SUPERSYMMETRY

WHY and HOW

J. Iliopoulos, LPTENS, Pohang, June 2009

List of topics

- Why a new symmetry

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- Why a new symmetry
- What is SUPERSYMMETRY

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- Why a new symmetry
- What is SUPERSYMMETRY
- SUSY's wonderful properties

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- SUSY's wonderful properties
- The breaking of Supersymmetry
- Where is Supersymmetry
- Conclusions in two years

THE STANDARD MODEL

HAS BEEN ENORMOUSLY SUCCESSFUL

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Gauge theories have come to stay

The purpose of this talk is not to abolish but to fulfill

The trial of scalars

-Gauge theories contain three independent worlds
Radiation-Matter-Higgs
Increasing arbitrariness. No connexion with geometry. New couplings. Loss of asymptotic freedom. Huge hierarchy problem.

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Radiation-Matter-Higgs
Increasing arbitrariness. No connexion with geometry. New couplings. Loss of asymptotic freedom. Huge hierarchy problem.

Remedy: Throw away the scalars!
In QCD the spontaneous breaking of chiral symmetry does not involve fundamental scalar fields.
Technicolor,.....

The defence of scalars

- A better solution: Connect the three worlds

The defence of scalars

- A better solution: Connect the three worlds
- Supersymmetry

Supersymmetry

An infinitesimal transformation acting on a set of fields $\phi^i(x), i = 1, ..., m$

$$\delta\phi^i(x) = \epsilon^a (T_a)^i_j \phi^j(x)$$

-If the ϵ's are c-numbers, the transformation mixes only fields with the same spin and obeying the same statistics.

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-If they have non-zero integer spin they can mix scalars with vectors, or spin-1/2 with spin-3/2 fields.
(Relativistic $SU(6)$, Impossible)

Supersymmetry

-If they are anti-commuting, zero spin, parameters, they will mix fermions with bosons without changing their spins.

$$\text{Particles} \longleftrightarrow \text{Ghosts}$$

(B.R.S. transformations)

Supersymmetry

-If they are anti-commuting, zero spin, parameters, they will mix fermions with bosons without changing their spins.

Particles $\longleftrightarrow$ Ghosts

(B.R.S. transformations)

-If they are anti-commuting spinors, they connect physical bosons with physical fermions.

Supersymmetry

The supersymmetry algebra

- $A_m \; m = 1, ..., D$: The generators of a Lie algebra

  $Q_\alpha \; \alpha = 1, ..., d$: The elements of a d-dimensional representation

$$[A_m, A_n] = f^l_{mn} A_l \quad ; \quad [A_m, Q_\alpha] = s^\beta_{m\alpha} Q_\beta$$

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- A graded superalgebra : Find numbers $r^m_{\alpha\beta}$ such that:

$$[Q_\alpha, Q_\beta]_+ = Q_\alpha Q_\beta + Q_\beta Q_\alpha = r^m_{\alpha\beta} A_m$$

The supersymmetry algebra

-The Lie algebra is the Poincaré algebra with generators P_μ and $M_{\mu\nu}$

-The grading representation is given by a Majorana spinor Q_α

$$[P_\mu, Q_\alpha] = 0 \quad ; \quad [Q_\alpha, M_{\mu\nu}] = i\gamma^{\mu\nu}_{\alpha\beta} Q_\beta$$

$$[Q_\alpha, \bar{Q}_\beta]_+ = -2\gamma^\mu_{\alpha\beta} P_\mu$$

$\gamma^{\mu\nu} = \frac{1}{4}[\gamma^\mu, \gamma^\nu]$.

-Admits the Lorentz group $SL(2, C)$ as an automorphism

Using Weyl spinors

$$[Q_\alpha, Q_\beta]_+ = [\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}]_+ = 0 \quad ; \quad [Q_\alpha, \bar{Q}_{\dot\beta}]_+ = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$$

Generalisation

-Start from the Poincaré algebra $\times$ a compact internal symmetry G with generators A_i.
-If the Q's belong to a certain representation of G

$$[A_i, A_j] = f_{ij}^k A_k \quad ; \quad [P_\mu, Q_\alpha^m] = 0 \quad ; \quad [Q_\alpha^m, M_{\mu\nu}] = i\gamma_{\alpha\beta}^{\mu\nu} Q_\beta^m$$

$$[A_i, Q_\alpha^m] = s_{in}^m Q_\alpha^n \quad ; \quad [Q_\alpha^m, \bar{Q}_\beta^n]_+ = -2\delta^{mn}\gamma_{\alpha\beta}^\mu P_\mu$$

-Admits $SL(2,C) \times G$ as a group of automorphisms.

All possible supersymmetries of the S matrix
(Haag, Lopusansky, Sohnius)
The only possible extension

$$[Q^m_\alpha, Q^n_\beta]_+ = \epsilon_{\alpha\beta} Z^{mn}$$

where Z^{mn} are a set of central charges, $i.e.$ operators which commute with every operator in the algebra

Representations

(In terms of one-particle states)

-The Q's commute with $P_\mu \longrightarrow$ they do not change the momentum of the one-particle state

-P^2 commutes with all the operators of the algebra $\rightarrow$ all the members of a supermultiplet will have the same mass

Representations

(i) Massive case

-Go to the rest frame

$$[Q_\alpha, \bar{Q}_{\dot{\beta}}]_+ = 2M\delta_{\alpha\dot{\beta}}$$

$$\downarrow$$

$Q/\sqrt{2M}$ and $\bar{Q}/\sqrt{2M}$ satisfy the anti-commutation relations for creation and annihilation operators of free fermions. Starting from any one-particle state with spin S and projection S_z, we can build a four-dimensional Fock space with states

$$|S, S_z; n_1, n_2 >= Q_2^{n_2} Q_1^{n_1} |S, S_z > \qquad n_1, n_2 = 0, 1$$

Representations

(i) Massive case

We can define a parity operation under which the Majorana spinor Q_α, $\alpha = 1, ...4$ transforms as

$$(Q_\alpha)_P = (\gamma^0 Q)_\alpha$$

Then, the spin-parity content of the representation is:

$$(S-1/2)^\eta \quad ; \quad S^{i\eta} \quad ; \quad S^{-i\eta} \quad ; \quad (S+1/2)^{-\eta}$$

where $\eta = \pm i, \pm 1$ for S integer or half-integer

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- S=1 : a vector, a pseudo-vector, a spinor, a 3/2 spinor
- S=3/2 : a vector, a tensor, two 3/2 spinors.

The generalisation to include internal symmetries is straightforward. The difference is that now we have more creation operators and the corresponding Fock space has 2^{2N} independent states, where N is the number of spinorial charges.

Representations

(ii) Massless case

Choose the frame $P_\mu = (E,0,0,E)$

$$[Q_\alpha, \bar{Q}_{\dot{\beta}}]_+ = 2E(1-\sigma_z) = 4E\delta_{\alpha 2}\delta_{\dot{\beta}2}$$

$$\downarrow$$

Only Q_2 and $\bar{Q}_{\dot{2}}$ can be considered as creation and annihilation operators. Starting from a one-particle state with helicity $\pm\lambda$, we obtain the state with helicity $\pm(\lambda+1/2)$

Examples:

λ = 1/2 : one spin 1/2 and one spin 1 (both massless)

λ = 3/2 : one spin 3/2 and one spin 2 (both massless)

Representations

- A well- established theoretical prejudice is that, if one excludes gravitation, there exist no elementary particles with spin higher than one

$$\downarrow$$

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  N = 4 is the largest supersymmetry which may be interesting for particle physics without gravitation.

- If we include gravitation, N = 8 is the maximum allowed supersymmetry
- All representations contain equal number of bosonic and fermionic states. All states in an irreducible representation have the same mass

Representations

(In terms of local fields)

- A “chiral” super-multiplet:

$$\phi_1(x,\theta) = A(x) + \theta\psi(x) + \theta\theta F(x)$$

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- A "chiral" super-multiplet:

$$\phi_1(x,\theta) = A(x) + \theta\psi(x) + \theta\theta F(x)$$

- A "vector" super-multiplet:

$$\begin{aligned}\phi(x,\theta,\bar\theta) = \; & C \quad + i\theta\chi - i\bar\theta\bar\chi + \frac{i}{2}\theta\theta(M+iN) - \frac{i}{2}\bar\theta\bar\theta(M-iN) \\ - & \quad \theta\sigma_\mu\bar\theta v^\mu \\ + & \quad i\theta\theta\bar\theta_{\dot\alpha}(\bar\lambda^{\dot\alpha} - \frac{i}{2}\sigma^{\dot\alpha}_{\mu\alpha}\partial^\mu\chi^\alpha) - i\bar\theta\bar\theta\theta(\lambda + \frac{i}{2}\sigma_\mu\partial^\mu\bar\chi) \\ + & \quad \frac{1}{2}\theta\theta\bar\theta\bar\theta(D + \frac{1}{2}\Box C)\end{aligned}$$

The transformation properties are:

$$\delta A = \xi\psi \;\; ; \;\; \delta\psi = 2i\sigma_\mu\bar{\xi}\partial^\mu A + 2\xi F \;\; ; \;\; \delta F = i\partial^\mu\psi\sigma_\mu\bar{\xi}$$

Three remarks:
1) We have more fields than the physical one-particle states
$\longrightarrow$ Some of the fields must turn out to be auxiliary fields.
2) The field F transforms, under supersymmetry, with a total derivative.
3) We can build a “tensor calculus”

Field Theory

The simplest field theory model: It involves only one chiral super-multiplet.

$$
\begin{aligned}
\mathcal{L} = \quad & - \quad \frac{1}{2}[(\partial A)^2 + (\partial B)^2 + i\bar{\psi}\gamma_\mu\partial^\mu\psi - F^2 - G^2] \\
& + \quad m[FA + GB - \frac{i}{2}\bar{\psi}\psi] \\
& + \quad g[F(A^2 - B^2) + 2GAB - i\bar{\psi}(A - \gamma_5 B)\psi]
\end{aligned}
$$

$A \longrightarrow 1/2(A + iB) \qquad F \longrightarrow 1/2(F + iG) \qquad \psi_{Weyl} \longrightarrow \psi_{Maj.}$

Field Theory

F and G are auxiliary fields and can be eliminated using the equations of motion:

$$F + mA + g(A^2 - B^2) = 0 \quad ; \quad G + mB + 2gAB = 0$$

$$\begin{aligned}\mathcal{L} = & - \frac{1}{2}[(\partial A)^2 + (\partial B)^2 + i\bar{\psi}\gamma_\mu \partial^\mu \psi] - \frac{1}{2}m^2(A^2 + B^2) - \frac{i}{2}m\bar{\psi}\psi] \\ & - mgA(A^2 + B^2) - ig\bar{\psi}(A - \gamma_5 B)\psi - \frac{1}{2}g^2(A^2 + B^2)^2\end{aligned}$$

It is a renormalisable theory.
It describes Yukawa, trilinear and quartic couplings among a Majorana spinor, a scalar and a pseudoscalar.

Field Theory

All fields have a common mass. All interactions are described in terms of a single coupling constant. Supersymmetry implies the conservation of a spin 3/2 current

$$J^\mu = \gamma^\lambda \partial_\lambda (A - \gamma_5 B)\gamma_\mu \psi - (F + \gamma_5 G)\gamma_\mu \psi$$

Strictly speaking one could add a term linear in the field F:

$$\mathcal{L} \longrightarrow \mathcal{L} + \lambda F$$

It has no effect in the model because it can be eliminated by a shift in the field A.

Field Theory

Renormalisation:
Supersymmetry $\rightarrow$ Ward identities $\rightarrow$

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- Non-renormalisation Theorems

Field Theory

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- This non-renormalisation theorem makes supersymmetry so central in all attempts to go beyond the Standard Model. It is the only known way to solve the technical part of the Hierarchy problem.
- The vanishing of the coupling constant renormalisation counterterm is special to this particular model.

Supersymmetry and gauge invariance

The supersymmetric extension of quantum electrodynamics
If v_μ is the photon field and ϕ_1 and ϕ_2 the real and imaginary parts of a charged field, an infinitesimal gauge transformation is given by:

$$\delta v_\mu = \partial_\mu \Lambda \quad ; \quad \delta\phi_1 = e\Lambda\phi_2 \quad ; \quad \delta\phi_2 = -e\Lambda\phi_1$$

Supersymmetry $\rightarrow$ replace v_μ by a whole vector multiplet
Choose the matter fields to be a charged chiral multiplet
Supersymmetric QED describes simultaneously the interaction of photons with charged scalars, pseudoscalars and spinors.
In the gauge transformation the function Λ must be replaced by a whole chiral multiplet.

Supersymmetry and gauge invariance

$$
\begin{aligned}
\mathcal{L} = \; & - \frac{1}{4}(\partial_\mu v_\nu - \partial_\nu v_\mu)^2 - \frac{i}{2}\bar{\lambda}\gamma^\mu\partial_\mu\lambda + \frac{1}{2}D^2 \\
& - \frac{1}{2}[(\partial A_1)^2 + (\partial A_2)^2 + (\partial B_1)^2 + (\partial B_2)^2 - F_1^2 - F_2^2 - G_1^2 - \\
& + i\bar{\psi}_1\gamma_\mu\partial^\mu\psi_1 + i\bar{\psi}_2\gamma_\mu\partial^\mu\psi_2] \\
& + m[F_1A_1 + F_2A_2 + G_1B_1 + G_2B_2 - \frac{i}{2}\bar{\psi}_1\psi_1 - \frac{i}{2}\bar{\psi}_2\psi_2 \\
& + ev_\mu[i\bar{\psi}_1\gamma^\mu\psi_2 - A_1\partial^\mu A_2 + A_2\partial^\mu A_1 - B_1\partial^\mu B_2 + B_2\partial^\mu B_1] \\
& - \frac{1}{2}e^2v_\mu v^\mu[A_1^2 + A_2^2 + B_1^2 + B_2^2] + eD(A_1B_2 - A_2B_1) \\
& - ie\bar{\lambda}[(A_1 + \gamma_5B_1)\psi_2 - (A_2 + \gamma_5B_2)\psi_1]
\end{aligned}
$$

Supersymmetry and gauge invariance

v_μ is the photon field

λ is the field of the photino

The real Majorana spinors ψ_1 and ψ_2 can be combined together to form a complex Dirac spinor, the field of the electron

A_1, A_2, B_1 and B_2 are the real and imaginary parts of two complex, charged, spin-zero fields, a scalar and a pseudoscalar

They are the supersymmetric partners of the electron, sometimes called “selectrons”

As before, the fields F_1, F_2, G_1, G_2 and D are auxiliary

The Lagrangian is invariant under ordinary gauge transformations

Supersymmetry and gauge invariance

If we eliminate the auxiliary fields, we obtain the usual interaction of a photon with a charged scalar, pseudoscalar and spinor field including the seagull term and the quartic term among the scalar fields.

Supersymmetry has introduced only two new elements:

(i) The coupling constant in front of the quartic self-interaction of the spin-zero fields is not arbitrary, but it is equal to $e^2/2$

(ii) New terms describing Yukawa-type interactions between the Majorana spinor (the photino) and the spin 1/2 and zero fields of the matter multiplet with strength e.

A supersymmetry transformation can be compensated by a gauge transformation, so all physical results will be supersymmetric.

Supersymmetry and gauge invariance

Non-Abelian Yang-Mills theories
Group $SU(m) \to m^2 - 1$ gauge bosons W_μ which can be written as an $m \times m$ traceless matrix.
Their supersymmetric partners, the "gauginos", are $m^2 - 1$ Majorana spinors which we write as another $m \times m$ traceless matrix λ.
The resulting gauge invariant Lagrangian is:

$$\mathcal{L} = Tr[-\frac{1}{4}W_{\mu\nu}^2 - \frac{i}{2}\bar{\lambda}\gamma^\mu \mathcal{D}_\mu \lambda]$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \quad ; \quad \mathcal{D}_\mu \lambda = \partial_\mu \lambda + ig[W_\mu, \lambda]$$

Supersymmetry and gauge invariance

This Lagrangian describes the gauge invariant interaction of $m^2 - 1$ massless Majorana fermions belonging to the adjoint representation of $SU(m)$ with the gauge fields.
The surprising result is that it is automatically supersymmetric!
A supersymmetry transformation can be compensated by a gauge transformation.
The spin 3/2 conserved current is:

$$J^{\mu} = -\frac{1}{2} Tr(W_{\nu\rho}\gamma^{\nu}\gamma^{\rho}\gamma^{\mu}\lambda)$$

Supersymmetry and gauge invariance

We can introduce matter multiplets in the form of chiral superfields belonging to any desired representation of the group.
The one-loop β-function for an $SU(m)$ Yang-Mills supersymmetric theory with n chiral multiplets belonging to the adjoint representation is:

$$\beta(g) = \frac{m(n-3)}{16\pi^2} g^3$$

For $n < 3$, the theory is asymptotically free, although it contains scalar and pseudoscalar particles.

Supersymmetry and gauge invariance

A surprising and probably deep result:
Consider supersymmetric theories with N spinorial generators. For particle physics $N_{max} = 4$.
The most astonishing result is that the β-function of an $N = 4$ supersymmetric Yang-Mills theory based on any group $SU(m)$ vanishes to all orders, the effective coupling constant is scale independent and does not run.
For $N = 2$ we have an intermediate result: the β-function receives only one loop contributions.

The breaking of supersymmetry

Fermions and bosons are not degenerate in nature, so supersymmetry, if it is at all relevant, must be broken.
Supersymmetry is hard to break spontaneously.
We can show that spontaneous symmetry breaking occurs only when one, or more, of the auxiliary fields acquires a non-vanishing vacuum expectation value.
The potential of the scalar fields in the tree approximation has the form:

The breaking of supersymmetry

$$V(\phi) = -\frac{1}{2}\left[\sum F_i^2 + \sum G_i^2 + \sum D_i^2\right] + \left[\sum F_i F_i(\phi) + \sum G_i G_i(\phi) + \sum D_i D_i(\phi)\right]$$

where the functions $F_i(\phi)$, $G_i(\phi)$ and $D_i(\phi)$ are polynomials in the physical fields ϕ of degree not higher than second. The equations which eliminate the auxiliary fields are:

$$F_i = F_i(\phi) \quad ; \quad G_i = G_i(\phi) \quad ; \quad D_i = D_i(\phi)$$

so the potential, in terms of the physical fields, reads:

The breaking of supersymmetry

$$V(\phi) = \frac{1}{2}\left[\sum F_i^2(\phi) + \sum G_i^2(\phi) + \sum D_i^2(\phi)\right]$$

The important observation is that V is non-negative and vanishes only for:

$$F_i(\phi) = 0 \quad ; \quad G_i(\phi) = 0 \quad ; \quad D_i(\phi) = 0$$

It comes from the anti-commutation relations:

$2H = |Q_1|^2 + |Q_2|^2$

A supersymmetric state is always stable.

The breaking of supersymmetry

An example:
Supersymmetric QED. Add a linear term:

$$\mathcal{L} \longrightarrow \mathcal{L} + \xi D$$

Still supersymmetric.

The system of eqs $F_i = G_i = D = 0$ give:

$$mA_1 = 0 \quad ; \quad mA_2 = 0 \quad ; \quad mB_1 = 0 \quad ; \quad mB_2 = 0$$
$$e(A_1 B_2 - A_2 B_1) + \xi = 0$$

The breaking of supersymmetry

No solution $\rightarrow$ Supersymmetry is spontaneously broken.

$$-\frac{1}{2}(m^2+\xi e)(\tilde{A}_1{}^2+\tilde{B}_1{}^2)-\frac{1}{2}(m^2-\xi e)(\tilde{A}_2{}^2+\tilde{B}_2{}^2)$$

The electron has mass m but its partners are split.

λ is the spin-1/2 Goldstone particle

The physics of the Goldstino

-Can one of the neutrinos be an approximate, Goldstino? There is a low-energy theorem, known as “Adler’s zero” satisfied by any Goldstone particle.

A Goldstino $\eta(x)$ has the quantum numbers of the divergence of J_μ, the conserved supersymmetry current

For any two physical states $|a>$ and $|b>$

$$k^\mu < a|J_\mu|b >= 0$$

When $k \to 0$ only the one η state contributes

$$\lim_{k_\mu \to 0} M(k) = 0$$

$M =$ the amplitude $M(a \to b + \eta)$

The physics of the Goldstino

Neutrinos do not satisfy this theorem.

Where is the Goldstino?

(i) “invisible”

(ii) Super-Higgs mechanism

The physics of the Goldstino

(i) R-parity For example:

$$(-)^R = (-)^{2S}(-)^{3(B-L)}$$

$R = 0$ for all known particles, $R = \pm 1$ for their supersymmetric partners.

Can the Goldstino be the partner of a known particle? (ex. the photino)?

$$J_\mu(x) = d\gamma_\mu\gamma_5\eta(x) + \hat{J}_\mu(x)$$

$$d\gamma_5\gamma_\mu\partial^\mu\eta = \partial^\mu\hat{J}_\mu$$

The eq. of motion of the Goldstino.

The physics of the Goldstino

Absence of sp. sym. br. $\rightarrow d = 0$ and $\partial^\mu \hat{J}_\mu = 0$.

Sp. sym. Br. $\rightarrow \partial^\mu \hat{J}_\mu \propto \Delta m^2$

$$f_\eta = \pm \frac{\Delta m^2}{d}$$

f_η is the coupling constant of the Goldstino to a spin-0-spin-1/2 pair. If η is the photino, $f_\eta \propto e$

$$m^2(s_e) + m^2(t_e) = 2m^2(e)$$

The Goldstino is not partner of a known boson.

The physics of the Goldstino

$$\sum_J (-)^{2J}(2J+1)m_J^2 = 0$$

(ii) Super-Higgs mechanism

In the normal Higgs phenomenon

$(m = 0, spin = 1 + m = 0, spin = 0) = (m \neq 0, spin = 1)$

In a super-Higgs mechanism

$(m = 0, spin = 3/2 + m = 0, spin = 1/2) = (m \neq 0, spin = 3/2)$

Natural in supergravity theories.

Supersymmetry and the S.M.

Assume N=1

The physical degrees of freedom in the S.M. are
Bosonic degrees of freedom = 28
Fermionic degrees of freedom = 90 (or 96, with ν_R's)

Super-S.M. introduces new particles

In N=1 all the particles of a given supermultiplet must belong to the same representation of the gauge group. For the S.M.

(i) The gauge bosons (1,8), (3,1), (1,1) $\longrightarrow$ No fermions
(ii) The Higgs take a vev $\longrightarrow$ no fermion quantum nb.

Supersymmetry and the S.M.

Super-S.M. associates
known bosons $\longleftrightarrow$ unknown fermions
known fermions $\longleftrightarrow$ unknown bosons

In S.S.M. complex conjugation on the scalars induces a helicity change of the corresponding spinors. $\rightarrow$
The same Higgs doublet cannot give masses to both up and down quarks $\rightarrow$
A richer spectrum of physical Higgs particles.
3 neutral + 2 charged (the scalar partners of $W's$)

Supersymmetry and the S.M.

Use three types of multiplets:

(i) Chiral multiplets for matter and Higgs fields
1 Weyl fermion + 2 scalars
(ii) Massless vector multiplets
1 Weyl fermion + 1 massless vector
(iii) Massive vector multiplets
1 vector + 1 Dirac fermion + 1 scalar

To solve the hierarchy problem $\Delta m \leq \mathcal{O}(1TeV)$

Supersymmetry and the S.M.

<table>
<tr><th>SPIN-1</th><th>SPIN-1/2</th><th colspan="2">SPIN-0</th></tr>
<tr><td>Gluons</td><td>Gluinos</td><td colspan="2">no partner</td></tr>
<tr><td>Photon</td><td>Photino</td><td colspan="2">no partner</td></tr>
<tr><td>$W^{\pm}$</td><td>2 Dirac Winos</td><td>$w^{\pm}$</td><td rowspan="3">Higgs bosons</td></tr>
<tr><td>Z^0</td><td>2 Majorana Zinos</td><td>z</td></tr>
<tr><td></td><td>1 Majorana Higgsino</td><td>standard ϕ^0
pseudoscalar $\phi^{0\prime}$</td></tr>
<tr><td></td><td>Leptons</td><td colspan="2">Spin-0 leptons</td></tr>
<tr><td></td><td>Quarks</td><td colspan="2">Spin-0 quarks</td></tr>
</table>

Supergravity

Why supergravity

(i) All fundamental symmetries in nature are local (or gauge) symmetries.
(ii) The supersymmetry algebra contains the translations. Local supersymmetry contains general relativity. It may provide the road to a true unification.
(iii) Local supersymmetry provided the most attractive explanation for the absence of a physical Goldstino.
(iv) In a supersymmetric grand unified theory the unification scale approaches the Planck mass (10^{19} GeV) at which gravitational interactions can no more be neglected.

Supergravity

$N = 1$

The gauge fields are the metric tensor $g_{\mu\nu}(x)$ which represents the graviton and a spin-three-half Majorana "gravitino" $\psi_\mu(x)$.

The Lagrangian of G.R. is:

$$\mathcal{L}_{\mathcal{G}} = -\frac{1}{2\kappa^2}\sqrt{-g}R = \frac{1}{2\kappa^2}eR$$

$$g_{\mu\nu}(x) = e_\mu^m(x)e_\nu^n(x)\eta_{\mu\nu}$$

Supergravity

Add to it the Rarita-Schwinger Lagrangian of a spin-3/2 massless field in interaction with gravitation:

$$\mathcal{L}_{\mathcal{RS}} = -\frac{1}{2}\sqrt{-g}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma$$

$$\mathcal{D}_\rho = \partial_\rho + \frac{1}{2}\omega_\rho^{mn}\gamma_{mn} \quad ; \quad \gamma_{mn} = \frac{1}{4}[\gamma_m, \gamma_n]$$

$\omega_\rho^{mn}(x)$ is the spin connection. It is expressed in terms of the vierbein and its derivatives.

Supergravity

The remarkable result is that the sum $\mathcal{L}_{\mathcal{G}} + \mathcal{L}_{\mathcal{RS}}$ gives a theory invariant under local supersymmetry transformations with parameter $\epsilon(x)$:

$$\delta e_\mu^m = \frac{\kappa}{2}\bar{\epsilon}(x)\gamma^m\psi_\mu$$

$$\delta\omega_\mu^{mn} = 0$$

$$\delta\psi_\mu = \frac{1}{\kappa}\mathcal{D}_\mu\epsilon(x) = \frac{1}{\kappa}(\partial_\mu + \frac{1}{2}\omega_\mu^{mn}\gamma_{mn})\epsilon(x)$$

Remember the analogous case with Yang-Mills.

Supergravity

Coupling to matter involves only two arbitrary functions.
Call z the set of complex scalar fields.
(i) $G(z, z^*)$, a real function, invariant under whichever gauge group we have used.
(ii) $f_{ij}(z)$, an analytic function which transforms as a symmetric product of two adjoint representations of the gauge group.

An important part of supersymmetry phenomenology consists in different choices of these two functions.

Supergravity

Spontaneous breaking of local supersymmetry results in a super-Higgs mechanism.
The gravitino absorbs the massless Goldstino and becomes a massive spin three-half field.
At ordinary energies gravitational interactions decouple and the spontaneously broken supergravity behaves like an explicitly but softly broken global supersymmetry.
The details of the final theory, like particle spectra, depend on the initial choice of the functions G and f, but the main features remain the same.
In a spontaneously broken supergravity we can arrange to have $E_{vac} = 0$ and hence $\Lambda = 0$.

Supergravity

$N = 8$

It is the only one which attempts a complete unification.
It is the largest supersymmetry we can consider.
The irreducible representation of one-particle states contains:

$$
\begin{array}{l}
1 \;\; spin - 2 \;\; graviton \\
8 \;\; spin - 3/2 \;\; Majorana \;\; gravitini \\
28 \;\; spin - 1 \;\; vector \;\; bosons \\
56 \;\; spin - 1/2 \;\; Majorana \;\; fermions \\
70 \;\; spin - 0 \;\; scalars
\end{array}
$$

A natural gauge symmetry is $SO(8)$

Supergravity

BUT

It has remarkable hidden symmetries.

(i) A global non-compact E_7 symmetry

(ii) a gauge $SU(8)$ symmetry whose gauge bosons are not elementary fields.

It is the classical limit of a general string theory

Supersymmetry: The next spectroscopy

The only model-independent prediction is the existence of the s-particles.
Too many arbitrary parameters.

Example: Supergravity breaking

Spontaneous breaking at the supergravity level $\longrightarrow$ The gravitino gets a mass, the Goldstino decouples.
Non-gravitational limit. Obtain a GUT with explicit but soft supersymmetry breaking.
Assume that at the GUT scale the breaking is as “simple” as possible.
Extrapolate to present energies using R.G.

Supersymmetry: The next spectroscopy

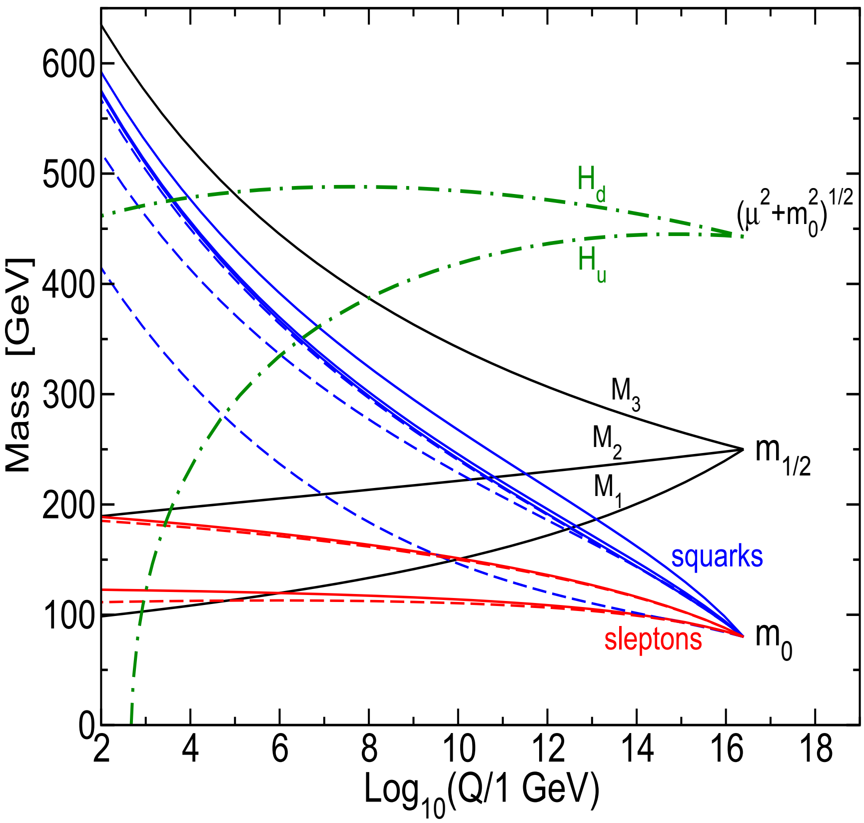
A possible set of parameters:
(i) A common mass parameter $m_{1/2}$ for all gauginos
(ii) m_0 for all squarks and sleptons ???
(iii) A tri-linear coupling among the various scalars, A
(iv) We need two Higgs doublets with v.e.v.'s v_1 and v_2. They are taken by the neutral components of the two doublets, but one has weak isospin $I_z = +1/2$ and the other $I_z = -1/2$. No CP violation. $tan\beta = v_1/v_2$
(v) A mixing term between the two Higgs fields $\mu H_1 H_2$

Masses and mixing parameters but no new coupling constants

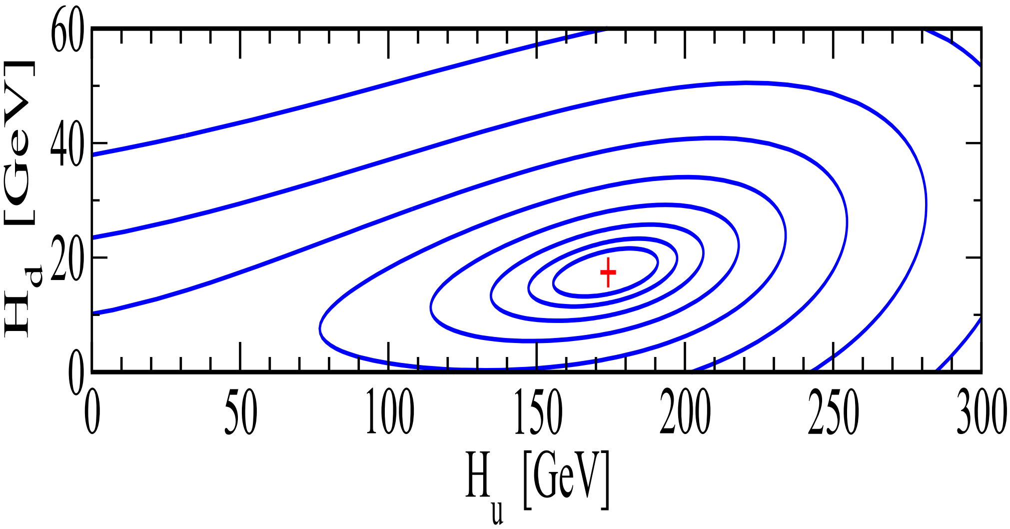
Supersymmetry: The next spectroscopy

The extrapolation is constrained:
The tree-level Higgs mechanism is not applicable in exact supersymmetry.
No electroweak breaking at GUT scale $\rightarrow$ all corresponding square masses must be positive or zero at that scale.
Correct breaking at m_W, no breaking in between, maintaining of the perturbative nature of the theory everywhere.
$\rightarrow$ a narrow window for the possible values of the Higgs parameters. Light Higgs
A bonus: A natural hierarchy:

$$m_W \sim M_{GUT}\, e^{-\frac{1}{\alpha_t}} \quad ; \quad \alpha_t = \frac{\lambda_t^2}{4\pi}$$



RG evolution of scalar and gaugino mass parameters in the MSSM with typical minimal
-inspired boundary conditions imposed at $Q_0 = 2.5 \times 10^{16}$ GeV. The parameter $\mu^2 + m^2_{H_u}$
e, provoking electroweak symmetry breaking.



gure 7.1: A contour map of the Higgs potential, for a typical case with $\tan\beta \approx -\cot\alpha \approx 10$.
he minimum of the potential is marked by +, and the contours are equally spaced equipotentials.
scillations along the shallow direction, with $H_u^0/H_d^0 \approx 10$, correspond to the mass eigenstate h^0, while
e orthogonal steeper direction corresponds to the mass eigenstate H^0.

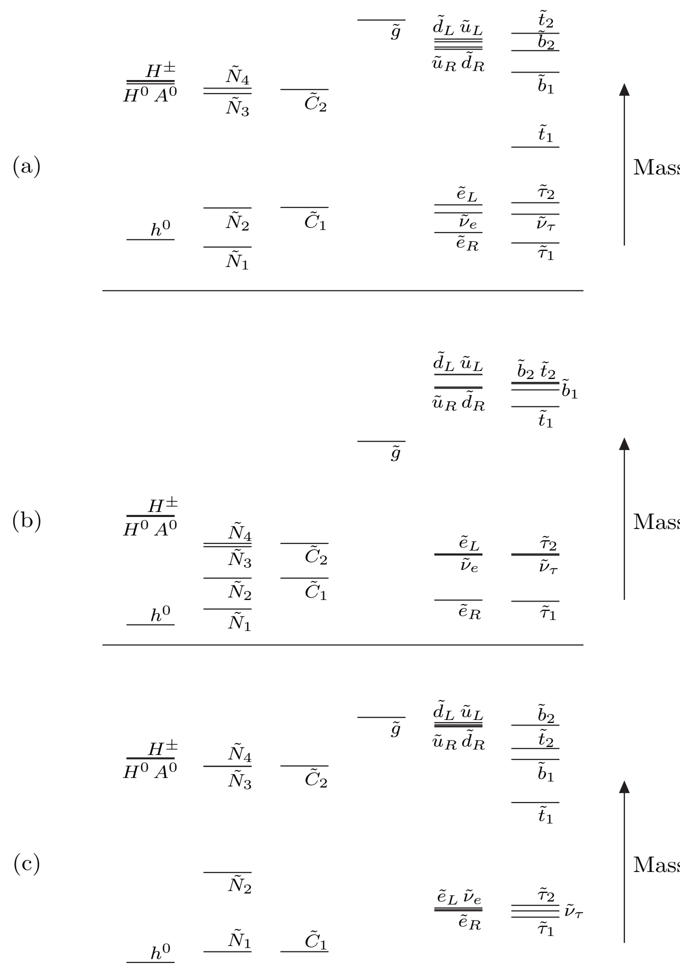


Figure 7.5: Three sample schematic mass spectra for the undiscovered particles in the MSSM, for (a) minimal supergravity with $m_0^2 \ll m_{1/2}^2$, (b) minimal GMSB with $N_5 = 1$, and (c) AMSB with an extra m_0^2 for scalars. These spectra are presented for entertainment purposes only! No warranty, expressed or implied, guarantees that they look anything like the real world.

Supersymmetry: The next spectroscopy

In the MSSM R-parity is conserved, therefore all new particles are produced in pairs and the lightest among them is stable.

If we relax the minimality assumption, no predictions.

Qualitatively, ordinary particles are expected to be lighter than their supersymmetric partners.

Squarks and gluinos heavier than sleptons and other gauginos.

Sneutrinos are predicted to be of the same order as those of the corresponding charged sleptons.

The LSP is identified with a linear combination of the neutral gauginos and Higgsinos.

It leaves no trace in the detector.

Supersymmetry: The next spectroscopy

m_{LSP} is a very important phenomenological parameter. Cosmology a rather loose bound $m_{LSP} < O(200)$ GeV. Direct determination very important.

m_ϕ at tree level is predicted $\sim m_Z$

Fortunately, radiative corrections, especially the t-quark loops, raise this limit considerably $\sim$ 130 GeV (max I have seen is 208 GeV).

The present LEP limit is 114.1 GeV with a tantalising possible signal at 115 GeV.

The breaking of $U(1) \otimes SU(2)$ causes mixings among the partners of opposite chirality fermions, so the final mass spectrum is the result of several diagonalisations, $\rightarrow$ squarks do not necessarily follow the mass hierarchy of their quark partners.

Supersymmetry: The next spectroscopy

Squarks are produced in hadron collisions either in pairs or in association with gluinos

Their decay modes are of the form $\tilde{q} \rightarrow q + LSP$ (quark + LSP) or, if phase space permits, $\tilde{q} \rightarrow q + \tilde{g}$ (quark + gluino).

The signature is missing P_T plus jets.

Sleptons behave similarly and give $\tilde{l} \rightarrow l + LSP$.

LEP limits for the charged ones $\sim$ 100 GeV

Tevatron results suggest that squarks may be much heavier, but model-dependent.

Gluinos may decay into a gluon and an LSP or into a quark-antiquark pair and an LSP.

The Tevatron gives limits for the gluino masses similar to those for squarks.

Supersymmetry: The next spectroscopy

Gauginos and Higgsinos mix among themselves and must be analysed together.
The charged ones are the supersymmetric partners of $W^\pm$ and $H^\pm$ and are described by a 2×2 mass matrix.
Among the neutral ones, the partners of W^3, B and the two CP-even neutral Higgses mix in a 4×4 matrix. The lightest of them is assumed to be the LSP.

Supersymmetry: The next spectroscopy

Supersymmetric particles may be spread all over from 50 GeV to 1 TeV.

Looking for them will be an important part of experimental search in the years to come.

Experiments have developed impressive tools to find them.

CONCLUSION

CONCLUSION

in two years