Cosmological neutrinos

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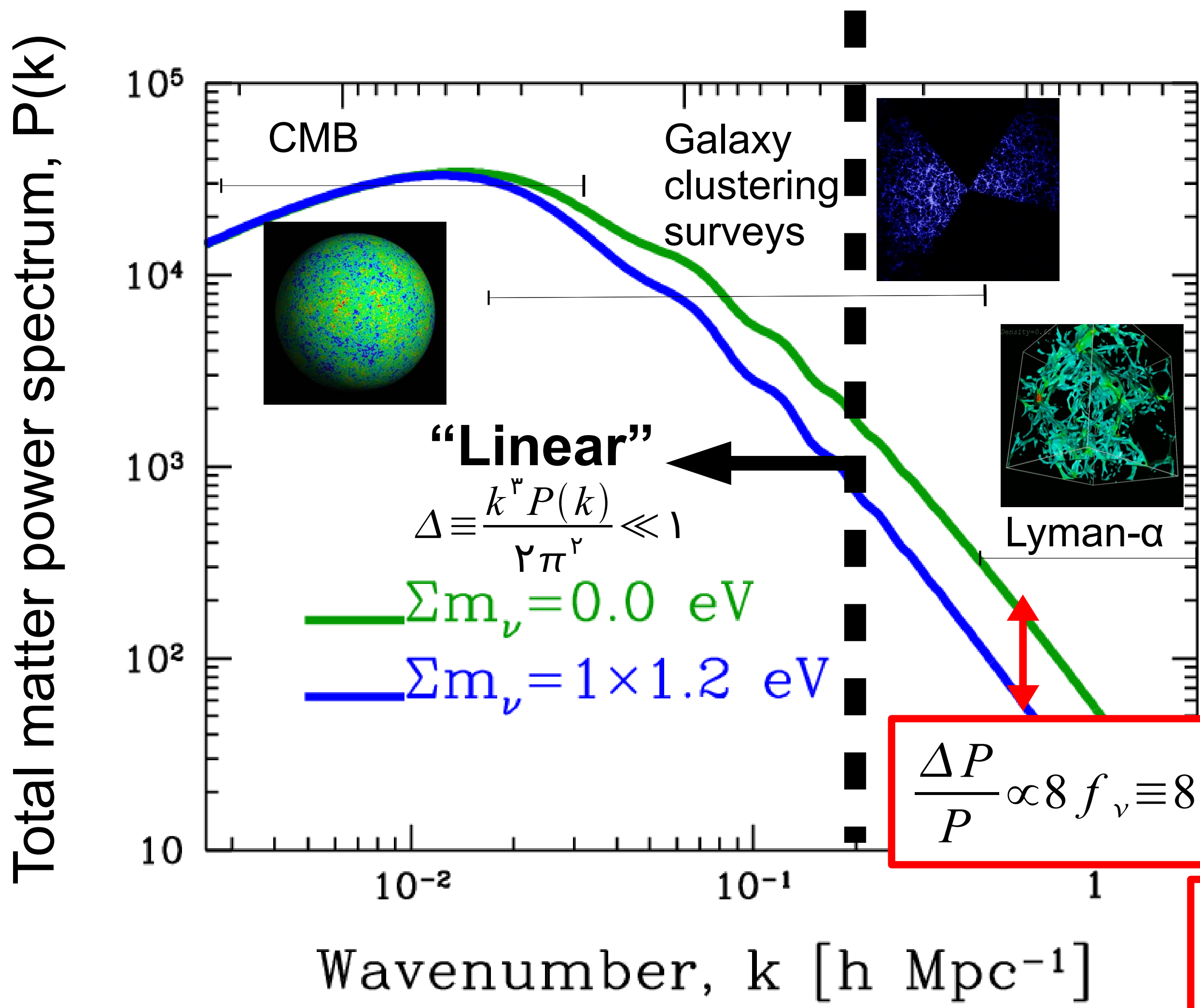
APCTP Focus Program, June 15 - 25, 2009

3. Neutrinos and structure formation: the nonlinear matter power spectrum

Measuring neutrino masses with cosmology...



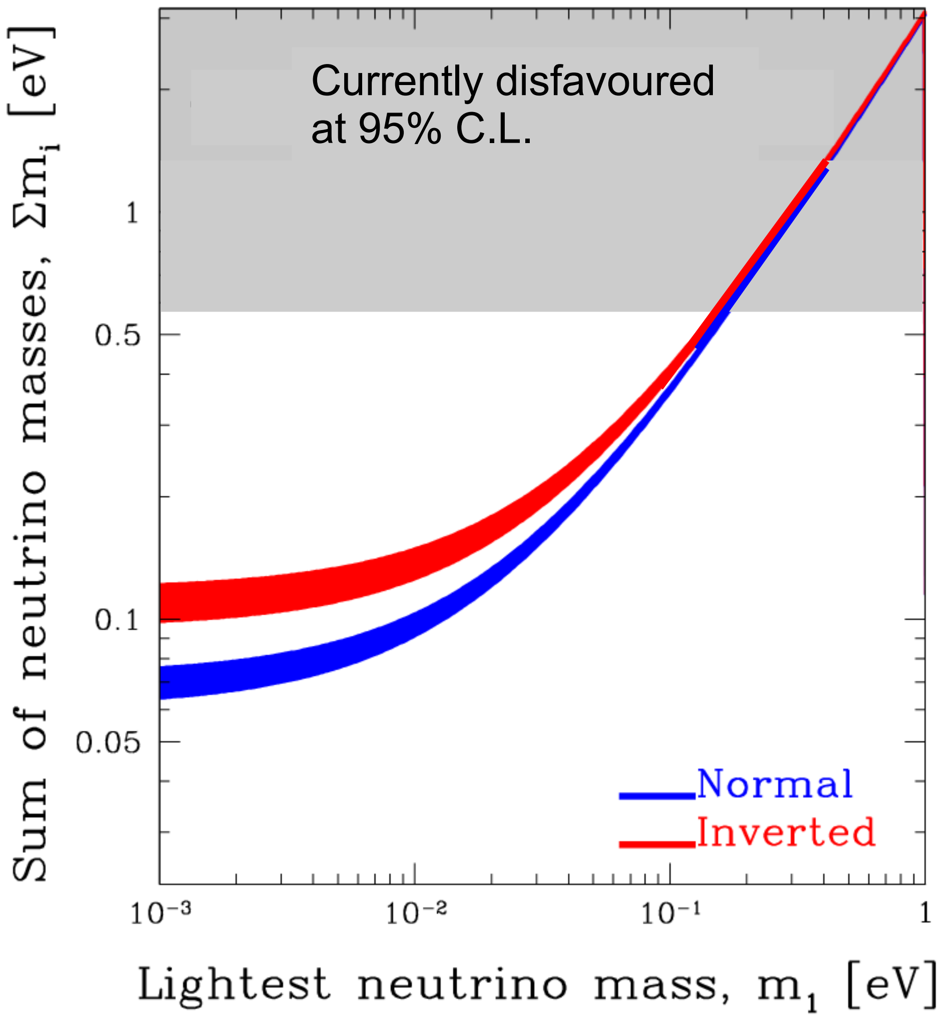
$$\Delta \equiv \frac{k^3 P(k)}{2\pi^2} \ll 1$$

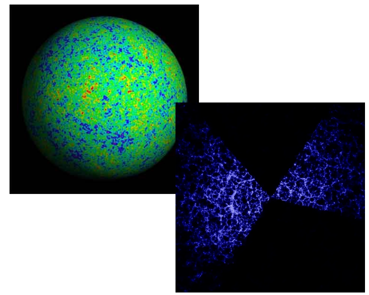
$$\frac{\Delta P}{P} \propto 8 f_\nu \equiv 8 \frac{\Omega_\nu}{\Omega_m}$$

f = Neutrino fraction

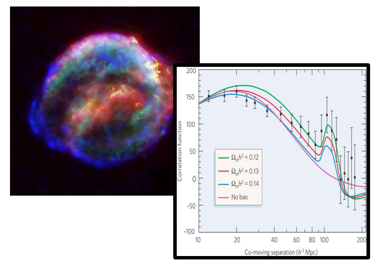
$$\Omega_\nu h^2 = \sum \frac{m_\nu}{93\,\mathrm{eV}}$$

Current constraints...

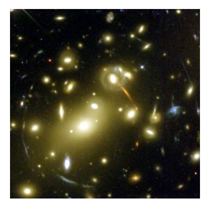




WMAP5 +
Galaxy clustering

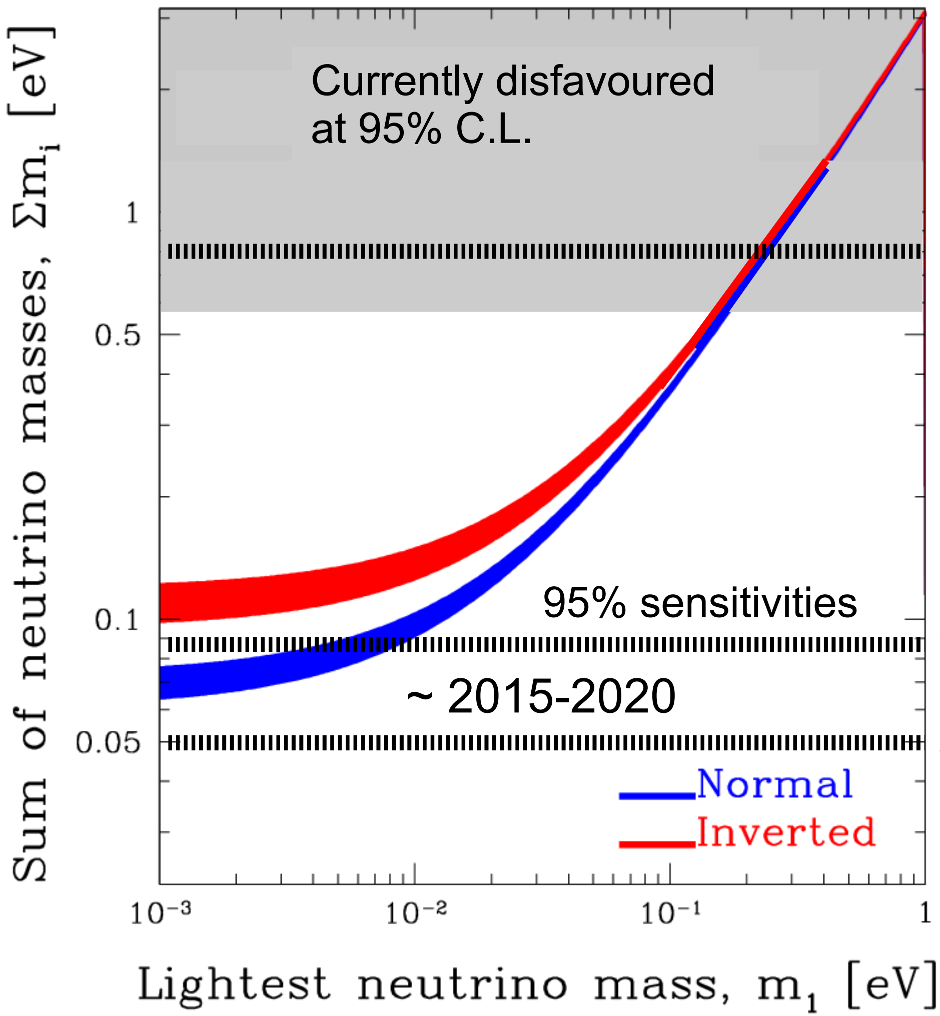


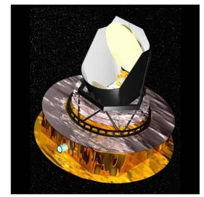
+ SN + BAO
Goobar et al. 2006



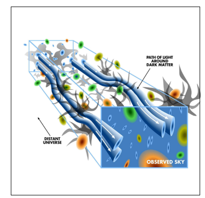
+ Weak lensing
Tereno et al. 2008
Ichiki et al. 2008

Projected sensitivities...

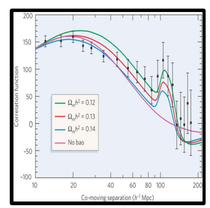




Planck (1 year)
Lesgourgues et al. 2006
Perotto, Lesgouruges,
Hannestad, Tu & Y[3]W 2006



+ Weak lensing with LSST
(tomography)
Hannestad, Tu & Y[3]W. 2006
Kitching et al. 2008



+ BAO
Ivezic et al. 2008
www.lsst.org

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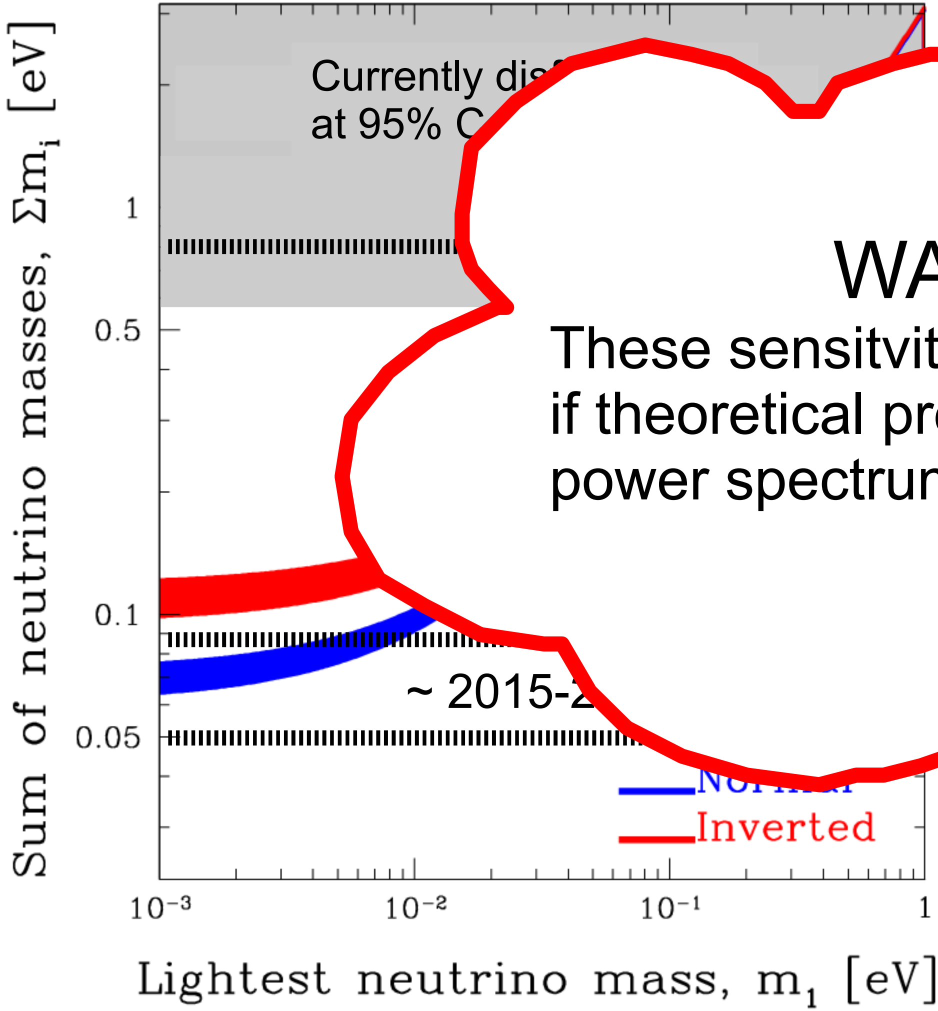

(1 year)
ues et al. 2006
uges,
W 2006

WARNING!

These sensitvities can be achieved only if theoretical predictions of the matter power spectrum are accurate to ~ 1%.

Huterer & Takada 2005

with LSST

& Y^{3}W. 2006
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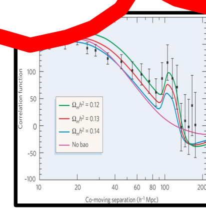

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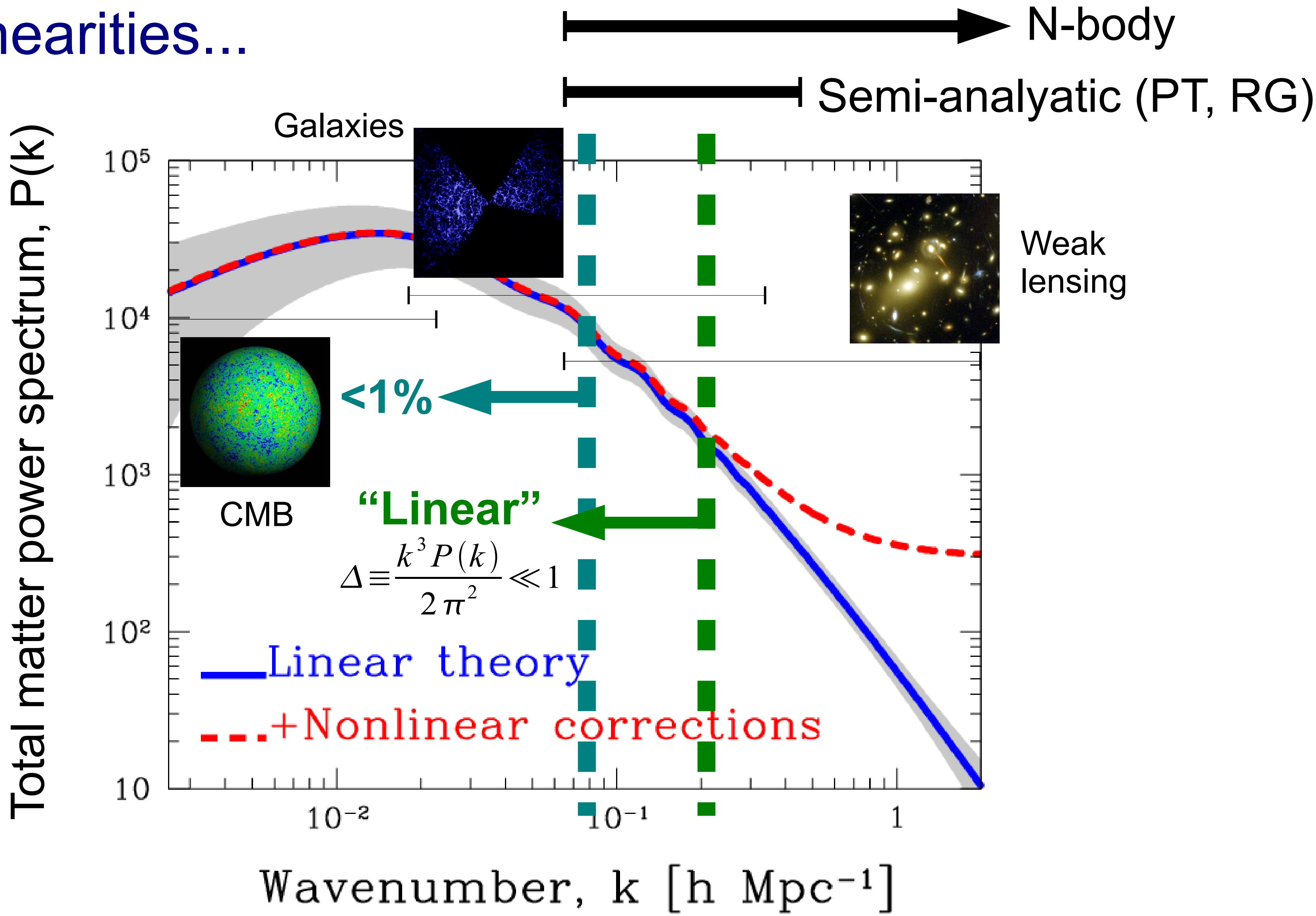
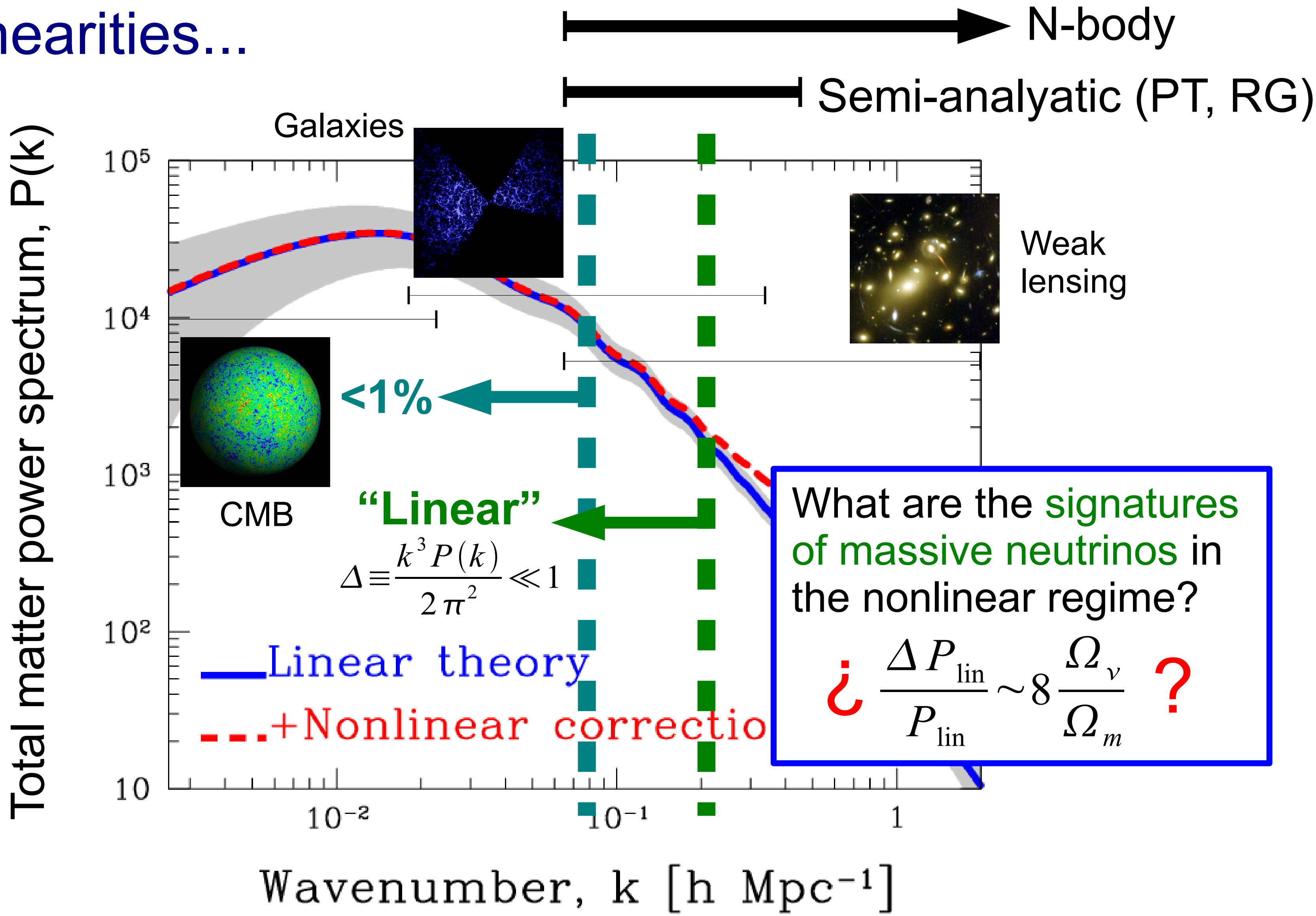
Nonlinearities...



Nonlinearities...



Plan...

- N-body simulations with massive neutrinos.
- Semi-analytic methods
  - Standard perturbation theory
  - Beyond perturbation theory

Useful references...

- E. Bertschinger, *Simulations of structure formation in the universe,* Ann. Rev. Astron. Astrophys. **36** (1998) 599.

- F. Bernardeau et al., *Large-scale structure of the universe and cosmological perturbation theory*, Phys. Rept. **367** (2002) 1.

3.1. N-body simulations…

Vlasov equation for gravitational clustering...

- Boltzmann equation without collisions for the ith particle species:

$$\frac{\partial f_i}{\partial \tau} + \frac{d\boldsymbol{x}}{d\tau} \cdot \frac{\partial f_i}{\partial \boldsymbol{x}} + \frac{d\boldsymbol{q}}{d\tau} \cdot \frac{\partial f_i}{\partial \boldsymbol{q}} = 0$$

  - Phase space density $f_i(\boldsymbol{x}, \boldsymbol{q}, \tau)$
  - Comoving spatial coordinates $\boldsymbol{x} = \boldsymbol{r}/a$
  - Comoving momentum $\boldsymbol{q} = a\,\boldsymbol{p}$
  - Conformal time $d\tau = dt/a$

- At late times (z < 100), all particles species are nonrelativistic.
- Nonlinearity expected only on subhorizon scales.

→ Vlasov equation with **Newtonian gravity**:

$$\frac{\partial f_i}{\partial \tau} + \frac{\boldsymbol{q}}{m_i a} \cdot \nabla f_i - a m_i \nabla \Phi \cdot \frac{\partial f_i}{\partial \boldsymbol{q}} = 0$$

Poisson equation $\nabla^2 \Phi(\boldsymbol{x}, \tau) = 4 \pi G a^2 \sum_i \bar{\rho}_i(\tau) \delta_i(\boldsymbol{x}, \tau)$

Gravitational potential

Mean energy density

Density fluctuation

$$\delta_i(\boldsymbol{x}, \tau) \equiv \frac{m_i}{a^3} \int d^3\boldsymbol{q}\, f_i(\boldsymbol{x}, \boldsymbol{q}, \tau) - f_i^0(q)$$

- The Vlasov equation expresses conservation of phase space density along characteristics $\{\boldsymbol{x}(\tau), \boldsymbol{q}(\tau)\}$.

→ An element drawn initially from $\{\boldsymbol{x}_i, \boldsymbol{q}_i\} \rightarrow \{\boldsymbol{x}_i + d\,\boldsymbol{x}, \boldsymbol{q}_i + d\,\boldsymbol{q}\}$

$$dN = f\,(\boldsymbol{x}_i, \boldsymbol{q}_i, \tau)\, d^3 x\, d^3 q$$

will move according to:

$$\frac{d\,\boldsymbol{x}}{d\,\tau} = \frac{\boldsymbol{q}}{a\,m_i} \qquad \frac{d\,\boldsymbol{q}}{d\,\tau} = -a\,m_i \nabla \Phi(\boldsymbol{x}, \tau)$$

↓

Tracing **all** characteristics originating from all possible initial $\{x_i, q_i\}$

=

Solving the Vlasov equation

→ N-body simulation

Cosmological N-body simulations...

Samples of phase space elements

- Represent fluid by discrete particles in a box.
  - Box size typically $[O(100)\ \mathrm{Mpc}]^3$ to $[O(1000)\ \mathrm{Mpc}]^3$.

- Let particles move under each other's gravity according to:

$$\frac{d\boldsymbol{x}}{d\tau}=\frac{\boldsymbol{q}}{a\,m}, \qquad \frac{d\boldsymbol{q}}{d\tau}=-a\,m\nabla\Phi$$

$$\nabla^2\Phi(\boldsymbol{x},\tau)=4\pi G a^2\sum_i \bar{\rho}_i(\tau)\delta_i(\boldsymbol{x},\tau)$$

- Expansion of the universe absorbed by the comoving coordinates; also residual factors of a in the equations.

Cosmological N-body simulations...

ples of phase
lements

- Represent flui
  - Box size
- Let partic

$\nabla^2 \varphi$

CAUTION!!

This description is valid **only** for collisionless particles (e.g., CDM, neutrinos).

Baryons interact "strongly" and dissipate energy: other descriptions required.

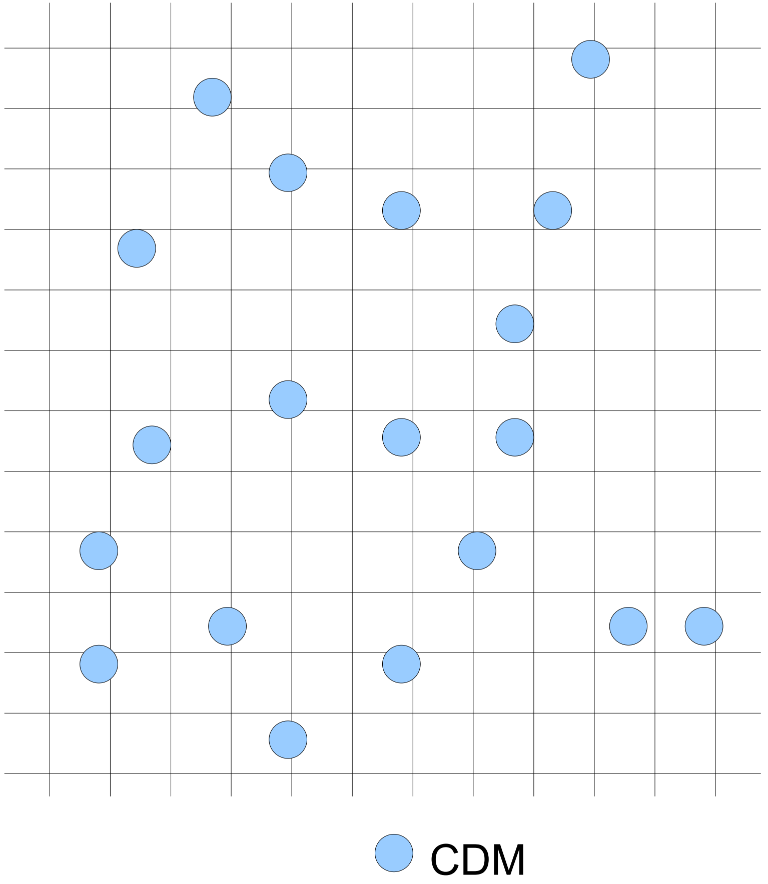
- Expansion of the universe absorbed by the comoving coordinates; also residual factors of a in the equations.

Particle-Mesh code...

- N-body Particle-Mesh code:
  - **Particles**: motion described by Newtonian dynamics.

$$\frac{d\boldsymbol{x}}{d\tau}=\frac{\boldsymbol{q}}{am}, \qquad \frac{d\boldsymbol{q}}{d\tau}=-am\nabla\Phi$$

  - **Mesh**: grid points on which gravitational potential is evaluated.



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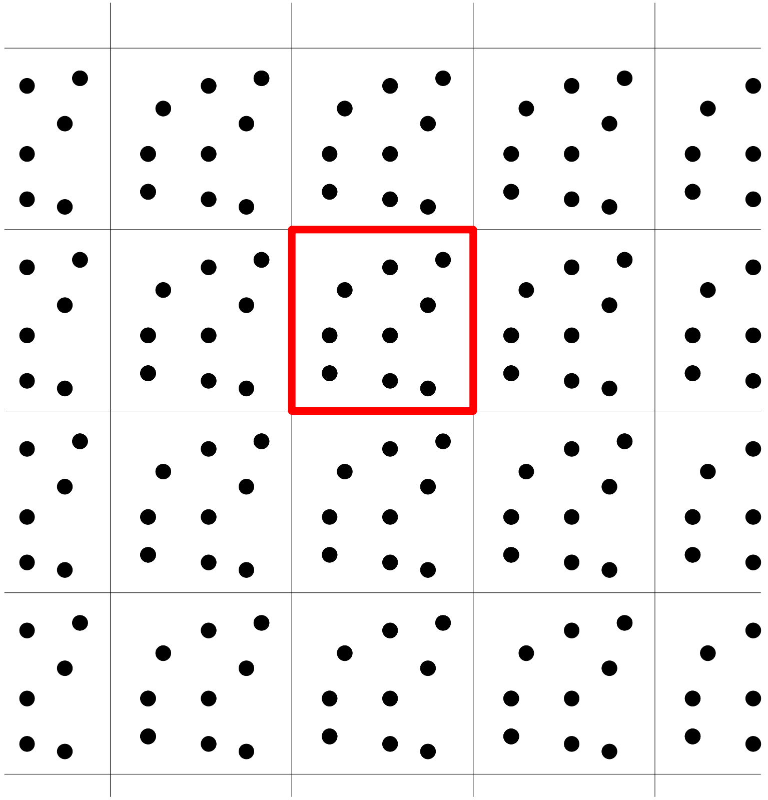
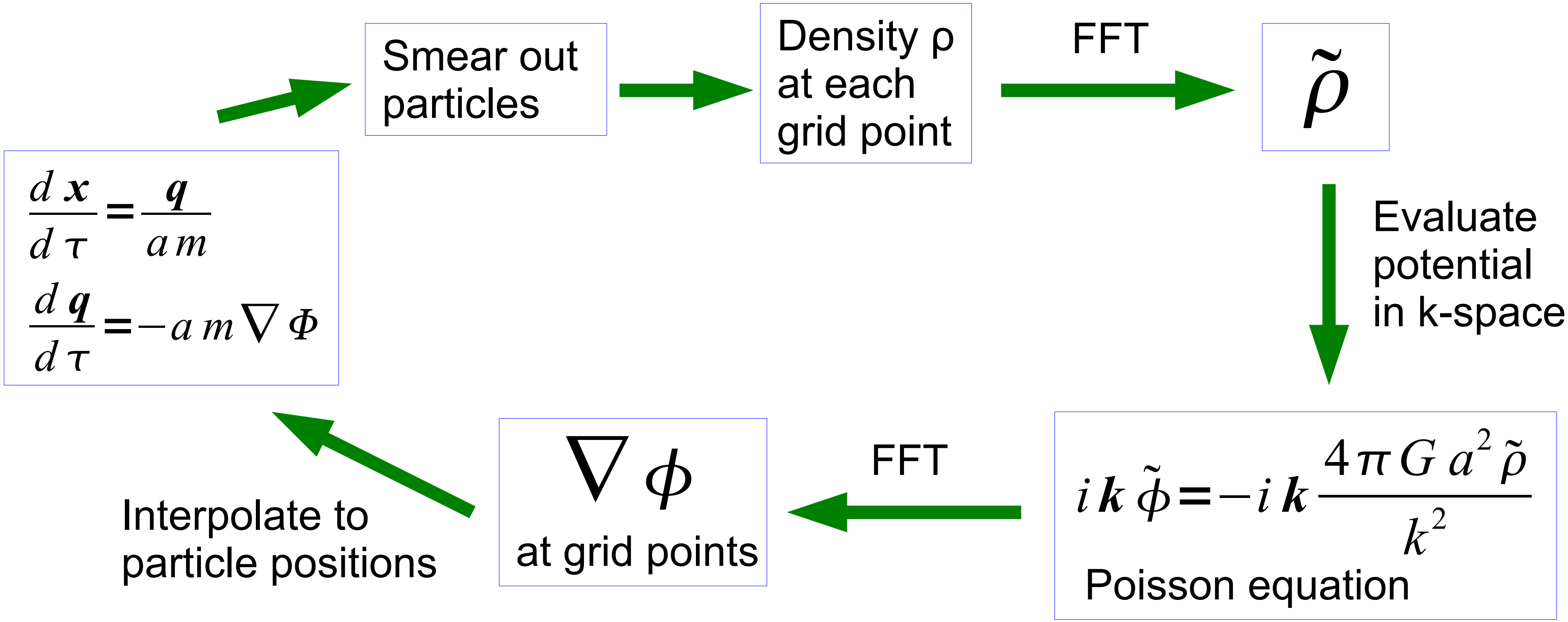
  - **Mesh**: grid points on which gravitational potential is evaluated.
  - Periodic boundary conditions.



- **Mesh**: grid points on which the gravitational potential is evaluated.



Initial conditions...

- Simulation usually begins at z ~ 50 → 100.
  - Perturbations are still small and can be calculated from linear perturbation theory.
    - Linear growth function: $\delta(k,\tau) = \underline{D(k,\tau)}\,\delta(k,\tau_0)$
    - Linear power spectrum: $\underline{P(k)}\,\delta_D(\boldsymbol{k}+\boldsymbol{q}) = \langle \delta(\boldsymbol{k})\,\delta(\boldsymbol{q}) \rangle$
  - These are used to generate initial conditions.

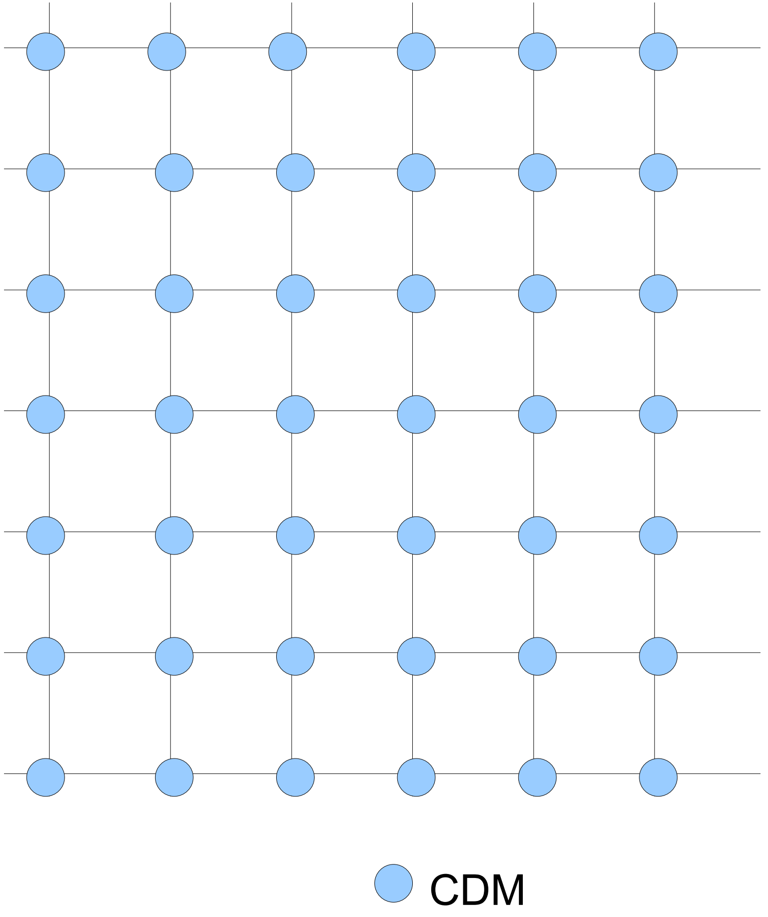
- Put a particle on each grid point $\boldsymbol{x}_i$

- Generate random perturbations $\hat{\delta}(\boldsymbol{x}_i, \tau_i)$ according to the linear power spectrum at τ_i.

- Apply Zel'dovich approximation:

Displacement vector

$$\nabla \cdot \boldsymbol{\psi} = -\frac{\hat{\delta}(\boldsymbol{x}_i, \tau_i)}{D(\tau_i)}$$



D( ) = Linear growth factor; assume no k-dependence

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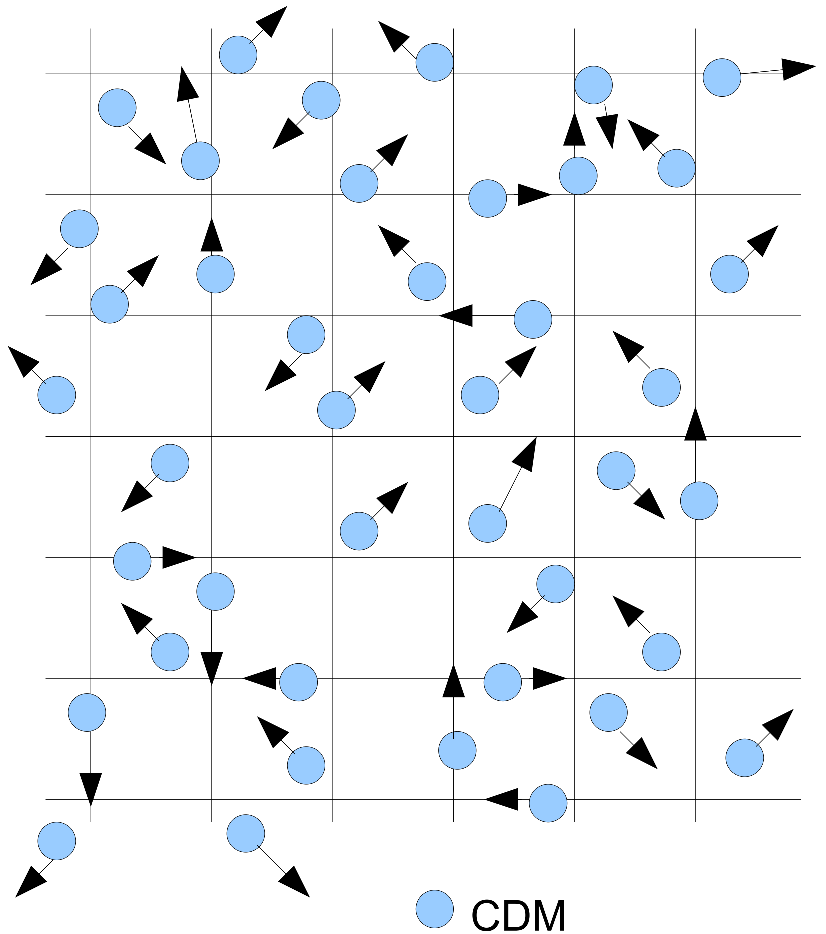
Initial positions & velocities

$$\boldsymbol{x} = \boldsymbol{x}_i + D(\tau_i)\,\boldsymbol{\psi}(\boldsymbol{x}_i)$$

$$\boldsymbol{v} = \frac{dD}{d\tau}\boldsymbol{\psi}(\boldsymbol{x}_i)$$



D( ) = Linear growth factor; assume no k-dependence

Simulating massive neutrinos...

- Neutrinos have thermal motion.
  - Unperturbed phase space distribution:

$$f(q) = \frac{1}{\exp(q/T_0) + 1}$$

Present day neutrino temperature

$$T_0 = 1.95 \text{ K}$$

  - Average velocity:

$$\langle v \rangle = \left\langle \frac{q}{m_\nu a} \right\rangle \simeq 81 (1+z) \left(\frac{\text{eV}}{m_\nu} \right) \text{ km s}^{-1}$$

- For m = 1 eV, <v> is 10% the speed of light at z = 100.

- Same equations of motion as for CDM.
- Neutrino thermal motion is factored into the initial conditions.
- **One possible implementation**:
  - A neutrino particle initially on grid point $\boldsymbol{x}_i$ has initial velocity:

$$\boldsymbol{v} = \frac{dD}{d\tau}\boldsymbol{\psi}(\boldsymbol{x}_i) + \underline{v_{\text{thermal}}\hat{\boldsymbol{r}}}$$

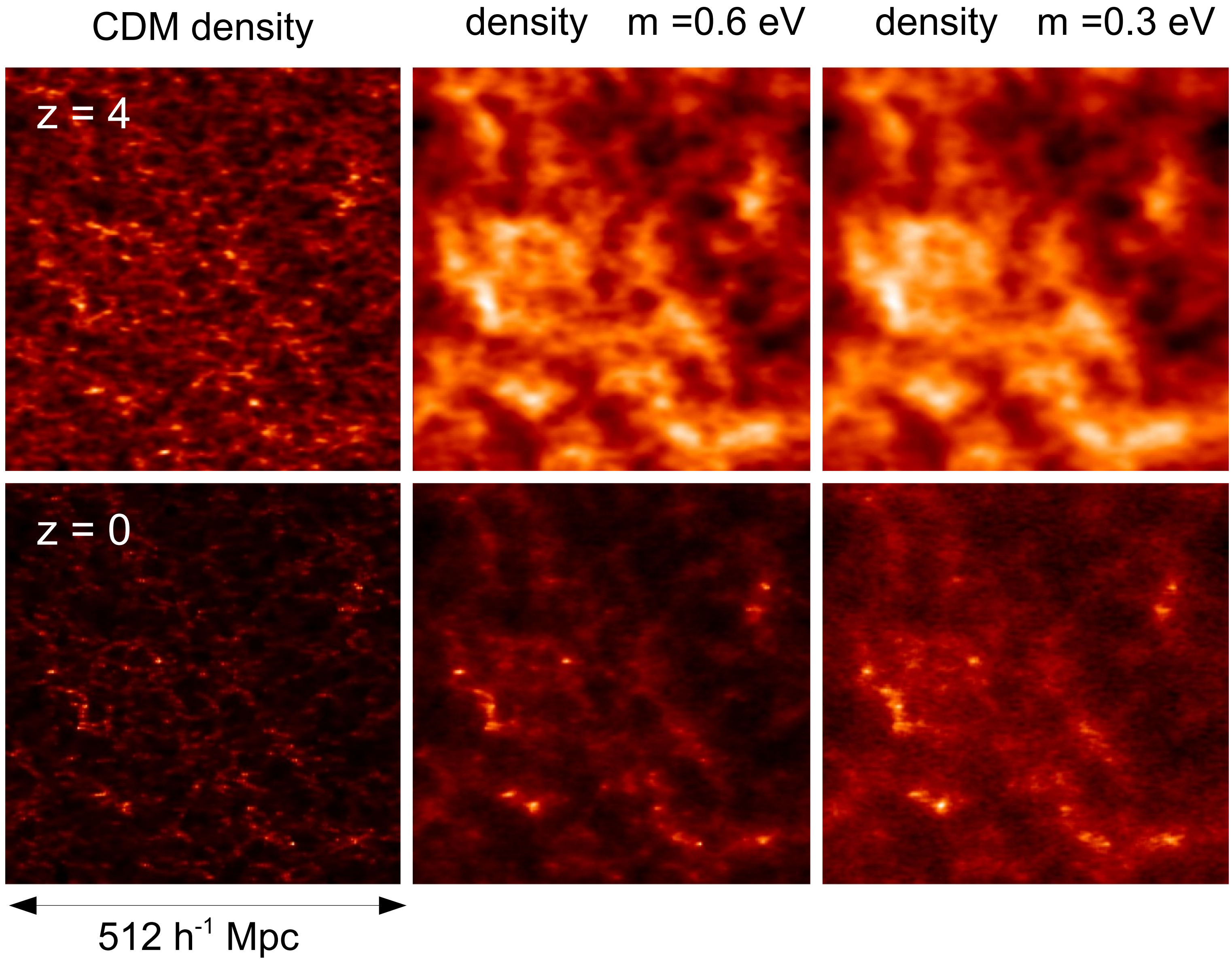
- $\hat{\boldsymbol{r}}$ = random direction vector
- v_{thermal} = thermal velocity drawn randomly from a Fermi-Dirac distribution

Brandbyge, Hannestad, Haugbølle & Thomsen 2008

Simulating CDM+massive neutrinos...

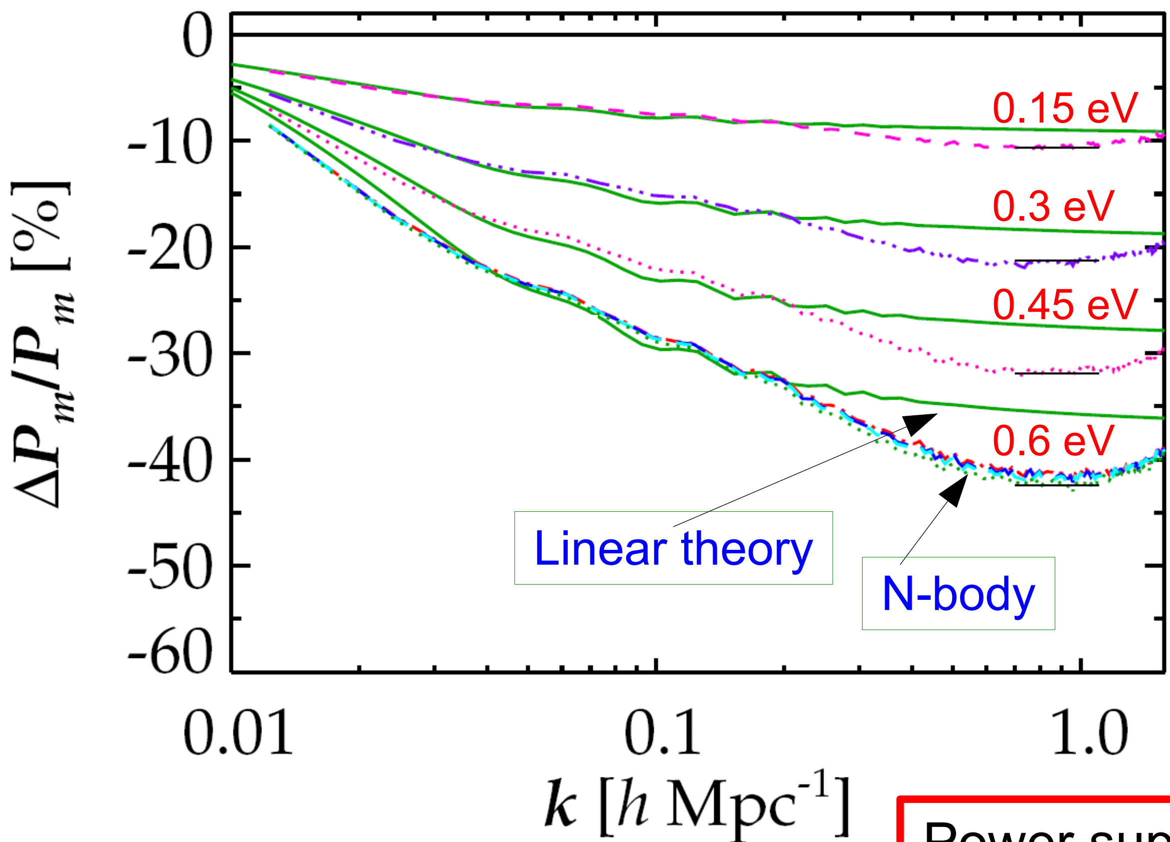
- Particle representation for **both** CDM and neutrinos.
  - Modified GADGET-2. Springel 2005
  - 512^3 CDM particles.
  - **512^3 neutrino particles**, drawn from Fermi-Dirac distribution.
  - Simulation box size = $(512\ h^{-1}\ \mathrm{Mpc})^3$.
  - Starting redshift $z_i = 49$.

Brandbyge, Hannestad, Haugbølle & Thomsen 2008


CDM density
density m =0.6 eV
density m =0.3 eV
z = 4
z = 0
512 h-1 Mpc

Change in the total matter power spectrum relative to the f = 0 case:

$$\frac{\Delta P_m}{P_m} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$



Linear perturbation theory:

$$\frac{\Delta P_m}{P_m} \sim 8 \frac{\Omega_\nu}{\Omega_m}$$

With nonlinear corrections:

$$\frac{\Delta P_m}{P_m} \sim \underline{9.8} \frac{\Omega_\nu}{\Omega_m}$$

Power suppression due to neutrino free-streaming is larger than predicted by linear perturbation theory.

- The challenge for simulating massive neutrinos:
  - Large velocity means accurate time-stepping.
  - Need a large number of particles to fully sample the thermal distribution; otherwise noisy.
  - The **smaller** the neutrino mass the **worse** the problem.

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  - Large velocity means accurate time-stepping.
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  - The **smaller** the neutrino mass the **worse** the problem.

- Small neutrino mass: A blessing in disguise?
  - **Small mass** → large velocity → **large free-streaming scale**

→ Neutrino density perturbations at small scales likely to stay close to predictions from linear theory.

→ An approximation scheme: Grid-based linear neutrino perturbations.

Next

Grid-based linear neutrino perturbations...

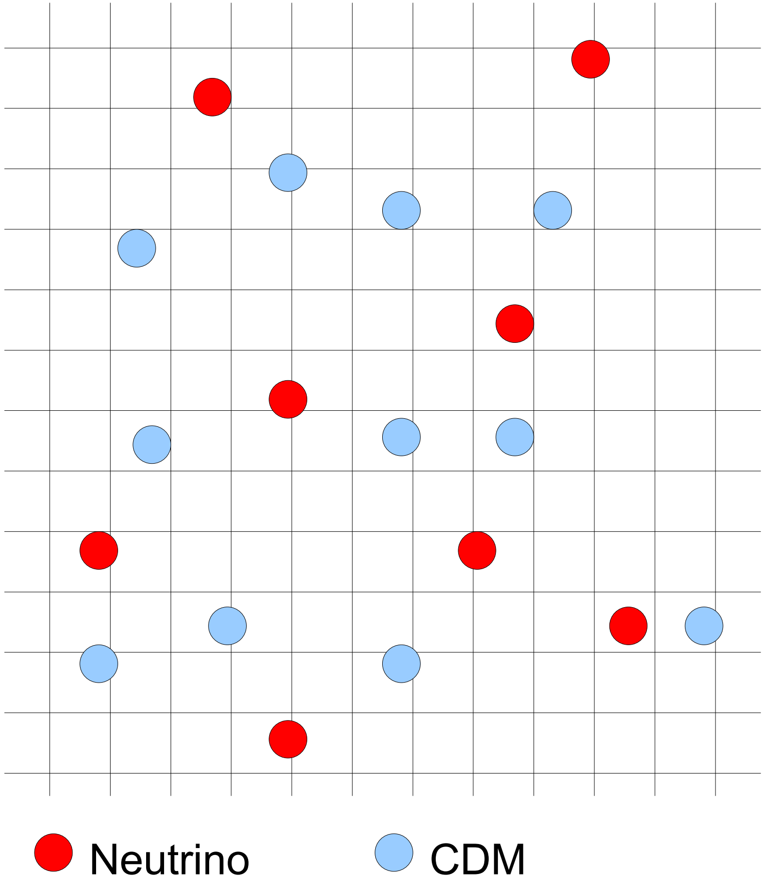
- N-body Particle-Mesh code:
  - **Particles**: motion described by Newtonian dynamics.

$$\frac{d\boldsymbol{x}}{d\tau}=\frac{\boldsymbol{q}}{am}, \qquad \frac{d\boldsymbol{q}}{d\tau}=-am\nabla\Phi$$

  - **Mesh**: grid points on which gravitational potential is evaluated.

$$\hat{\Phi}(\boldsymbol{k},\tau)=-\frac{4\pi G a^2\hat{\rho}(\boldsymbol{k},\tau)}{k^2}$$

$$\Phi(\boldsymbol{x},\tau)=\mathrm{FFT}[\hat{\Phi}(\boldsymbol{k},\tau)]$$



Brandbyge & Hannestad 2008

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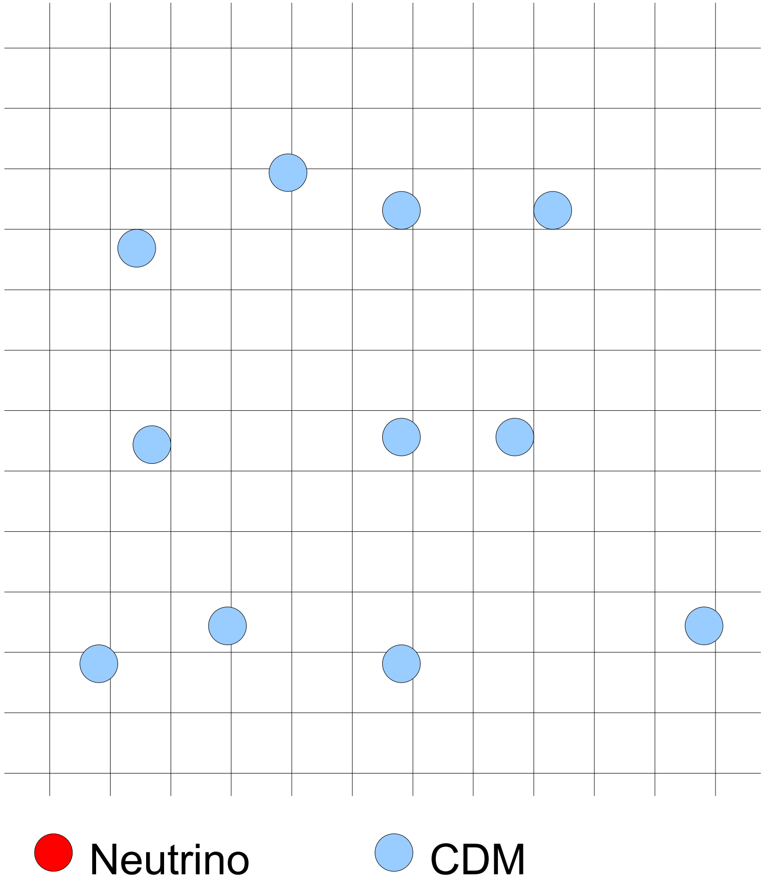

Brandbyge & Hannestad 2008

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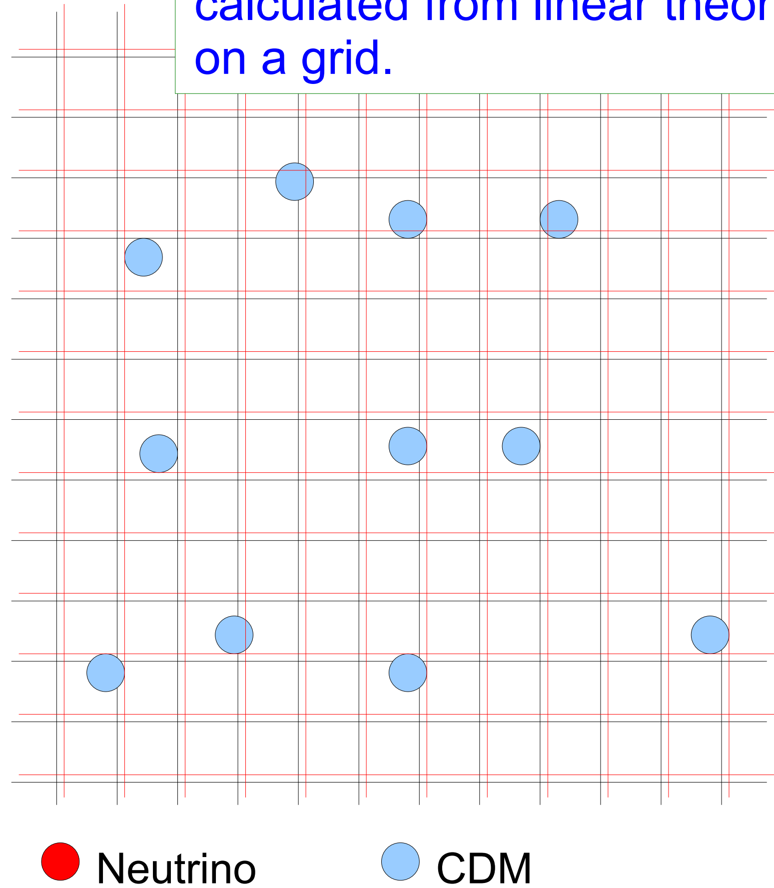
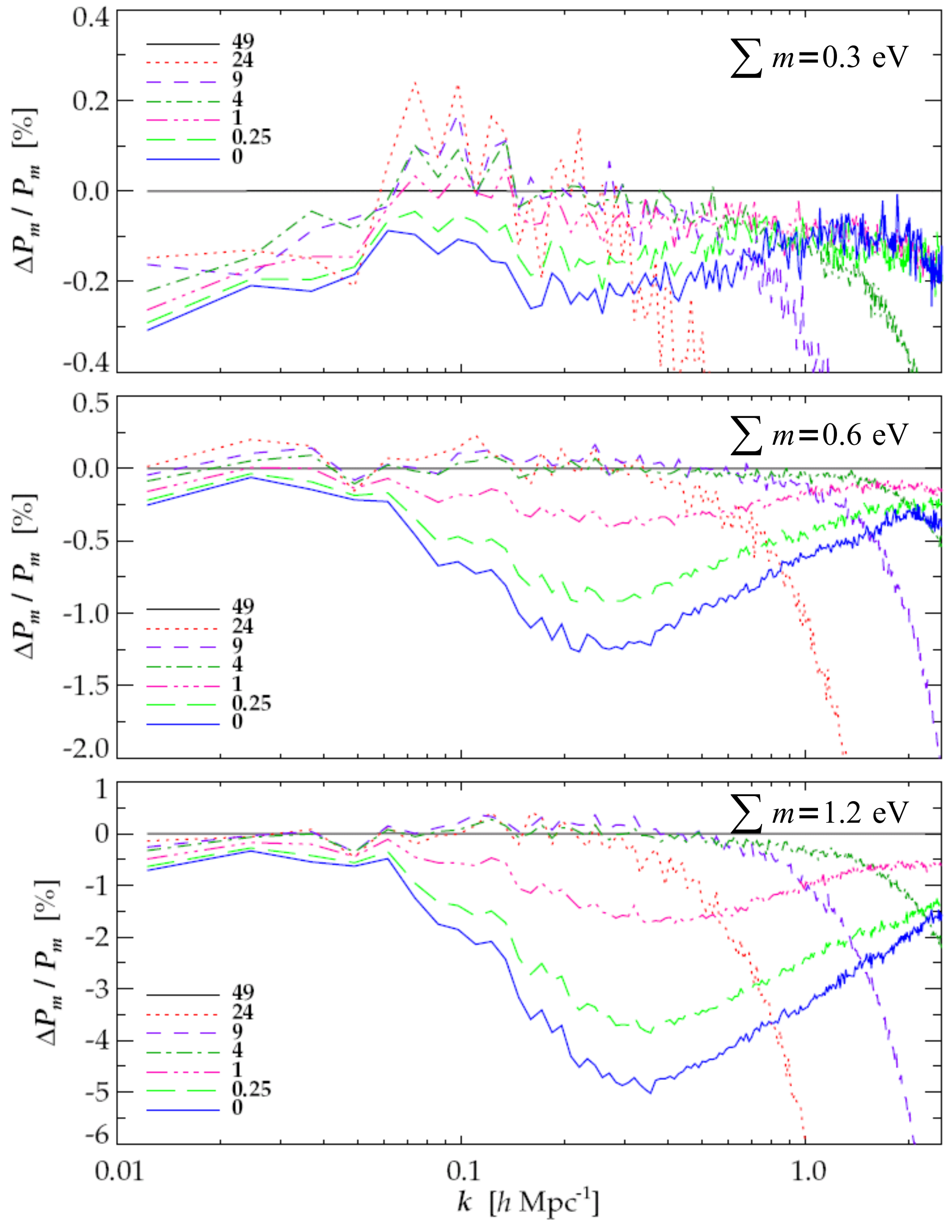
$$\Phi(\mathbf{x},\tau)=\mathrm{FFT}[\hat{\Phi}(\mathbf{k},\tau)]+\underline{\quad{}^{\mathrm{lin}}(x,\tau)}$$

Put neutrino perturbations calculated from linear theory on a grid.



Brandbyge & Hannestad 2008



- Difference in the total matter power spectrum:

$$\frac{\Delta P_m}{P_m} \equiv \frac{P^{\mathrm{Grid}} - P^{\mathrm{N\,body}}}{P^{\mathrm{N\,body}}}$$

- For **m < 0.5 eV**, linear grid calculation is accurate to better than 1% at z=0.
  - <0.1% accuracy at high z (plus no white noise).

- Simulation run-time O(10) times shorter than using a full particle representation for the neutrinos.

3.2. Perturbation theory and RG techniques...

Linear perturbation theory...

= Density perturbations
= Divergence of velocity field

- Recall from the previous talk:

CDM

$$\dot{\delta}_c + \theta_c = 0$$

Linear continuity eqn

$$\dot{\theta}_c + H\,\theta_c + \nabla^2 \Phi = 0$$

Linear Euler eqn

$$\theta \equiv \nabla \cdot \boldsymbol{u}$$

Neutrinos

$$\frac{\partial f}{\partial \tau} + \frac{\boldsymbol{q}}{m_\nu a} \cdot \nabla f - a\, m_\nu \nabla \Phi \cdot \frac{\partial f_0}{\partial \boldsymbol{q}} = 0$$

Linear Vlasov eqn

$$\int d^3\boldsymbol{q}\, f(\boldsymbol{x}, \boldsymbol{q}, \tau) = \bar{\rho}_\nu(\tau)\left[1 + \delta_\nu(\boldsymbol{x}, \tau)\right]$$

$$\nabla^2 \Phi = \frac{3}{2} H^2 \sum_i \Omega_i(\tau)\, \delta_i(\boldsymbol{x}, \tau)$$

Poisson eqn

- We apply nonlinear corrections **only** to the CDM component by
  - We solving nonlinear versions of the continuity and Euler equations.

$$\dot{\delta}_c + \theta_c = 0$$ Linear continuity eqn

$$\dot{\theta}_c + H\,\theta_c + \nabla^2 \Phi = 0$$ Linear Euler eqn

$$\theta \equiv \nabla \cdot \boldsymbol{u}$$

  - Pioneering works: Juszkiewisz 1981, Vishniac 1983, Fry 1984, Goroff et al. 1986

- We do **not** correct for nonlinearities in the neutrino perturbations
  - There are some ideas about how to do it. Shoji & Komatsu 2009, Y^3W in prep.

Beyond linear theory: Exact fluid description for CDM...

Continuity

$$\dot{\delta}_c + \nabla \cdot [(1+\delta_c)\boldsymbol{u}_c] = 0$$

Euler

$$\dot{\boldsymbol{u}}_c + H\,\boldsymbol{u}_c + (\boldsymbol{u}_c \cdot \nabla)\boldsymbol{u}_c + \nabla \Phi + \frac{1}{\rho_c}\nabla_i(\rho_c \sigma_{ij}^2) = 0$$

Pressure and anisotropic stess

- cf. linear approximation:

$$\dot{\delta}_c + \theta_c = 0$$

$$\dot{\theta}_c + H\,\theta_c + \nabla^2 \Phi = 0, \qquad \theta_c \equiv \nabla \cdot \boldsymbol{u}_c$$

Assumption 1: No pressure or anisotropic stress...

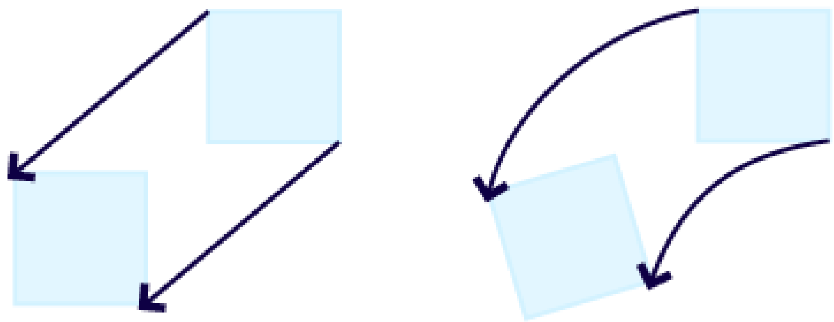
- We assume **no** pressure and anisotropic stress:

$$\sigma_{ij} = 0$$

  - i.e., CDM flow is single-stream and has zero velocity dispersion.
    - **Justifiable** at the initial stages of nonlinear evolution.
    - **Breaks down** in a fully nonlinear, virialised system ($k > 1\ \mathrm{Mpc}^{-1}$ at $z=0$).

Assumption 2: Curl-free flow...

- We track **only** the divergence of the velocity field: $\theta \equiv \nabla \cdot \boldsymbol{u}$
  - Neglect vorticity (i.e., curl): $\boldsymbol{w} \equiv \nabla \times \boldsymbol{u}$



No vorticity With vorticity

  - We expect vorticity on sufficiently nonlinear scales.

Assumption 2: Curl-free flow...

- We track **only** the divergence of the velocity field: $\theta \equiv \nabla \cdot \boldsymbol{u}$
  - Neglect vorticity (i.e., curl): $\boldsymbol{w} \equiv \nabla \times \boldsymbol{u}$

- Vorticity equation of motion:

$$\dot{\boldsymbol{w}} + H\,\boldsymbol{w} - \nabla \times (\boldsymbol{u} \times \boldsymbol{w}) = \nabla \times \left(\frac{1}{\rho} \nabla_i \sigma_{ij}^2 \right)$$

  - In the linear regime, vorticity always decays.
  - In the nonlinear regime, if σ_{ij} remains zero, **no** vorticity will be generated.

The master equations...

- In Fourier space:

Vertex

$$\gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\left[1+\frac{\boldsymbol{q}_1\cdot\boldsymbol{q}_2}{2q_1q_2}\left(\frac{q_1}{q_2}+\frac{q_2}{q_1}\right)\right]$$

Continuity eqn

$$\dot{\delta}_c(\boldsymbol{k},\tau)+\theta_c(\boldsymbol{k},\tau) = -\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{121}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\delta_c(\boldsymbol{q}_2,\tau)$$

Euler eqn

Mode coupling

$$\dot{\theta}_c(\boldsymbol{k},\tau)+H\theta_c(\boldsymbol{k},\tau)-k^2\Phi(\boldsymbol{k},\tau) = -\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\theta_c(\boldsymbol{q}_1,\tau)\theta_c(\boldsymbol{q}_2,\tau)$$

Poisson eqn

$$k^2\Phi = -\frac{3}{2}H^2\sum_i \Omega_i(\tau)\delta_i(\boldsymbol{k},\tau)$$

Vertex

$$\gamma_{222}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \delta_D(\boldsymbol{k}-\boldsymbol{q}_{12})\frac{q_{12}^2(\boldsymbol{q}_1\cdot\boldsymbol{q}_2)}{2q_1^2q_2^2}$$

= Density perturbations
= Velocity divergence

Starting point of **most** semi-analytic calculations in the literature.

- Define:

$$\varphi(\boldsymbol{k},\tau) \equiv \begin{pmatrix} \delta_c(\boldsymbol{k},\tau) \\ -\theta_c(\boldsymbol{k},\tau) \end{pmatrix}$$

- Rewrite Continuity, Euler and Poisson equations as:

$$\partial_\tau \varphi_a(\boldsymbol{k},\tau) + \Pi_{ab}\varphi_b(\boldsymbol{k},\tau) = \int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{abc}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\varphi_b(\boldsymbol{q}_1,\tau)\varphi_c(\boldsymbol{q}_2,\tau)$$

$$\Pi = \begin{bmatrix} 0 & -1 \\ H & -\frac{3}{2}H^2(\Omega_c + \sum_{i\neq c}\Omega_i\delta_i/\delta_c) \end{bmatrix}$$

- Define:

$$\varphi(\boldsymbol{k},\tau) \equiv \begin{pmatrix} \delta_c(\boldsymbol{k},\tau) \\ -\theta_c(\boldsymbol{k} \ldots \end{pmatrix}$$

- Rewrite Continu…

$$\partial_\tau \varphi_a(\boldsymbol{k} \ldots \boldsymbol{q}_2, \tau)$$

WARNING!

Semi-analytic approaches are valid only in the mildly nonlinear regime ($k < 1$ Mpc^{-1}); they **cannot** describe the fully nonlinear regime ($k > 1$ Mpc^{-1}).

$$\Pi = \begin{bmatrix} 0 & \ldots \\ H & -\frac{3}{2} H^2 (\Omega_c + \sum_{i \neq c} \Omega_i \delta_i / \delta_c) \end{bmatrix}$$

3.2.1 Standard perturbation theory...

Standard perturbation theory...

- Solve by perturbative expansion:

$$\varphi(\boldsymbol{k},\tau)=\sum_{n=1}^{\infty}\varphi^{(n)}(\boldsymbol{k},\tau)$$

- nth order equation of motion:

$$\partial_\tau \varphi_a^{(n)}(\boldsymbol{k},\tau)+\Pi_{ab}\varphi_b^{(n)}(\boldsymbol{k},\tau)=\int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{abc}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\sum_{m=1}^{n-1}\varphi_b^{(n-m)}(\boldsymbol{q}_1,\tau)\varphi_c^{(m)}(\boldsymbol{q}_2,\tau)$$

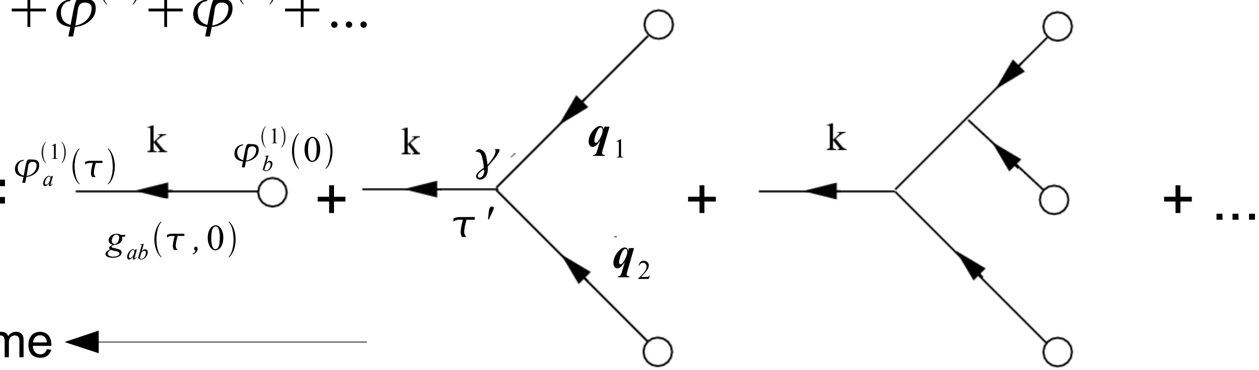
$$\varphi_a^{(n)}(\boldsymbol{k},\tau)=g_{ab}(\tau,0)\varphi_b^{(n)}(\boldsymbol{k},0)$$
$$+\int d^3\boldsymbol{q}_1\int d^3\boldsymbol{q}_2\int_0^\tau d\tau' g_{ab}(\tau,\tau')\gamma_{bcd}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\sum_{m=1}^{n-1}\varphi_c^{(n-m)}(\boldsymbol{q}_1,\tau')\varphi_d^{(m)}(\boldsymbol{q}_2,\tau')$$

Diagrams...

Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

$$\varphi(\boldsymbol{k}) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \dots$$

Density/
Velocity =



Linear

$$\varphi_a^{(1)}(\boldsymbol{k},\tau) = g_{ab}(\tau,0)\,\varphi_b^{(1)}(\boldsymbol{k},0)$$

Linear propagator
≈ Linear growth function

2nd order

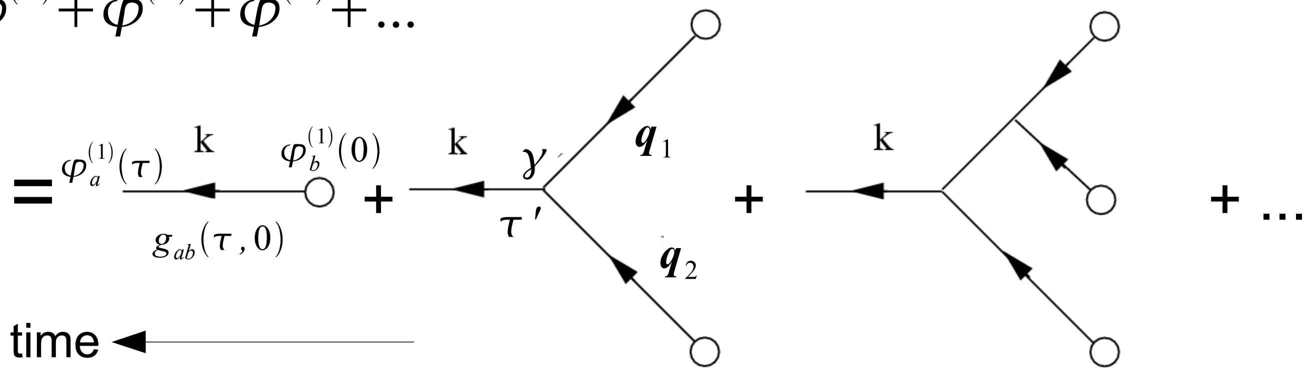
$$\varphi_a^{(2)}(\boldsymbol{k},\tau) = \int d^3\boldsymbol{q}_1 \int d^3\boldsymbol{q}_2 \int_0^\tau d\tau'\, g_{ab}(\tau,\tau')\,\gamma_{bcd}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2) \times g_{ce}(\tau',0)\,\varphi_e^{(1)}(\boldsymbol{q}_1,0)\,g_{df}(\tau',0)\,\varphi_f^{(1)}(\boldsymbol{q}_2,0)$$

Diagrams...

Crocce & Scoccimarro 2006
Matarrese & Pietroni 2007

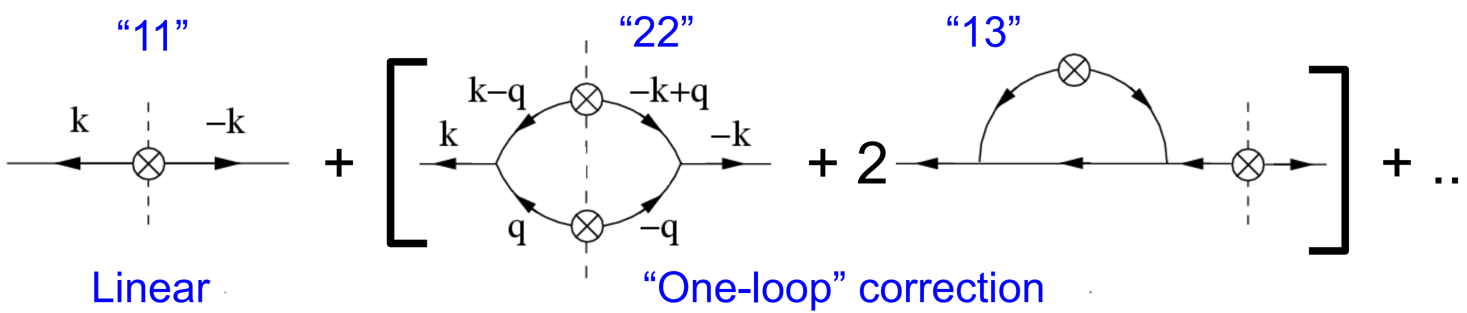
$$\varphi(\boldsymbol{k}) = \varphi^{(1)} + \varphi^{(2)} + \varphi^{(3)} + \ldots$$

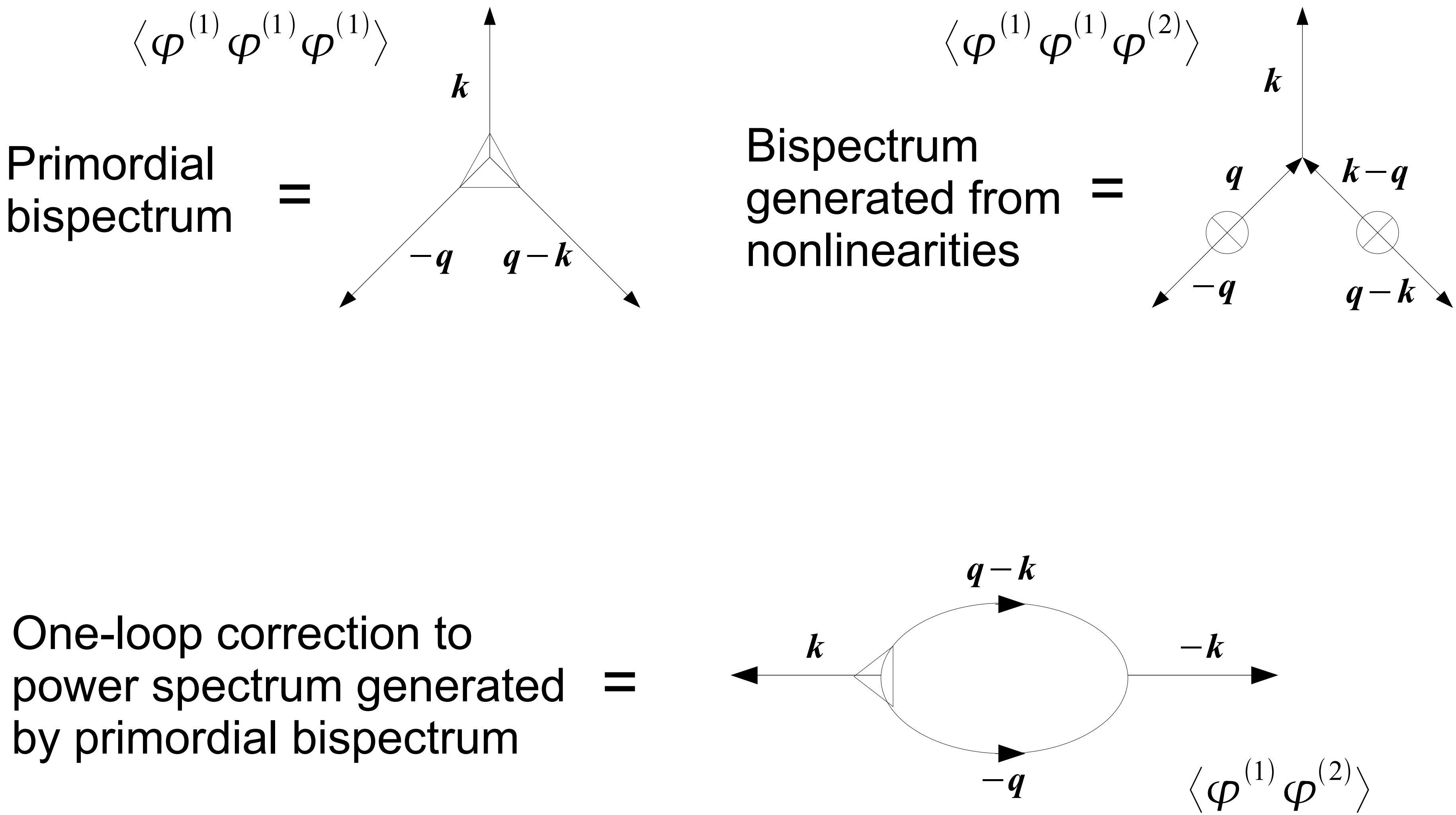
Density/
Velocity =



$$P(k)\,\delta_D(\boldsymbol{k}+\boldsymbol{k}') \equiv \langle \varphi(\boldsymbol{k})\,\varphi(\boldsymbol{k}') \rangle = \langle \varphi^{(1)}\varphi^{(1)} \rangle + [\langle \varphi^{(2)}\varphi^{(2)} \rangle + 2\langle \varphi^{(1)}\varphi^{(3)} \rangle] + \ldots$$

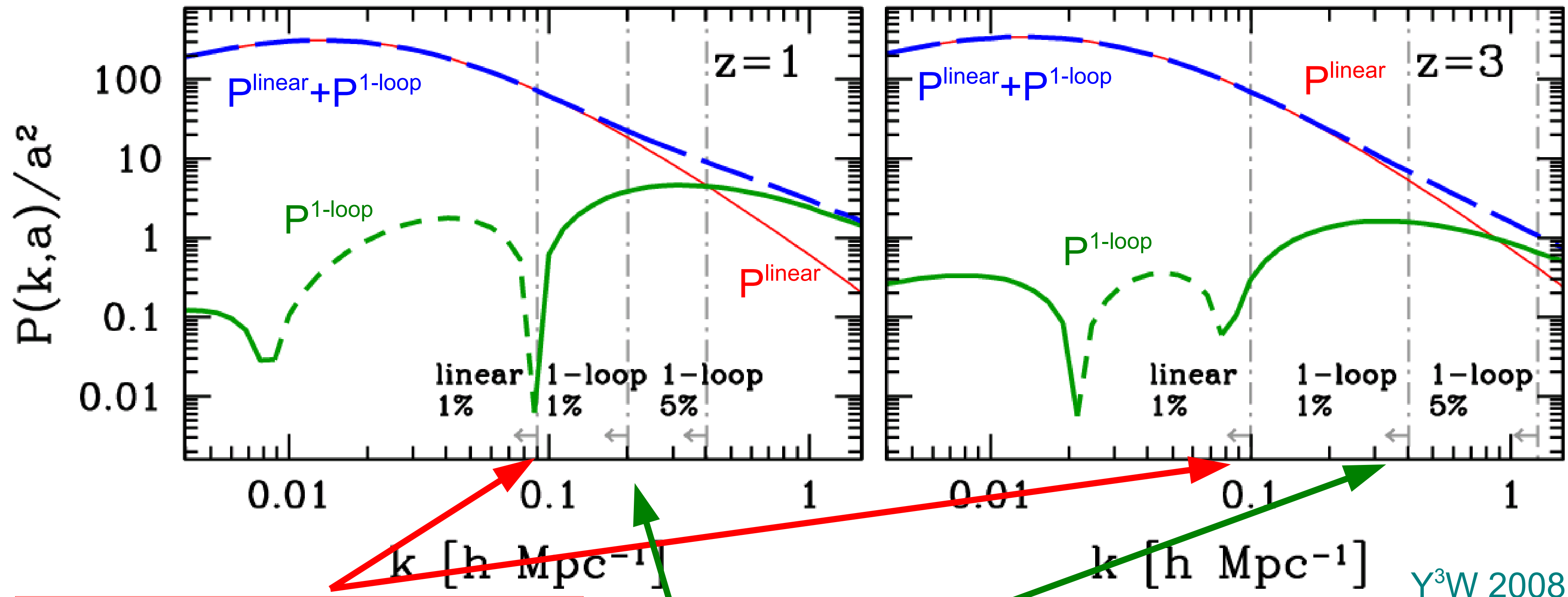
Power
spectrum =





$\langle \varphi^{(1)} \varphi^{(1)} \varphi^{(1)} \rangle$
Primordial bispectrum =
$\boldsymbol{k}$
$-\boldsymbol{q}$
$\boldsymbol{q}-\boldsymbol{k}$
$\langle \varphi^{(1)} \varphi^{(1)} \varphi^{(2)} \rangle$
Bispectrum generated from nonlinearities =
$\boldsymbol{k}$
$\boldsymbol{q}$
$\boldsymbol{k}-\boldsymbol{q}$
$-\boldsymbol{q}$
$\boldsymbol{q}-\boldsymbol{k}$
One-loop correction to power spectrum generated by primordial bispectrum =
$\boldsymbol{q}-\boldsymbol{k}$
$\boldsymbol{k}$
$-\boldsymbol{k}$
$-\boldsymbol{q}$
$\langle \varphi^{(1)} \varphi^{(2)} \rangle$

One-loop corrected total matter power spectrum...

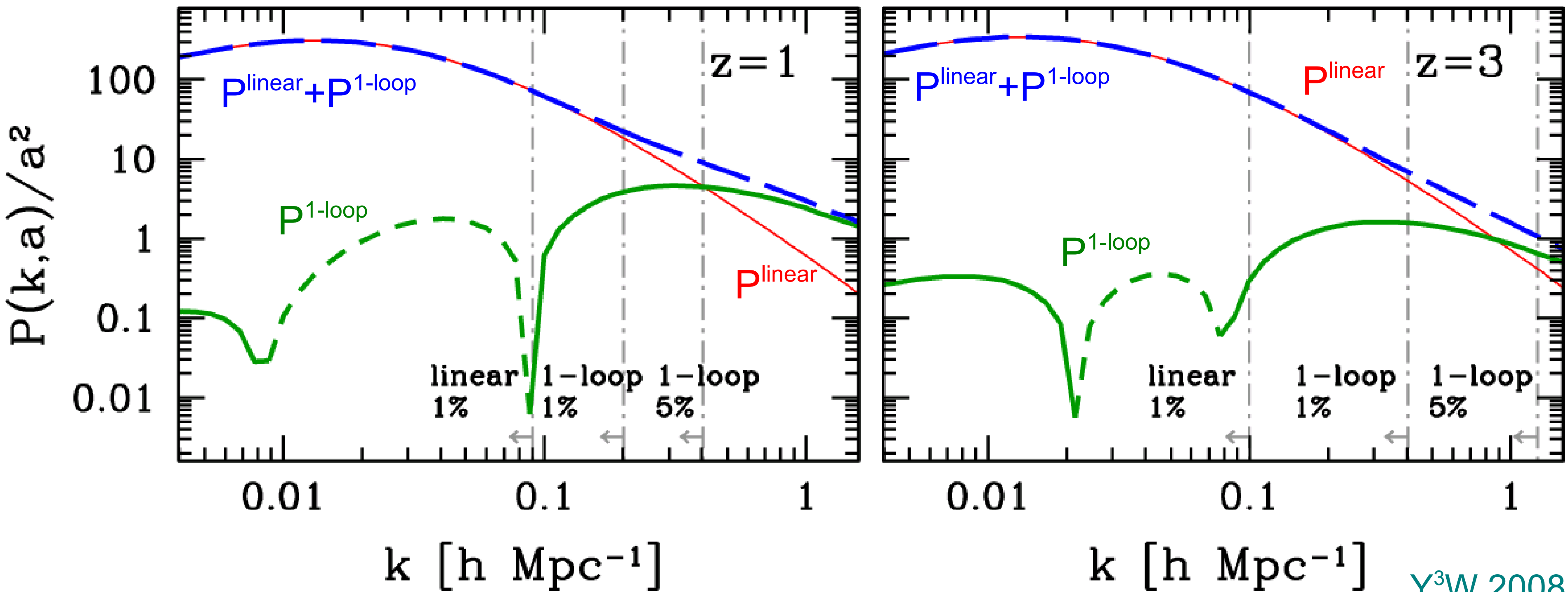


1-loop correction contributes 1% of total power spectrum.

Corrected spectrum agrees with N-body simulations to 1%.

One-loop correction improves the 1% accurate k range by a factor of 2 at z=1 and 4 at z=3.

One-loop corrected total matter power spectrum...



Y³W 2008

Total number of independent modes in a 3D survey $= \int_{k_{min}}^{k_{max}} V \frac{k^2\, dk}{2\pi^2} \sim k_{max}^3$

Maximum usable k

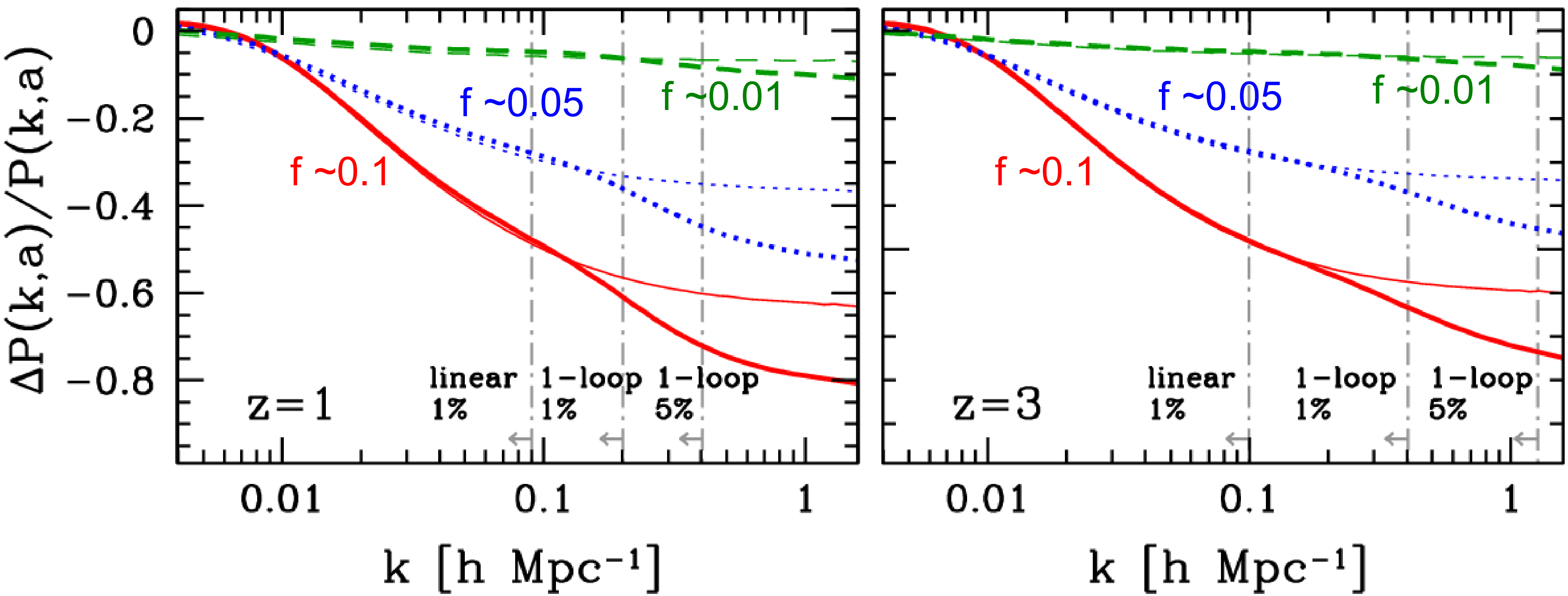
Statistical power $\propto k_{max}^{3/2}$

Change in power spectrum relative to the f = 0 case:

$$\frac{\Delta P}{P} \equiv \frac{P_{f_\nu \neq 0}(k) - P_{f_\nu = 0}(k)}{P_{f_\nu = 0}(k)}$$

Thin lines = linear

Thick lines = 1-loop corrected



Y^{3}W 2008

also Saito et al. 2008

3.2.2 Beyond standard perturbation theory...

Resummation and renormalisation group techniques...

- Many schemes have been proposed that go beyond standard perturbation theory:

  Crocce & Scoccimarro 2006, 2008

  Taruya & Hiramatsu 2007

  McDonald 2007

  Matarresse & Pietroni 2007, 2008

  Matsubara 2008

  Valageas 2007

  Pietroni 2008

  Hiramatsu & Taruya 2009

  etc..

The time renormalisation group flow approach... Pietroni 2008

- Recall the master equation:

$$\partial_\tau \varphi_a(\boldsymbol{k},\tau) + \Pi_{ab}\varphi_b(\boldsymbol{k},\tau) = \int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{abc}(\boldsymbol{k},\boldsymbol{q}_1,\boldsymbol{q}_2)\varphi_b(\boldsymbol{q}_1,\tau)\varphi_c(\boldsymbol{q}_2,\tau)$$

- Construct EoM for the 2-point function $\langle\varphi_a(\boldsymbol{k})\varphi_b(\boldsymbol{q})\rangle \equiv \delta_D(\boldsymbol{k}+\boldsymbol{q})P_{ab}(k)$:

Power spectrum

$$\begin{aligned}\partial_\tau\langle\varphi_a\varphi_b\rangle &= \langle\varphi_a(\partial_\tau\varphi_b)\rangle + \langle(\partial_\tau\varphi_a)\varphi_b\rangle \\ &= -\Pi_{bc}\langle\varphi_a\varphi_c\rangle - \Pi_{ac}\langle\varphi_b\varphi_c\rangle \\ &\quad + \int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \gamma_{bcd}\underline{\langle\varphi_a\varphi_c\varphi_d\rangle} + \gamma_{acd}\underline{\langle\varphi_b\varphi_c\varphi_d\rangle}\end{aligned}$$

3-point function

- EoM for the 3-point function $\langle \varphi_a(\boldsymbol{k})\varphi_b(\boldsymbol{q})\varphi_c(\boldsymbol{r})\rangle \equiv \delta_D(\boldsymbol{k}+\boldsymbol{q}+\boldsymbol{r}) B_{abc}(\boldsymbol{k},\boldsymbol{q},\boldsymbol{r})$:

Bispectrum

$$\partial_\tau \langle \varphi_a \varphi_b \varphi_c \rangle = -\Pi_{bd}\langle \varphi_a \varphi_c \varphi_d \rangle - \Pi_{ad}\langle \varphi_b \varphi_c \varphi_d \rangle - \Pi_{cd}\langle \varphi_a \varphi_b \varphi_d \rangle + \int d^3\boldsymbol{q}_1 d^3\boldsymbol{q}_2 \, \gamma_{bde} \underline{\langle \varphi_a \varphi_c \varphi_d \varphi_e \rangle} + \gamma_{ade} \underline{\langle \varphi_b \varphi_c \varphi_d \varphi_e \rangle} + \gamma_{cde} \underline{\langle \varphi_a \varphi_b \varphi_d \varphi_e \rangle}$$

4-point function

- In general, nonlinear coupling gives rise to an infinite hierarchy of differential equations for the n-point functions which must be **truncated**.

- **Truncate** hierarchy by setting the trispectrum to **zero**. Pietroni 2008
  - 4-point function:

2-point function

$$\langle \varphi_a \varphi_b \varphi_c \varphi_d \rangle = \langle \varphi_a \varphi_b \rangle \langle \varphi_c \varphi_d \rangle + \langle \varphi_a \varphi_c \rangle \langle \varphi_b \varphi_d \rangle + \langle \varphi_a \varphi_d \rangle \langle \varphi_b \varphi_c \rangle + \langle \varphi_a \varphi_b \varphi_c \varphi_d \rangle_{\mathrm{c}}$$

The connected piece:
the "trispectrum"

$$\langle \varphi_a(\boldsymbol{k}) \varphi_b(\boldsymbol{q}) \varphi_c(\boldsymbol{p}) \varphi_d(\boldsymbol{r}) \rangle_{\mathrm{c}} \equiv \delta_D(\boldsymbol{k}+\boldsymbol{q}+\boldsymbol{p}+\boldsymbol{r})\, \underline{Q_{abcd}(\boldsymbol{k},\boldsymbol{q},\boldsymbol{p},\boldsymbol{r})}$$

- **Final** set of equations to be solved numerically:

$$\partial_\tau P_{ab}(\mathbf{k},\tau) = -\Pi_{ac} P_{bc}(\mathbf{k},\tau) - \Pi_{bc} P_{ac}(\mathbf{k},\tau)$$
$$\int d^3\mathbf{q}\, \gamma_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) B_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau)$$
$$+ B_{acd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau)\gamma_{bcd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k})$$

P = Power spectrum

$$\partial_\tau B_{abc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau) = -\Pi_{ad}(\mathbf{k},\tau) B_{dbc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau)$$
$$-\Pi_{bd}(-\mathbf{q},\tau) B_{adc}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau)$$
$$-\Pi_{cd}(\mathbf{q}-\mathbf{k},\tau) B_{abd}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k};\tau)$$
$$+2[\gamma_{ade}(\mathbf{k},-\mathbf{q},\mathbf{q}-\mathbf{k}) P_{bd}(\mathbf{q},\tau) P_{ce}(\mathbf{k}-\mathbf{q},\tau)$$
$$+\gamma_{bde}(-\mathbf{q},\mathbf{q}-\mathbf{k},\mathbf{k}) P_{cd}(\mathbf{k}-\mathbf{q},\tau) P_{ae}(\mathbf{k},\tau)$$
$$+\gamma_{cde}(\mathbf{q}-\mathbf{k},\mathbf{k},-\mathbf{q}) P_{ad}(\mathbf{k},\tau) P_{be}(\mathbf{q},\tau)]$$

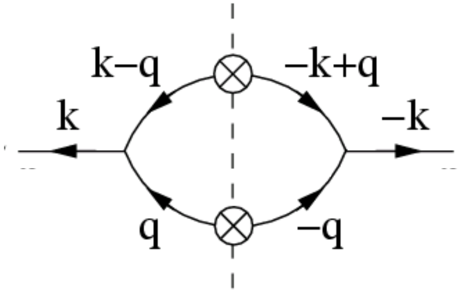
B = Bispectrum

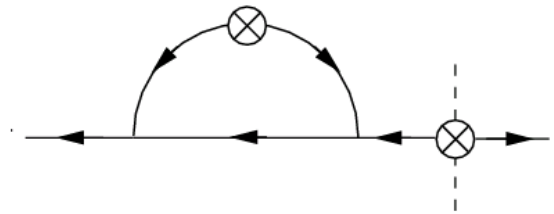
= Cosmology dependent matrix

= Vertex function

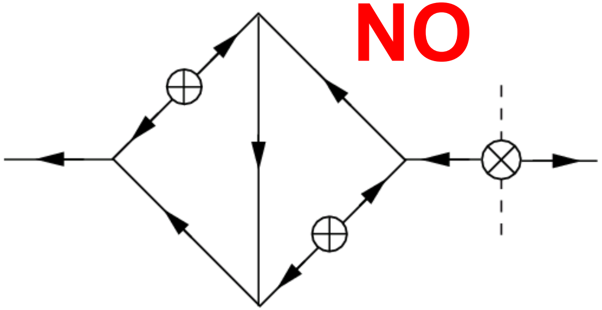
Pietroni 2008

- Truncation at the trispectrum is equivalent to summing to all orders diagrams that can be constructed from the one loop diagrams:

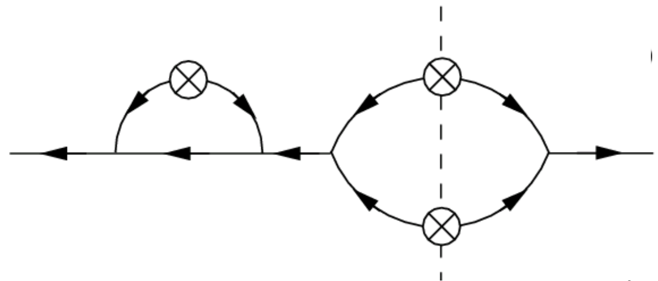




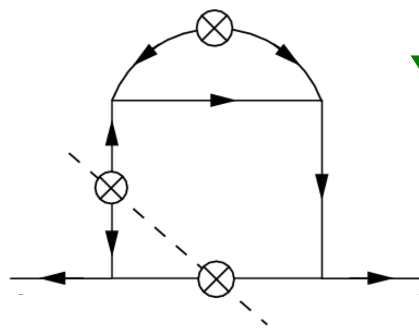
- e.g., two-loop:

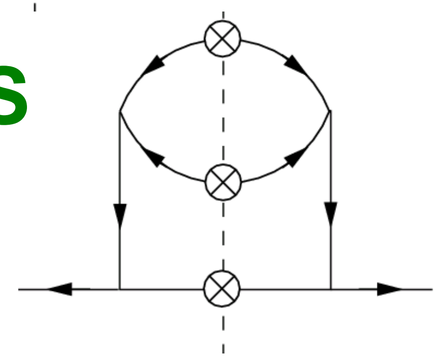


NO

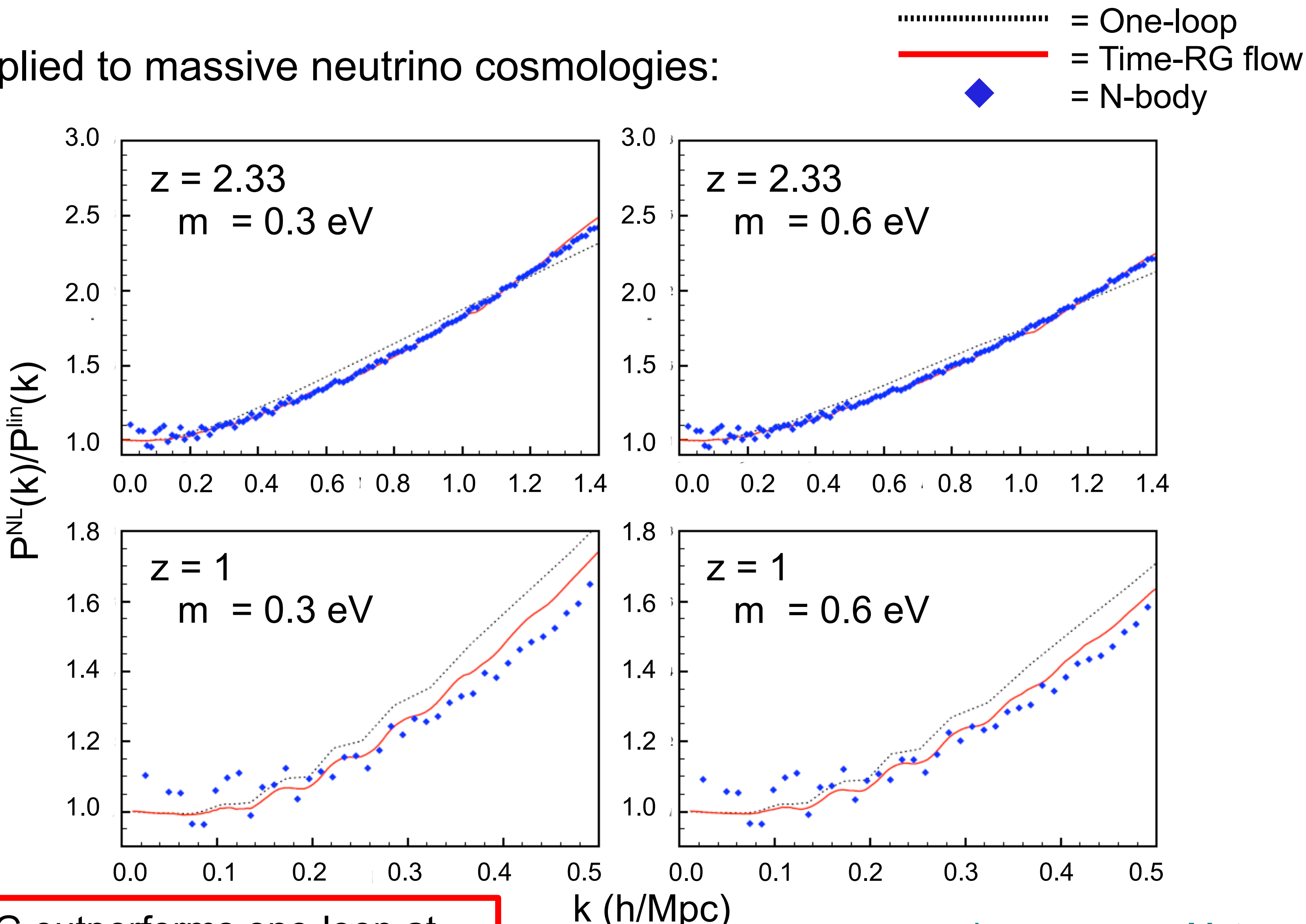


YES





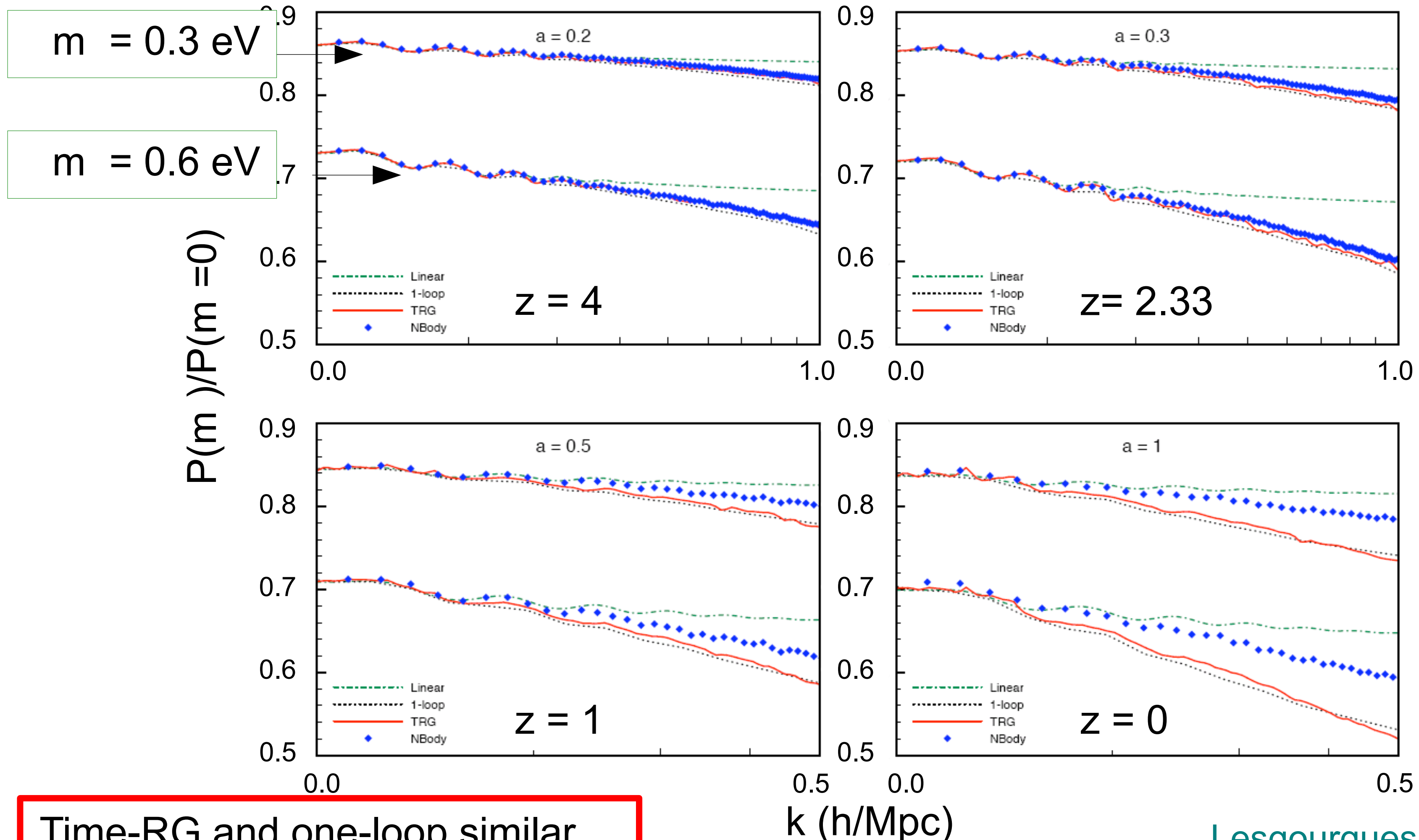
- Applied to massive neutrino cosmologies:



Time-RG outperforms one-loop at predicting absolute power spectrum.

Lesgourgues, Matarrese, Pietroni & Riotto 2009

- Applied to massive neutrino cosmologies:



Time-RG and one-loop similar at predicting power suppression.

Lesgourgues, Matarrese, Pietroni & Riotto 2009

Summary...

- Many future cosmological probes will derive their constraining power from measurements at nonlinear scales.

- We must make sure our theoretical predictions are reliable (1% accurate) at these scales.

  - N-body simulations are the definitive way to go.

  - Semi-analytic approaches (Higher order perturbation theory & renormalisation group techniques) are also of some (limited) use.