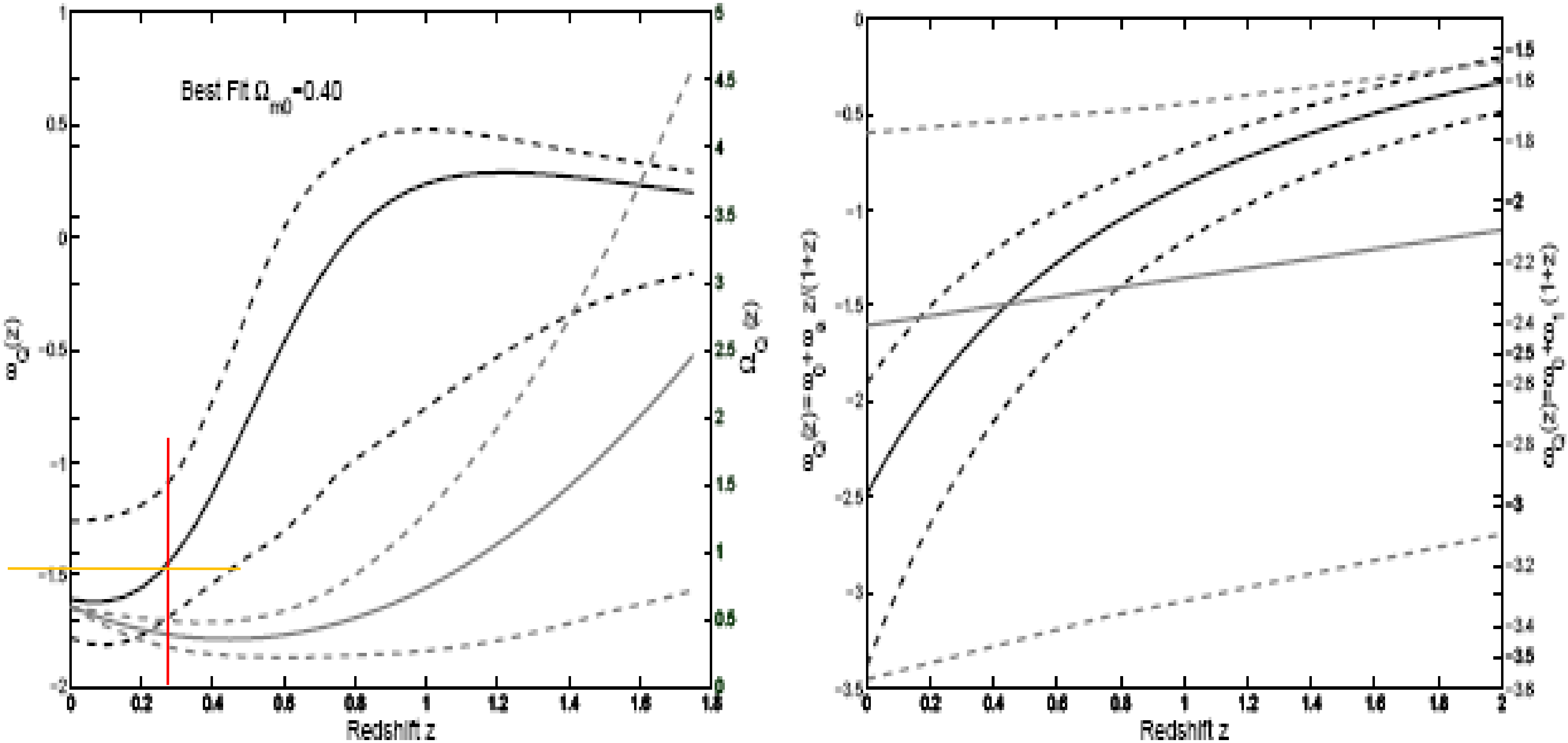


Fig. 4. The best supernova and WMAP data fits to the polynomial model and linear model. The left panel shows Riess gold sample and WMAP data fits to the two parameter polynomial model, the light black lines are for Ω_Q and the dark balck lines for ω_Q, the solid lines are from the best fit. The right panel shows Riess gold sample and WMAP data fits to the two parameter linear model of ω_Q, the solid lines are from the best fit, the light black lines are for the linear model and the dark black lines are for the stable model. The dashed lines define the 1σ boundaries.

Y. Gong, astro-ph/0401207

Model independent analysis of dark energy: Supernova fitting result

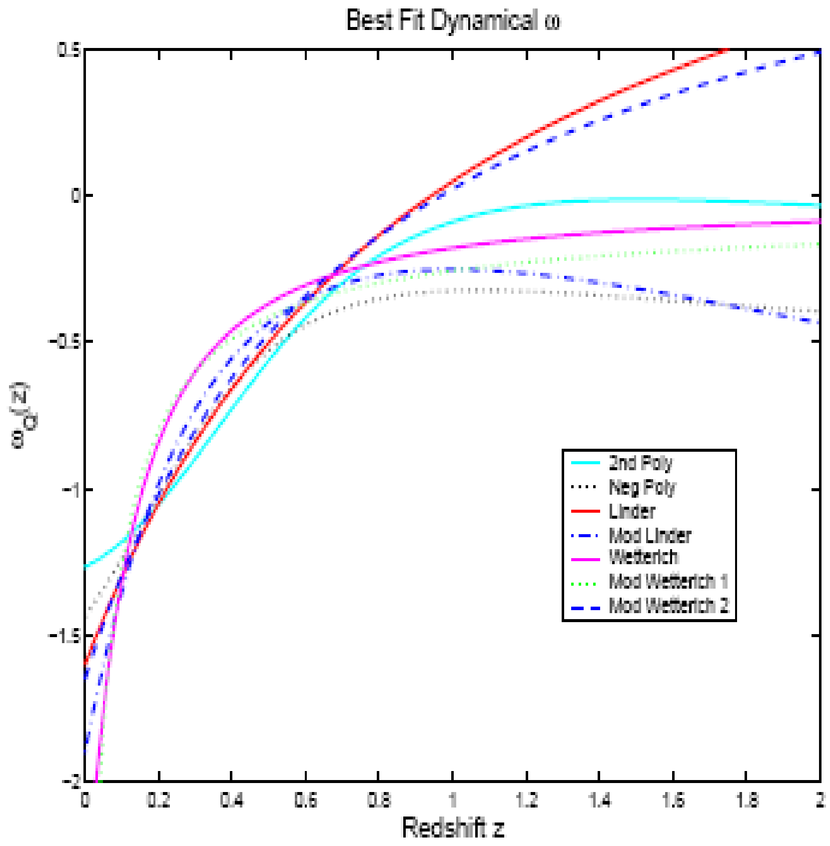


Figure 10. The evolution of $\omega_{\rm DE}$ for different parametrizations.

Y. Gong, astro-ph/0405446

3a. Comparison with observational data II: Latest data, independently of matters.

- Previously, I neglected matters, which occupy about 30 % of our current universe, to get Λ_W, ω, so this would be good within about 70 % accuracy, only !
- Is there any more improved analysis to achieve better accuracy, without neglecting matters ? Yes ! ...

- To this end, let me consider the series expansion of $w_{\rm D.E.}$: near the current epoch (a=1):

$$w_{\rm D.E.} = w_0 + w_a(1-a) + w_b(1-a)^2 + \cdots$$

$$w_0 = \frac{k^2 - 2k\bar{\omega} - 3\bar{\Lambda}_W^2}{3(k^2 + 2k\bar{\omega} + \bar{\Lambda}_W^2)}, \quad w_a = \frac{8k(\bar{\omega}k^2 + \bar{\omega}\bar{\Lambda}_W^2 + 2k\bar{\Lambda}_W^2)}{3(k^2 + 2k\bar{\omega} + \bar{\Lambda}_W^2)^2}$$

- This agrees exactly with Chevallier, Polarski, and Linder (CPL)'s parametrization !

- **By knowing w_0 and w_a from observational data, one can determine $\bar{\omega}$ $\bar{\Lambda}_W$ as**

$$\bar{\omega} = \frac{(1 - 2w_0 - 3w_0^2 - w_a)k}{(1 + 4w_0 + 3w_0^2 + w_a)},$$

$$\bar{\Lambda}_W^2 = \frac{(-1 + 9w_0^2 + 3w_a)k^2}{3(1 + 4w_0 + 3w_0^2 + w_a)}.$$

Remarks

- I do not need to know about matter contents, separately.
- Once $\bar{\omega}, \bar{\Lambda}_W$ are determined, the whole function $w_{\mathrm{D.E.}}(a)$ is completely determined !
-

Data analysis **without** assuming the flat universe

Parameters at $a=1$	Data analysis Ia [43]	Data analysis Ib [43]	Data analysis II [44]
w_0	-1.06	-1.10	-1.11
w_a	0.72	0.39	0.475
Ω_k	-0.000	-0.009	-0.0008
$\Omega_{\mathrm{D.E.}}$	0.699	0.730	0.739
Ω_{m}	0.301	0.279	0.262
H_0	65.5	67.6	72.4
$\bar{\omega}$	1.14	1.32	1.30
$\bar{\Lambda}_W$	2.10	2.44	2.29
μ	0.0000	0.0013	0.0004

$$|\bar{\omega}| < \bar{\Lambda}_W$$

TABLE I: A summary of the data sets *without* assuming the flat universe in a priori and their corresponding constant parameters, in the conventional units of H_0 (km s^{-1}Mpc^{-1}) and μ ($H_0 R_0 M_{\mathrm{P}}/L_{\mathrm{P}}$).

Data analysis Ia, Ib: CMB+BAO+SN

- K. Ichikawa, T. Takahashi [arXiv: 0710.3995v2 [astro-ph] 3 May 2008

Ia

CMB+BAO+Gold06	$\chi^2_{\rm min}$	Ω_m	h	Ω_k	w_0	w_1
No prior	158.3	0.301	0.655	0.000	-1.06	0.72
prior $h = 0.72 \pm 0.02$	161.3	0.263	0.702	0.014	-1.02	0.69
Prior $h = 0.62 \pm 0.02$	159.2	0.320	0.634	-0.007	-1.06	0.73

Ib

CMB+BAO+Davis07	$\chi^2_{\rm min}$	Ω_m	h	Ω_k	w_0	w_1
No prior	195.4	0.279	0.676	-0.009	-1.10	0.39
Prior $h = 0.72 \pm 0.02$	196.6	0.254	0.709	0.006	-1.14	0.80
Prior $h = 0.62 \pm 0.02$	197.3	0.315	0.638	-0.026	-0.93	-0.92

Table 6: The best fit values for Ω_m, h, Ω_k, w_0 and w_1 for the analysis presented in the panel (d)-(f) of Fig. 8. The minimum values of χ^2 are also shown.

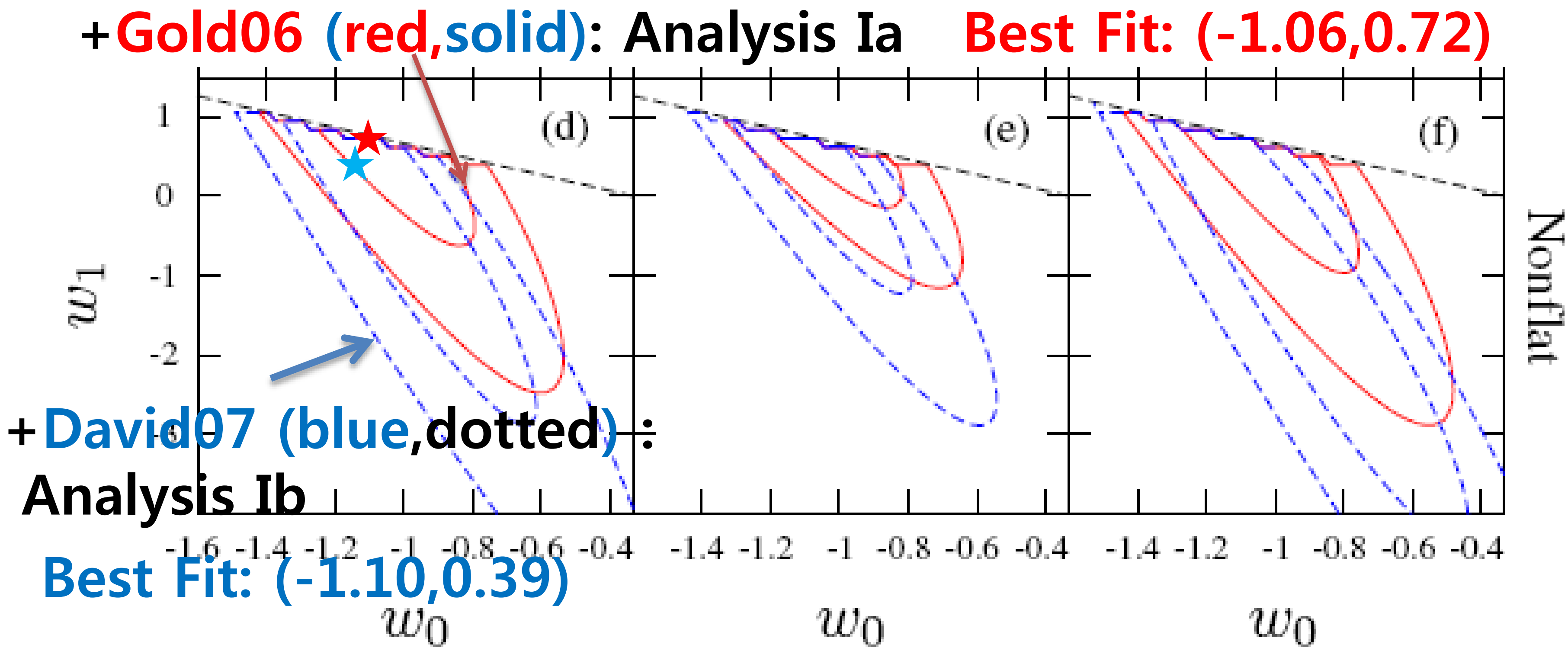


Figure 8: 1σ and 2σ constraints from CMB+BAO+SN in the w_0–w_1 plane marginalizing over Ω_m and h are shown for the cases with (a) no prior on the Hubble constant, (b) assuming a Gaussian prior on the Hubble constant $h = 0.72 \pm 0.02$ and (c) $h = 0.62 \pm 0.02$. In the panels (d)–(f), we allow a non-flat universe and marginalize over Ω_k in addition to Ω_m and h. The black dashed lines show the boundary of the prior Eq. (3). The constraints using the SN data sets from Gold06 (red solid line) and Davis07 (blue dashed line) are shown separately.

Data analysis II: CMB+BAO+SN

- J.-Q.Xia, et. al., arXiv:0807.3878v2 [astro-ph] 22 Aug 2008

TABLE I. Constraints on the dark energy EoS and some background parameters from the latest observations. Here we have shown the mean and the best fit values, which are obtained from the cases with and without the systematic uncertainties of Union compilation, respectively.

Parameter		w_0		w_1		Ω_{de}		H_0	
		with sys	w/o sys	with sys	w/o sys	with sys	w/o sys	with sys	w/o sys
ΛCDM	BestFit	-1	-1	0	0	0.735	0.741	71.0	71.6
$\Omega_k = 0$	Mean	-1	-1	0	0	0.738 ± 0.015	0.738 ± 0.014	71.4 ± 1.4	71.4 ± 1.3
WCDM	BestFit	-0.978	-0.955	0	0	0.738	0.735	71.4	70.6
$\Omega_k = 0$	Mean	-0.965 ± 0.080	-0.977 ± 0.056	0	0	0.736 ± 0.016	0.737 ± 0.014	70.8 ± 1.9	71.1 ± 1.4
RunW	BestFit	-1.09	-1.08	0.533	0.368	0.735	0.738	70.4	71.1
$\Omega_k = 0$	Mean	-0.946 ± 0.194	-0.993 ± 0.128	-0.133 ± 0.749	0.030 ± 0.582	0.734 ± 0.017	0.737 ± 0.014	70.7 ± 1.9	70.9 ± 1.5
RunW	BestFit	–	-1.11	–	0.475	–	0.739	–	72.4
$\Omega_k \neq 0$	Mean	–	-0.976 ± 0.148	–	-0.071 ± 0.848	–	0.736 ± 0.014	–	70.9 ± 1.9
RunW	BestFit	–	-1.04	–	0.290	–	0.742	–	71.3
w/o Pert.	Mean	–	-1.00 ± 0.114	–	0.103 ± 0.413	–	0.736 ± 0.012	–	70.8 ± 1.5

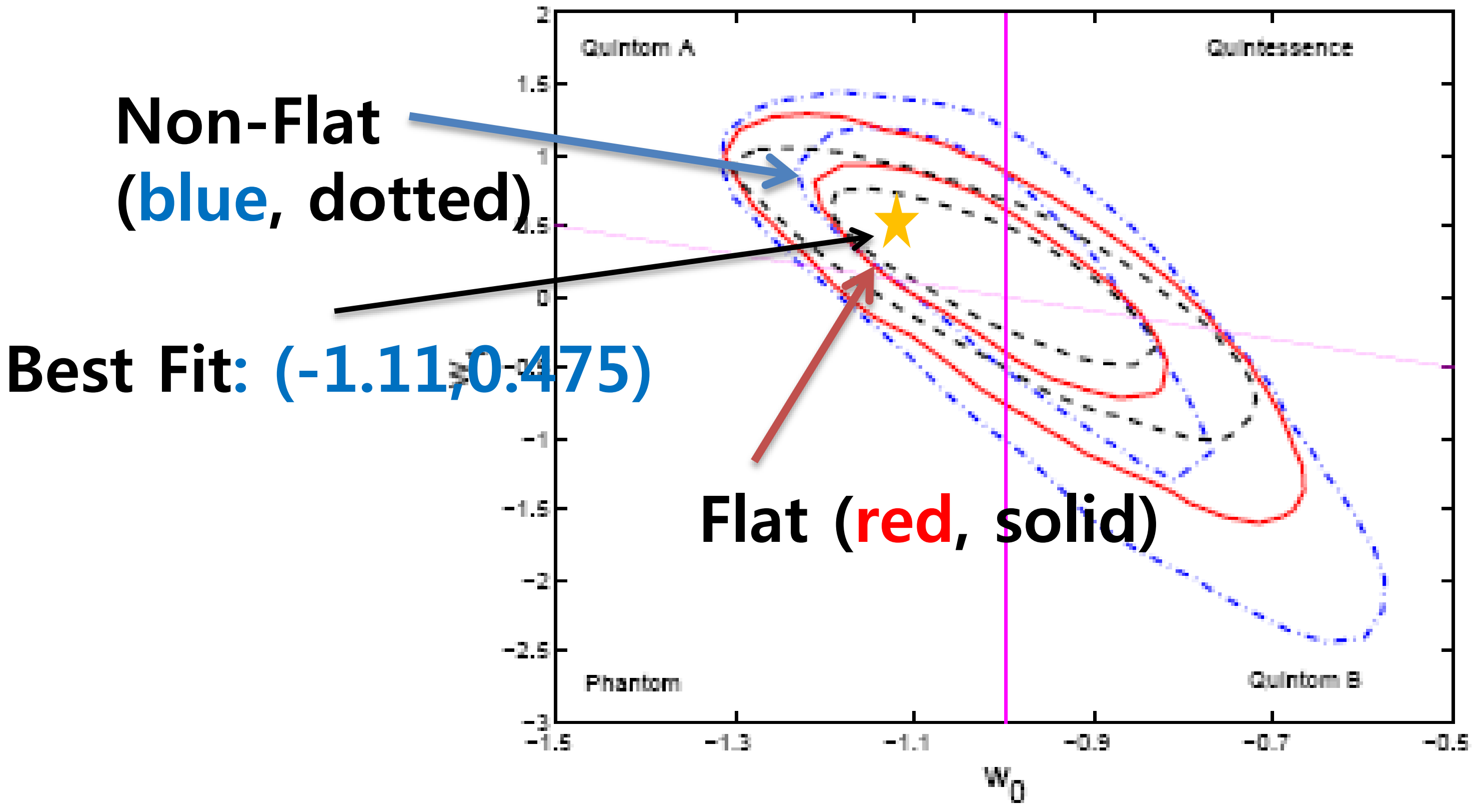


FIG. 2: Constraints on the dark energy EoS parameters w_0 and w_1 from the current observations, CMB+BAO+SN. The red solid lines and the blue dash-dot lines are obtained for the flat and non-flat universe, respectively. And the black dashed lines are obtained when (incorrectly) neglecting dark energy perturbations. The magenta solid lines stand for $w_0 = -1$ and $w_0 + w_1 = -1$. In this numerical calculation the systematic uncertainties of Union compilation is not considered.

The **whole** function of is determined as (a=1/(1+z))

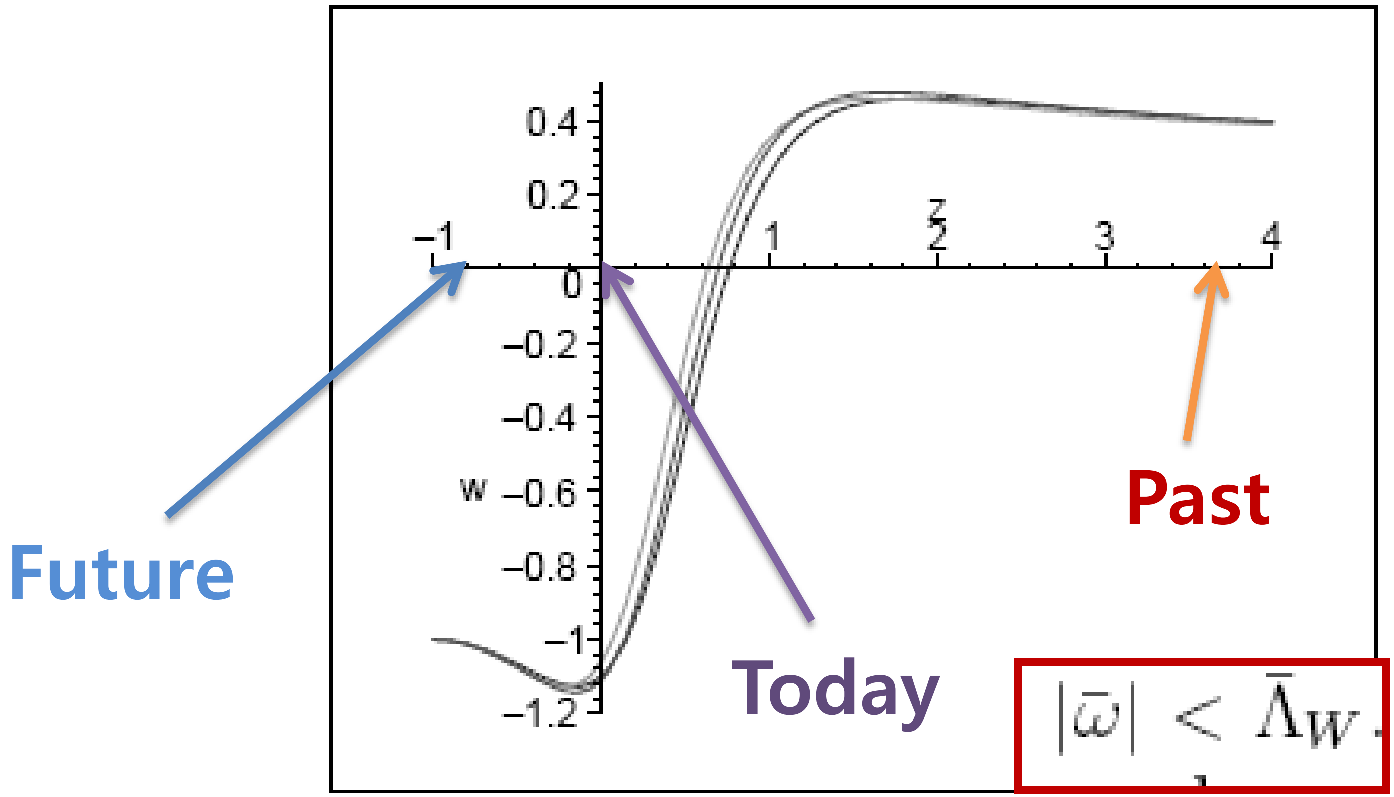
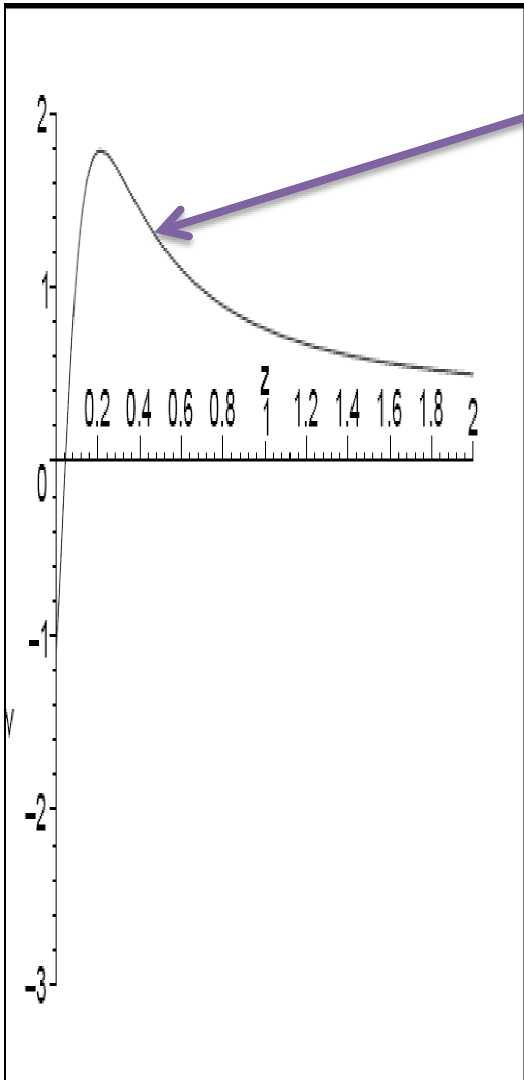
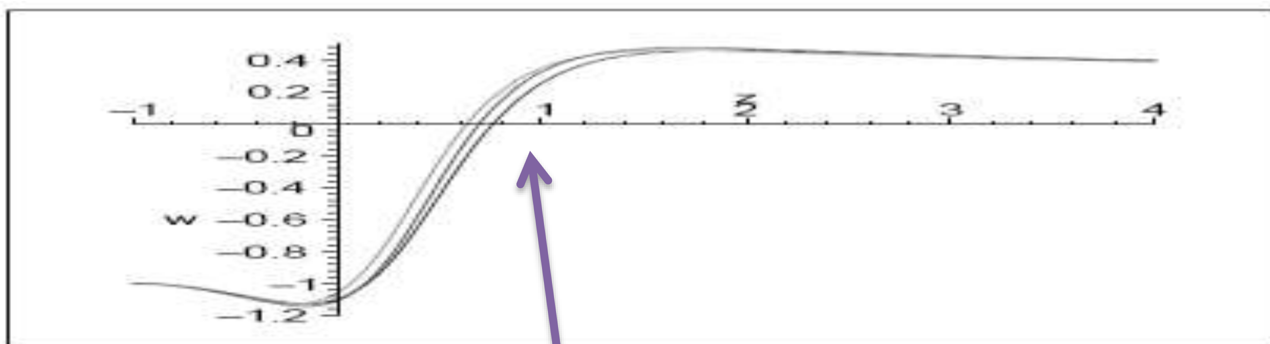


FIG. 3: Plots of equation of state parameters $w_{\mathrm{D.E.}}$ vs. redshift $z = 1/a - 1$ for latest data sets $(\bar{\omega}, \bar{\Lambda}_W) = (1.14, 2.10), (1.32, 2.44), (1.30, 2.29)$ from $(\omega_0, \omega_a) = (-1.06, 0.72), (-1.10, 0.39), (-1.11, 0.475)$ and $k = -1$.

Comparison with early estimate

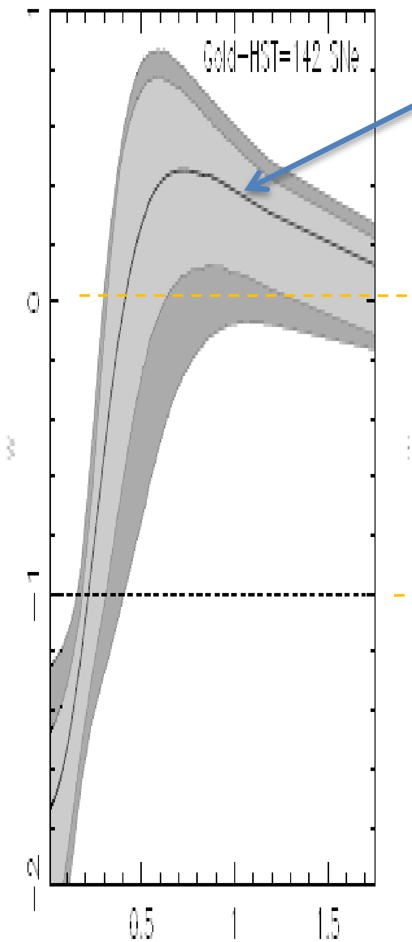


By neglecting matters

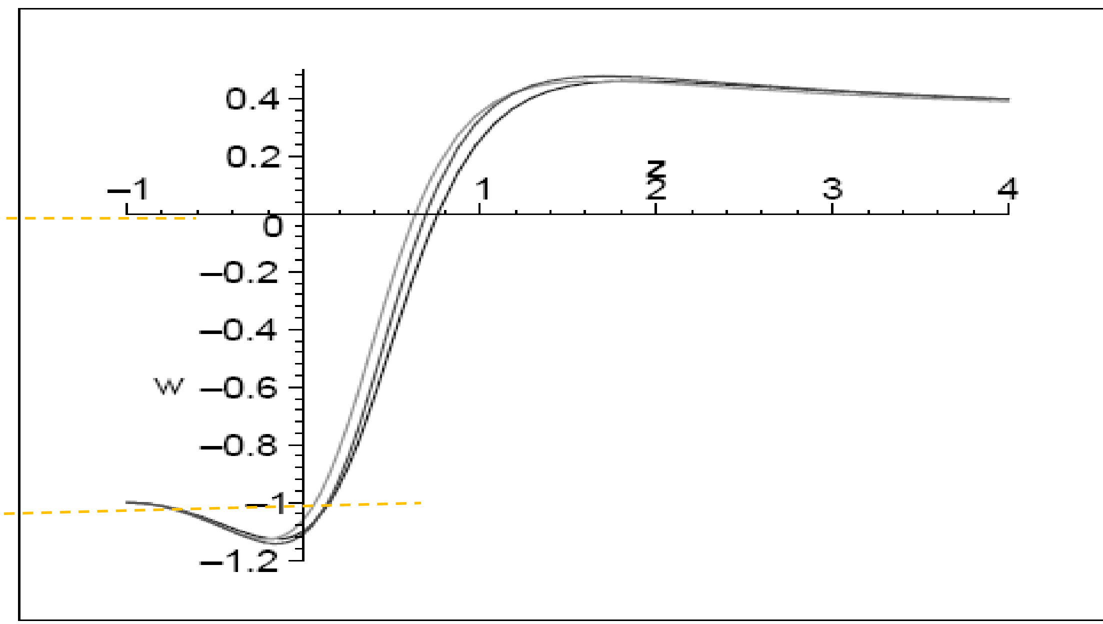


Estimated, independently of matters

Similar tendencies 1.



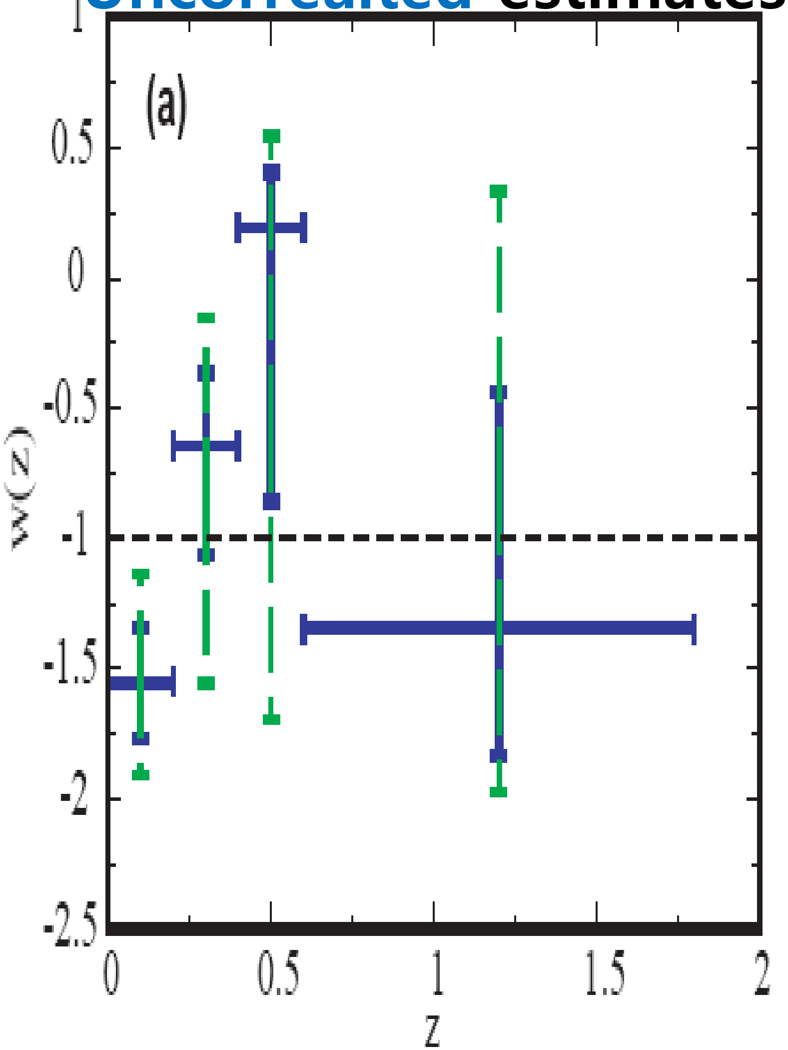
Best Fit: Gold-HST=142 SNe

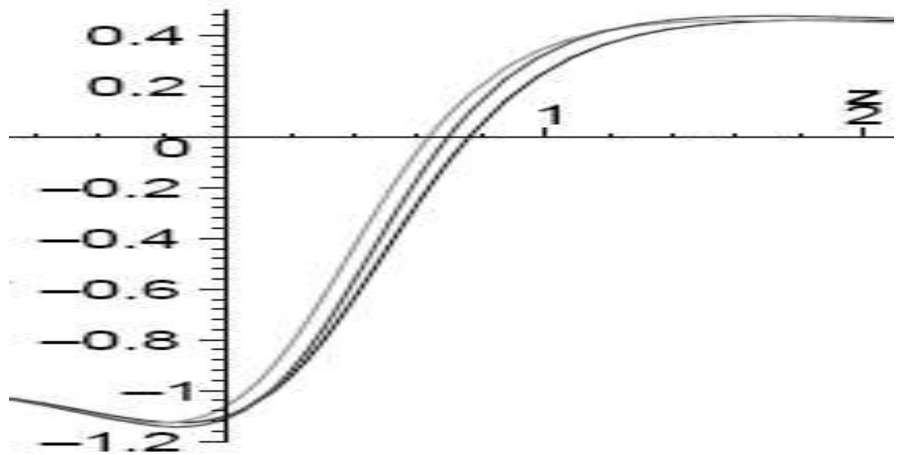


U. Alam et. al., astro-ph/0403687 (Flat universe is asumed)

Similar tendencies 2

Huterer and Cooray, PRD71, 023506 (2005):
Uncorrealted estimates (flat universe is assumed)





Remark

- For the consistency of our theory, we need

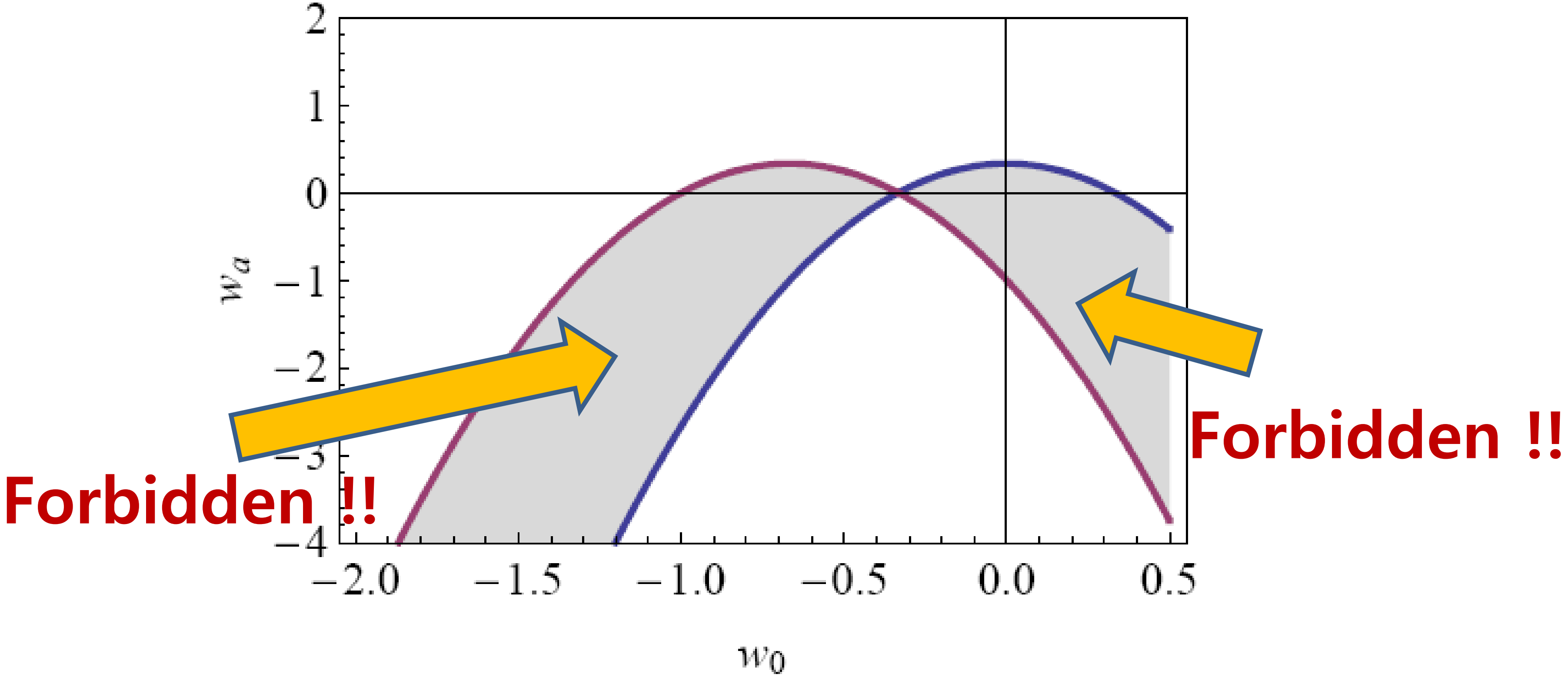
$$\bar{\Lambda}_W^2 = \frac{(-1+9w_0^2+3w_a)k^2}{3(1+4w_0+3w_0^2+w_a)} \geq 0$$

- Otherwise, we would have imaginary valued c^2 and G, though Λ would not !! :

$$c^2 \equiv i\frac{\kappa^4\mu^2|\Lambda_W|}{8(3\lambda-1)^2},\ G = \frac{\kappa^2 c^2}{16\pi(3\lambda-1)},\ \Lambda = \frac{3}{2}\Lambda_W c^2$$

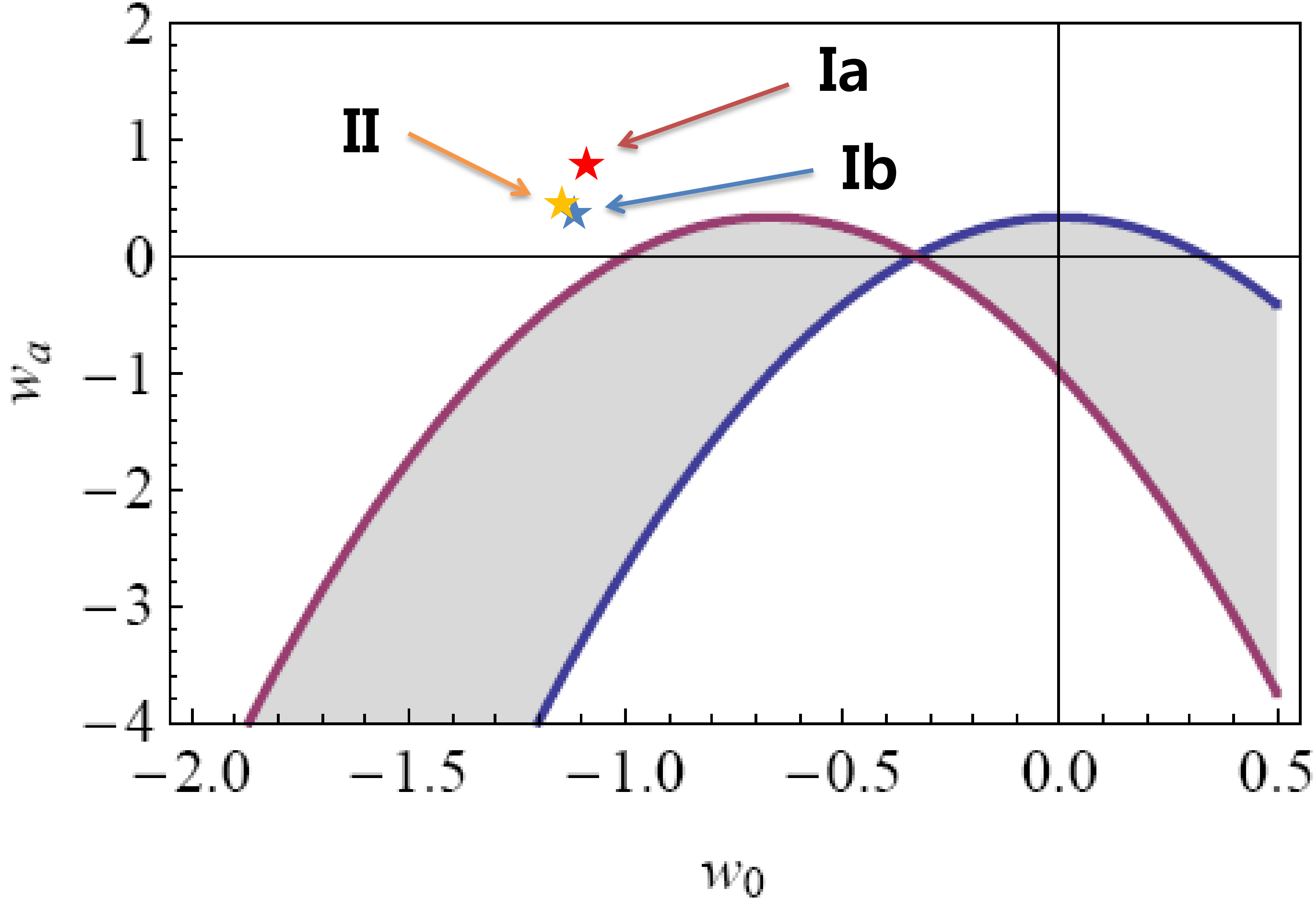
Consistency Conditions $\bar{\Lambda}_W^2 \geq 0$:

$$w_a > \frac{1}{3}(1-9w_0^2),\ -1-4w_0-3w_0^2$$

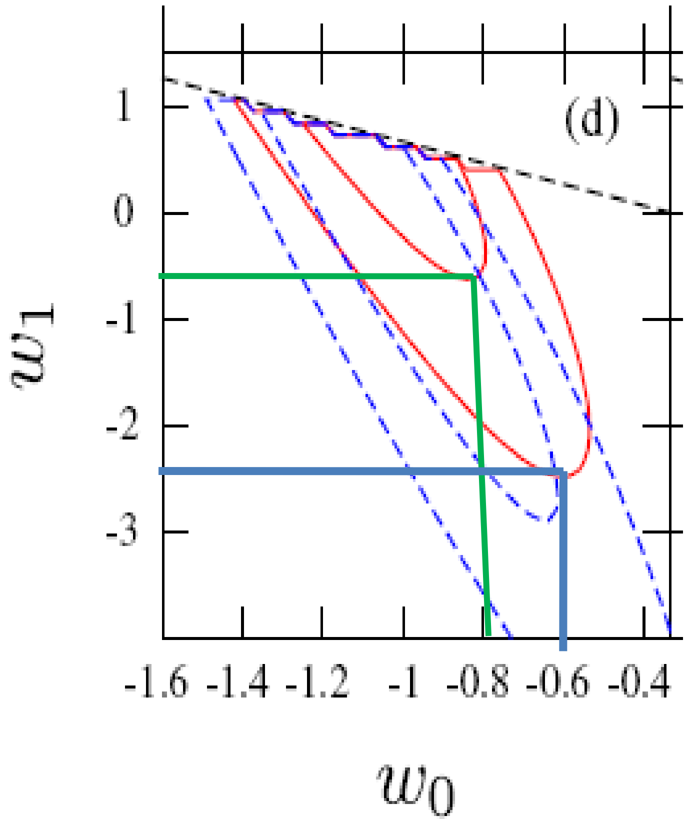


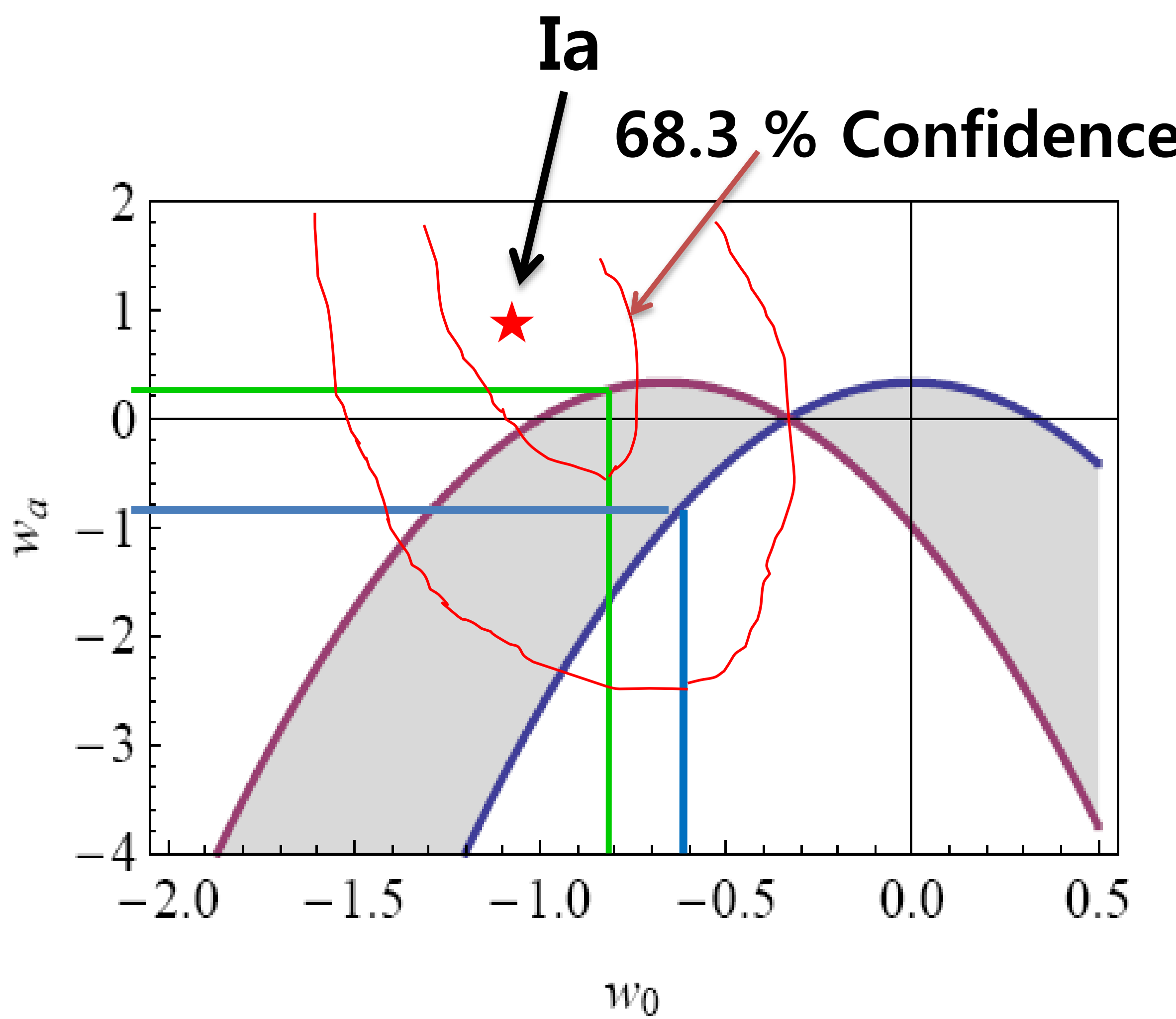
$$w_a < \frac{1}{3}(1-9w_0^2),\ -1-4w_0-3w_0^2.$$

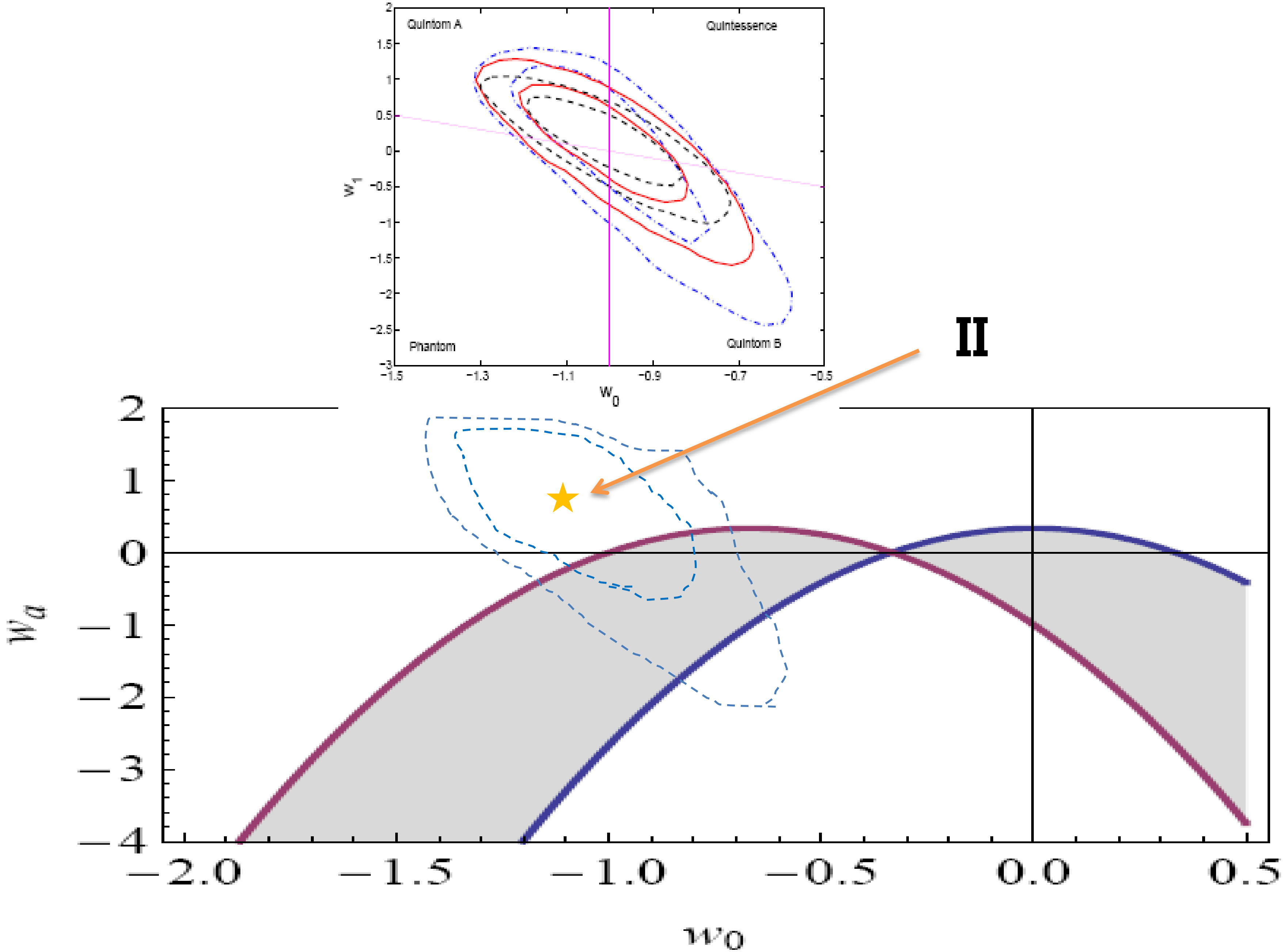
In our data sets



Within confidence levels







Quintom A
Quintessence
Phantom
Quintom B
w_1
w_0
II
w_a
w_0

- **Consistency condition may be tested near future by sharpening the data sets !**

Possible scenario I

- If k=0, i.e., flat universe is confirmed, there is no effect of the Horava gravity in FRW cosmology. This is predicted by inflationary cosmology but $k \neq 0$,i.e., non-flat universe can be still consistent with data !
- But, even in this case, it is still open problem to study its effect to anisotropic and non-Gaussianity.

Possible scenario IIa

- If $w_{\rm D.E.} < -1$ and $k \neq 0$, the original Horava gravity with the detailed balance, which predicts $-1 \leq w_{\rm D.E.} \leq 1/3$, may be ruled out .

- According to current observational data, this seems to be quite plausible and this is also consistent with other theoretical considerations

Possible scenario IIb

- Even if $w_{\rm D.E.} < -1, \; k \neq 0,$ and good agreements for small z are confirmed by determining ω and Λ_W, some disagreements or inconsistencies for higher z can occur.
- In this case, one might consider several further modifications:

(1) More detailed-balancing breaking terms with the additional parameters.

(2) Another definition of $\rho_{\rm D.E.}$ and $p_{\rm D.E.}$ by considering different definition c^2 .

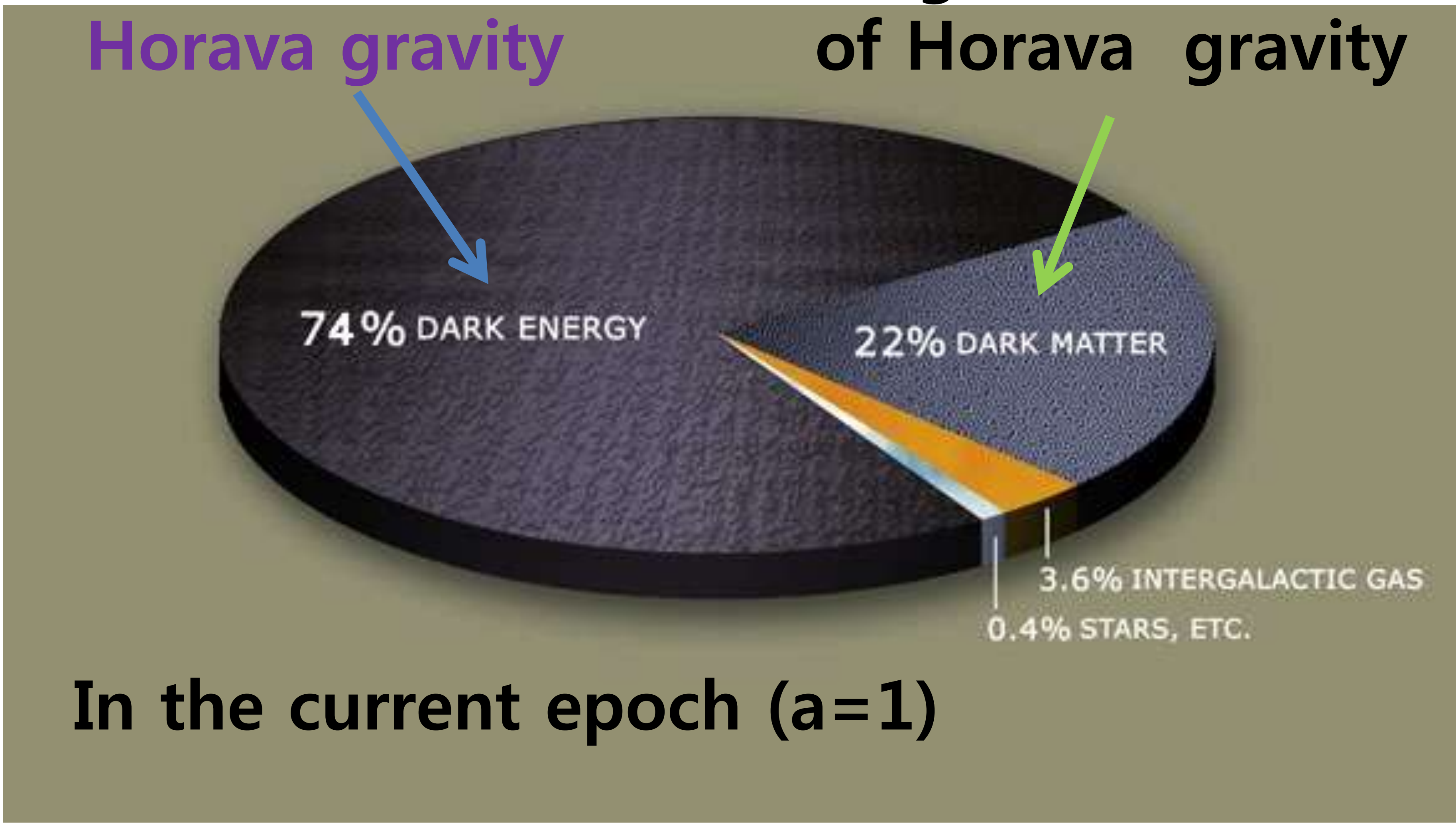
4. Open problems

- We need some more systematic fitting for the range of allowed constant parameters w, μ, Λ_W to see whether our theory is really consistent with our universe.

- Can we reproduce other complicated stories with matters ? If dark matters are given by the Horava gravity also, as Mukohyama proposed, I can say …

IR modified Horava gravity

Integration constant of Horava gravity



In the current epoch (a=1)

Estimated distribution of dark matter and dark energy in the universe

Slogan for gravitists

The Horava gravity is the theory of 97 % !!??

If yes, Slogan for gravitists

The Horava gravity is the theory of almost everything !