Relativistic Force-free MHD Turbulence in BH Magnetosphere

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(ApJ, 2005)

Introduction

In the BH magnetospheres filled with plasma via pair production, $B^2/8\pi \gg \rho c^2$.

c.f.) Goldreich & Julian (1969)

==> We can ignore the inertia of charge carriers.

Introduction

Suppose that $B^2/8\pi \gg \rho c^2$. Then,

$$\partial_\mu T^{\nu\mu}_{(f)} = 0$$

with $T^{\mu\nu}_{(f)} = F^{\mu}_{\alpha} F^{\alpha\nu} - \frac{1}{4}(F_{\alpha\beta}F^{\alpha\beta})g^{\mu\nu}$

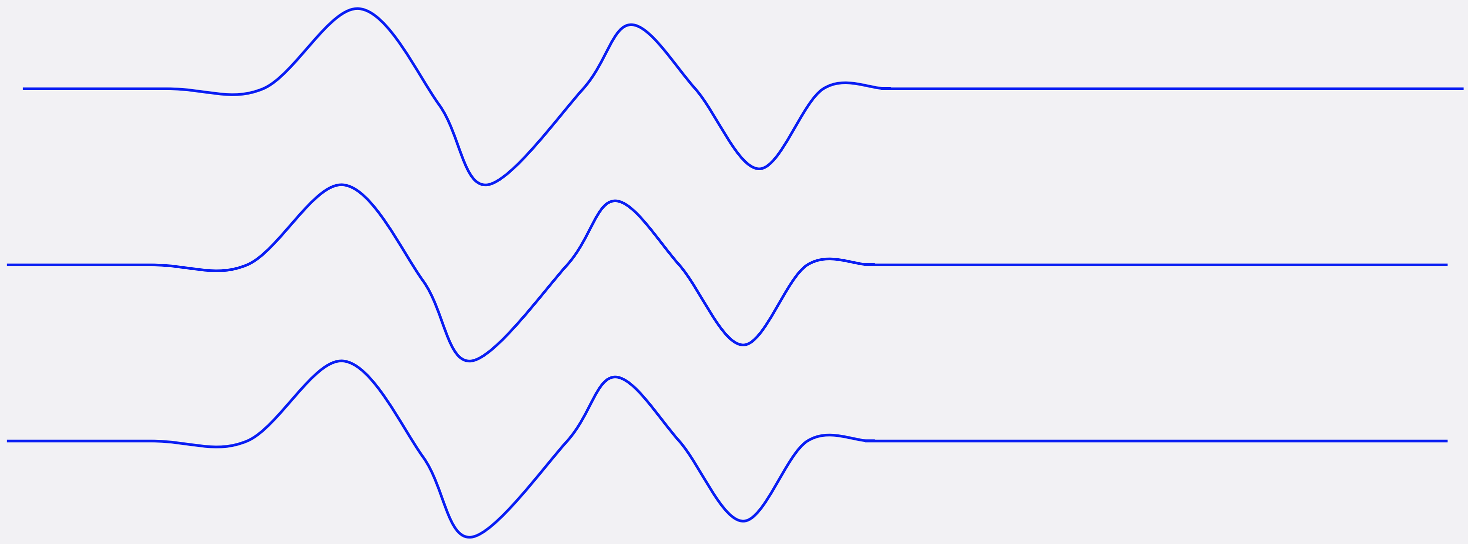
Maxwell eq.

$$\Rightarrow \quad \partial_\mu T^{\nu\mu}_{(f)} = -F_{\nu\mu}J^{\mu} = 0$$

:force-free

What do we want to understand?

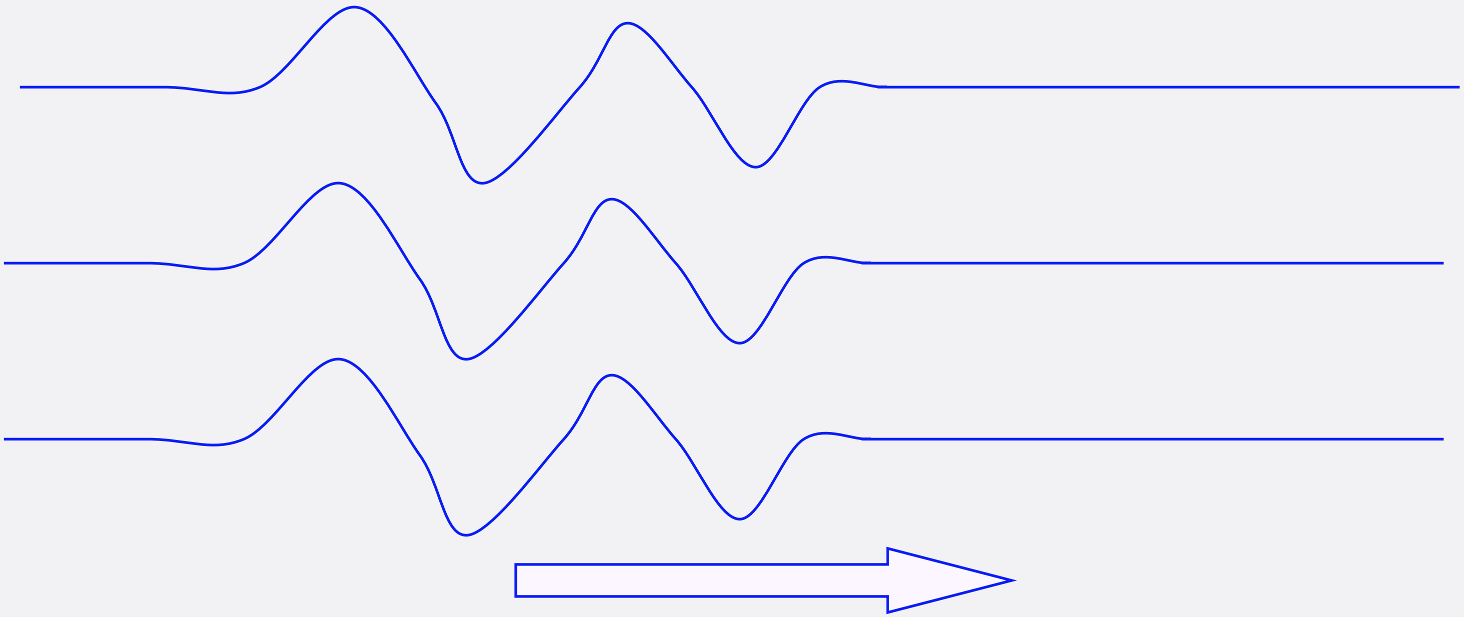
==> dynamics of wave packets



Suppose that we perturb magnetic field lines.
We will only consider Alfvenic perturbations.
(restoring force=tension)
We can make the wave packet move to one direction.
(We need to specify velocity)

Dynamics of one wave packet

Suppose that this packet is moving to the right.
What will happen?



V_A: Alfven speed

One wave packet

QuickTime?and a
DV/DVCPRO - NTSC decompressor
are needed to see this picture.

FFMHD
64^3

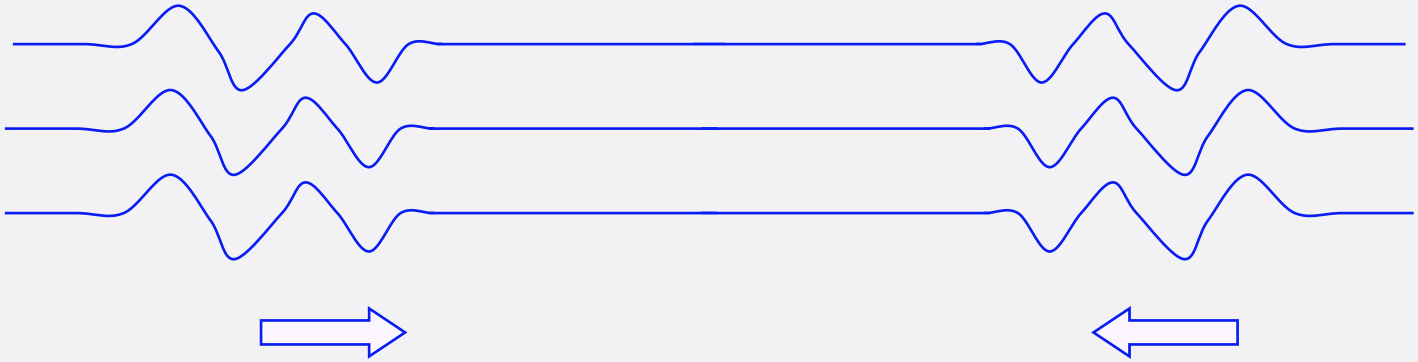
Nothing happens.

There is a good Chinese expression for this:

孤掌難鳴

Dynamics of two opposite-traveling wave packets

Now we have two colliding wave packets.
What will happen?

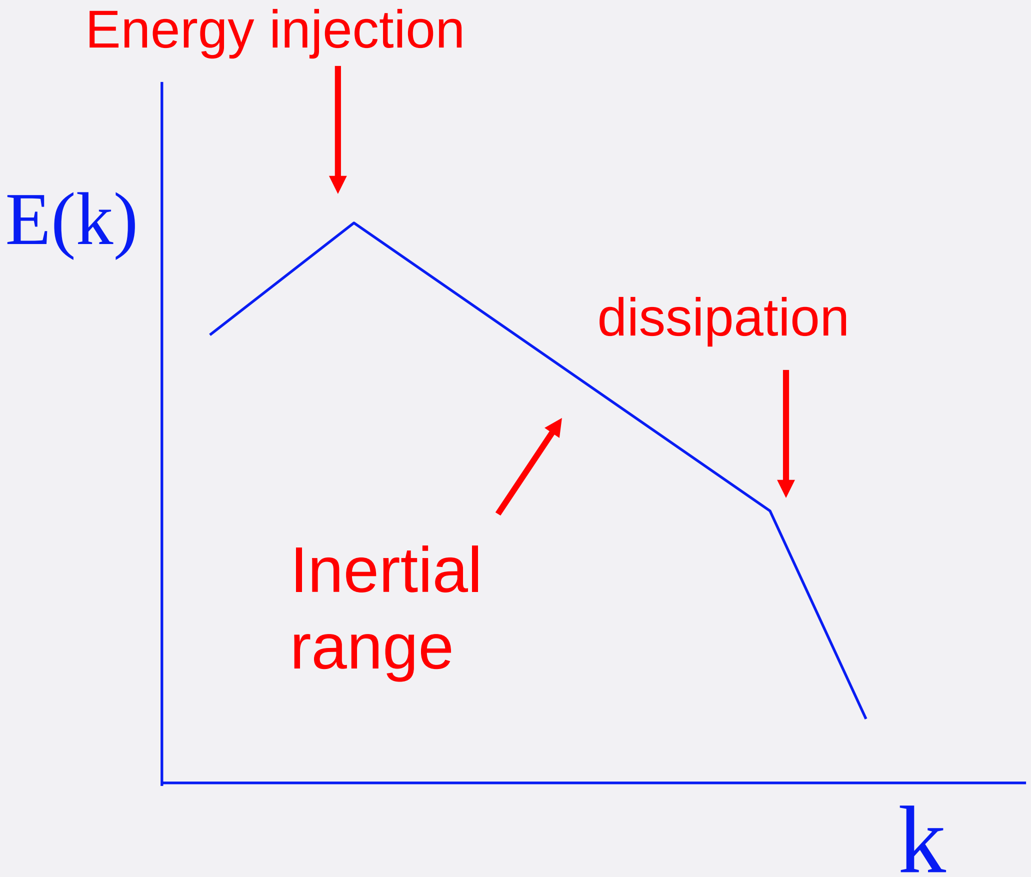


Two wave packets

QuickTime?and a
DV/DVCPRO - NTSC decompressor
are needed to see this picture.

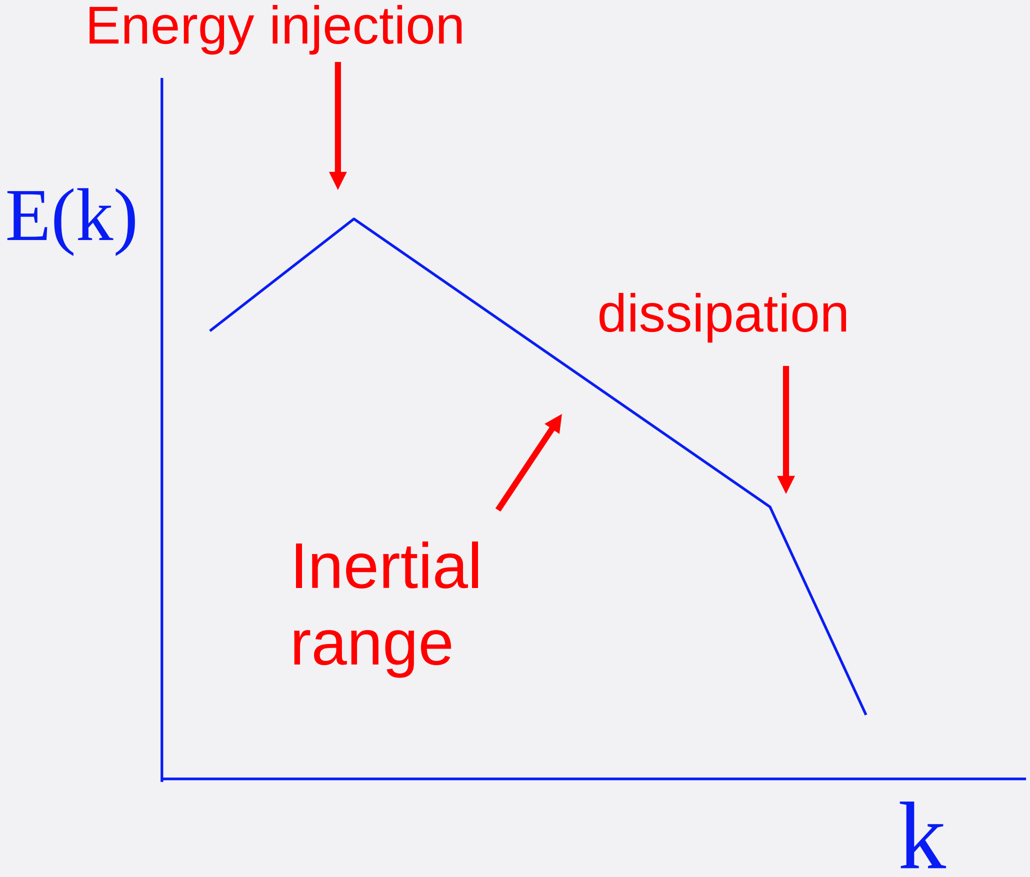
This is something we call turbulence

Energy spectrum is useful!



$E(k) \sim k^{-5/3}$
for hydrodynamic turbulence

Energy spectrum is useful!

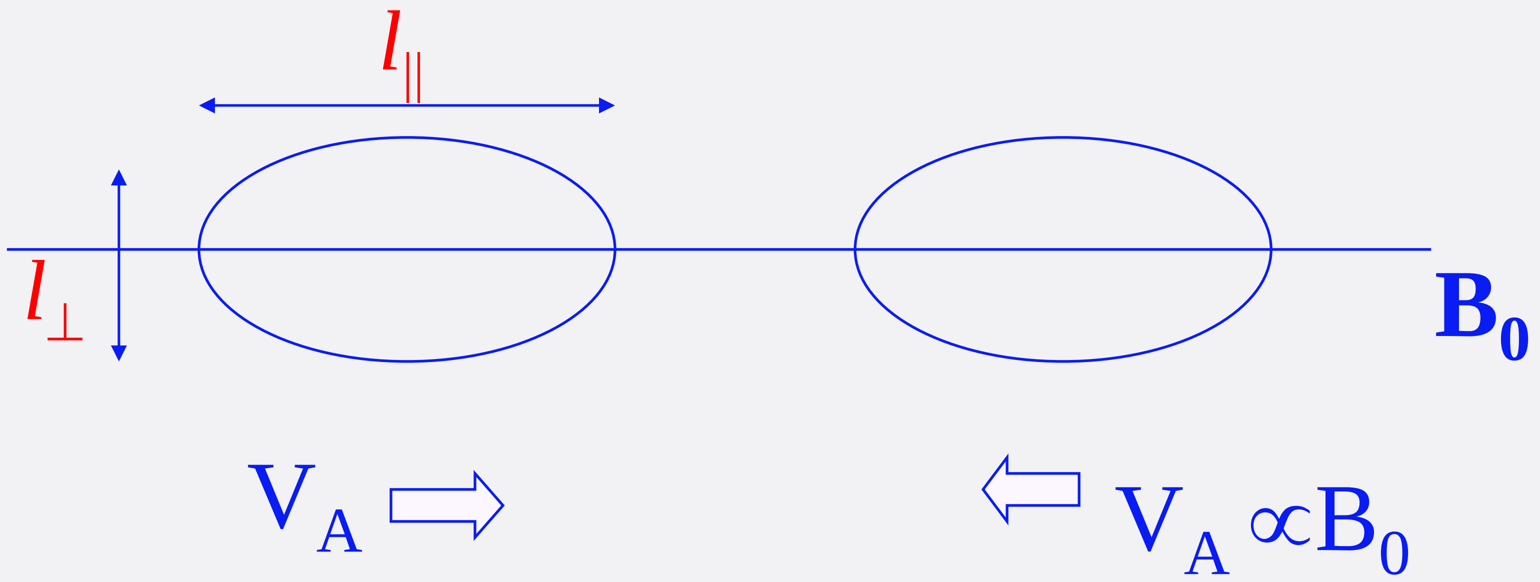


$E(k) \sim k^{-5/3}$
for hydrodynamic
turbulence

What is spectrum for
MHD turbulence?

Let's consider non-relativistic incompressible Alfvenic MHD first.

Goldreich & Sridhar (1995) considered dynamics of Alfvenic wave packets.



Let's consider non-relativistic incompressible Alfvenic MHD first.

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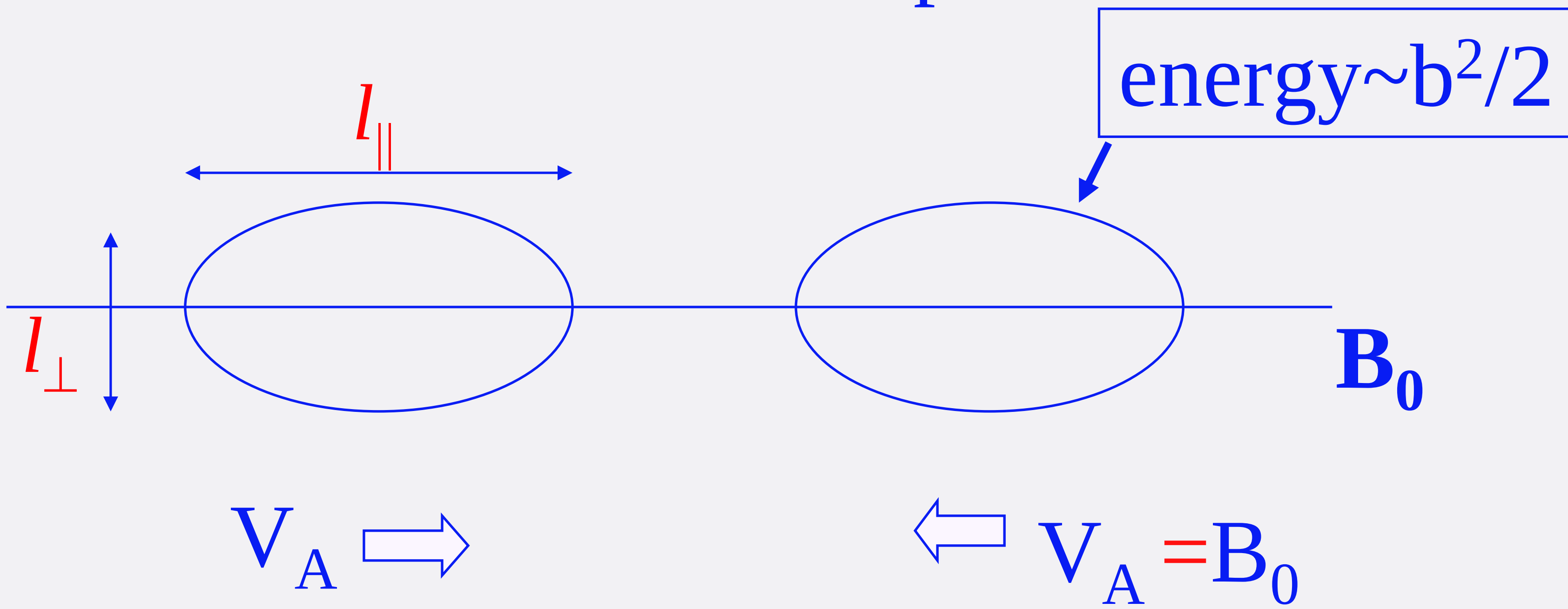


*From now on, B = actually $B/(4\pi\rho)^{1/2}$

When they collide, a packet loses energy of

$\Delta\mathcal{E} \sim (d\mathcal{E}/dt)\Delta t \sim (b^3/\,l_\perp\,)t_{coll} \sim (b^3/\,l_\perp\,)(l_\parallel/V_A)$.

Therefore $\Delta\mathcal{E}\,/\mathcal{E} \sim (b^3/\,l_\perp\,)(l_\parallel/V_A)\,/\,b^2$

$$= (b\,l_\parallel\,/\,l_\perp B_0)$$

$$= (l_\parallel/B_0)/(\,l_\perp\,/b\,)$$

$$= t_w/t_{eddy} = \chi$$

NOTE: $$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B}$$

$$\Rightarrow db/dt \sim b^2/l_\perp$$

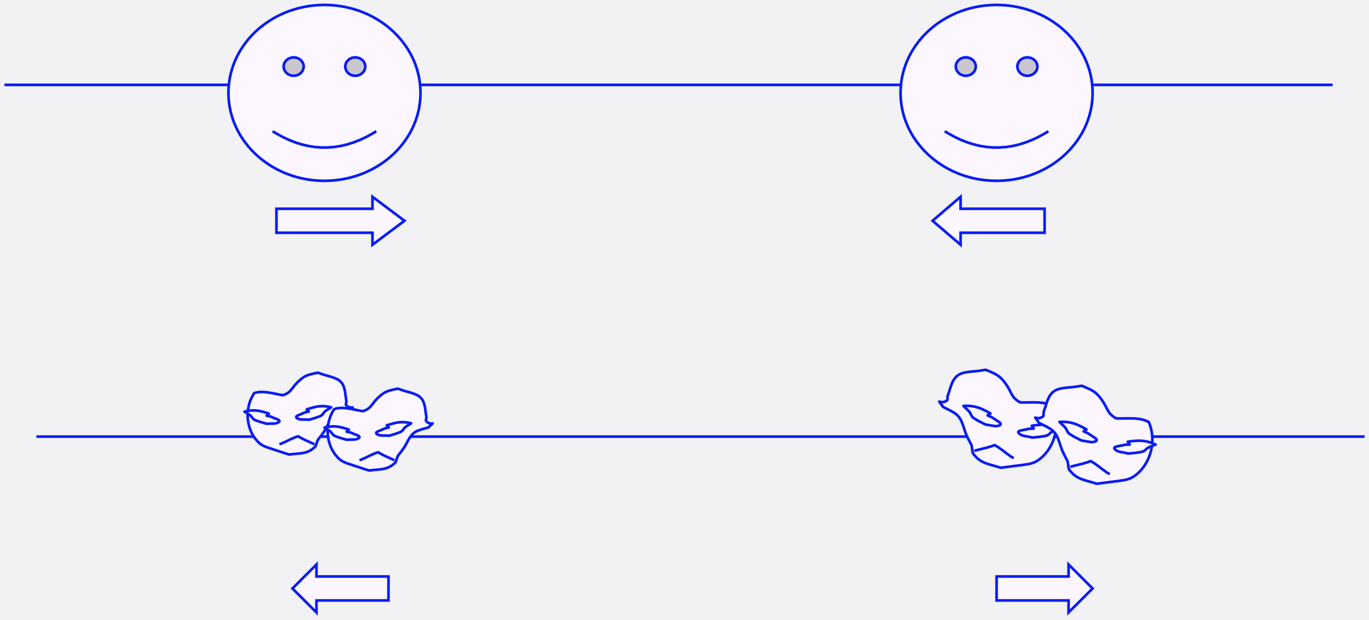
$$\Rightarrow d\mathcal{E}/dt \sim b^3/l_\perp$$

$\chi \sim t_w/t_{eddy} \sim (b\, l_\parallel / l_\perp B_0) \sim \Delta E / E$

- Suppose that $\chi \sim 1$.

e.g.) When $B_0 \sim b_l$ and $l_\parallel \sim l_\perp$, we have $\chi \sim 1$.

=>**1 collision is enough to complete cascade!**

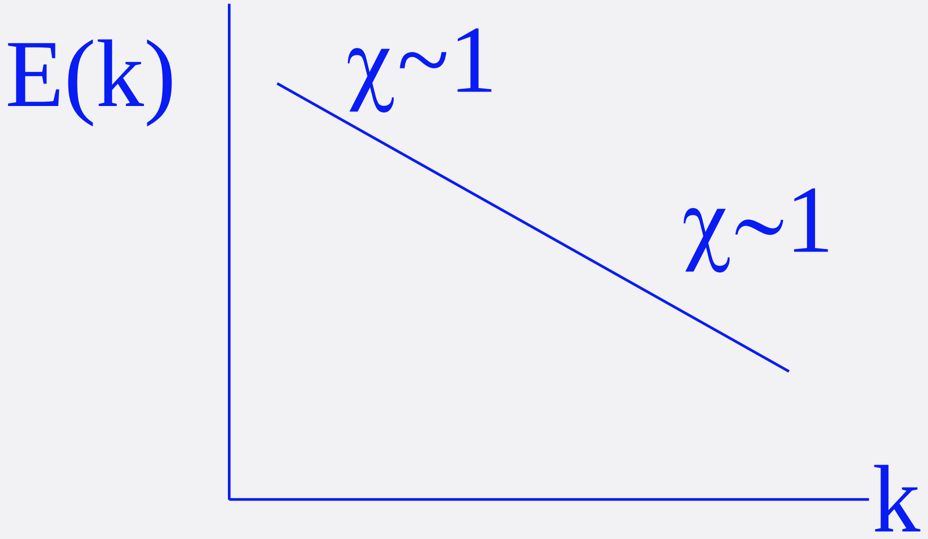


$$\chi \sim t_w / t_{eddy} \sim (b\, l_{\|} / l_{\perp} B_0) \sim \Delta E / E$$

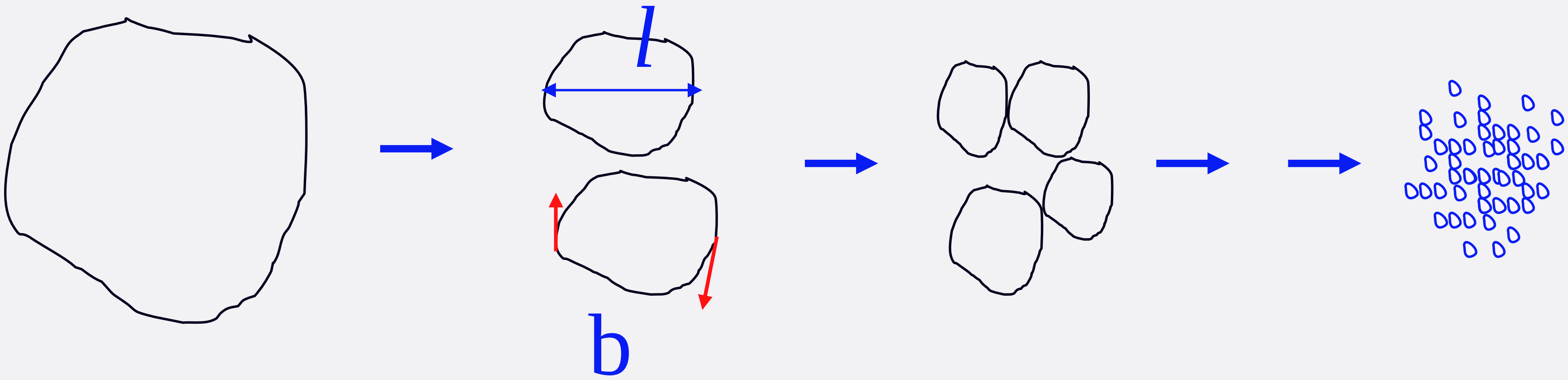
- Goldreich & Sridhar (1995) found that, when $\chi \sim 1$ on a scale, $\chi \sim 1$ on all smaller scales.

* $\chi \sim 1$ is called critical balance

*This regime is called strong turbulence regime



Energy Cascade



$$b^2/t_{cas} = \text{constant}$$

Goldreich-Sridhar model (1995)

- Critical balance

$$\frac{l_\perp}{b_{\perp l}} = \frac{l_\parallel}{B_0}$$

- Constancy of energy cascade rate

$$\frac{b_{\perp l}^2}{t_{cas}} = \text{const}$$

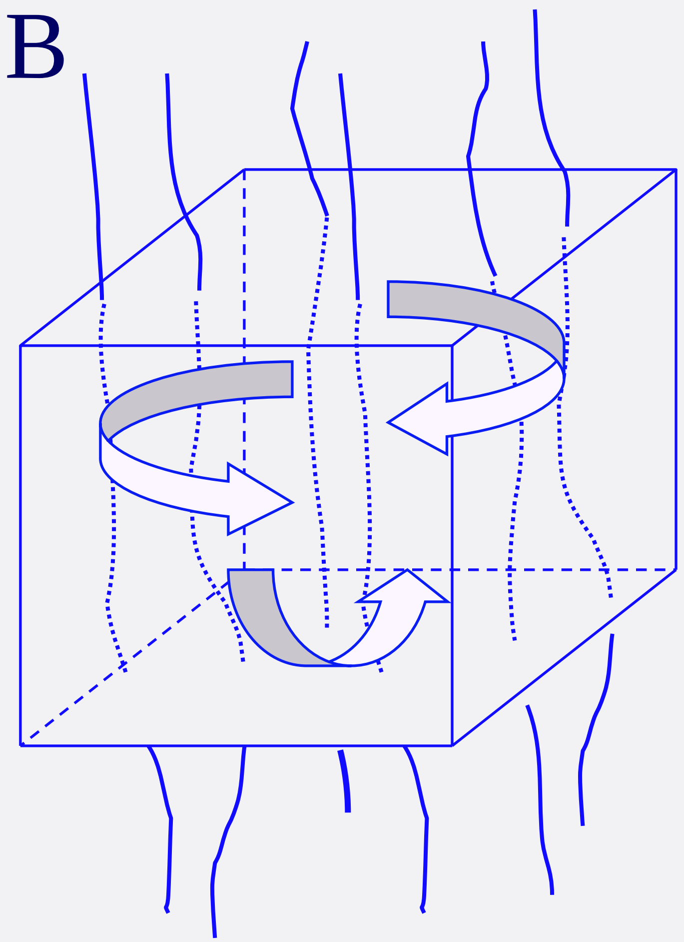
$$\frac{b_{\perp l}^2}{(l_\perp / b_{\perp l})} = \text{const}$$

$$b_\perp \sim l_\perp^{1/3}$$

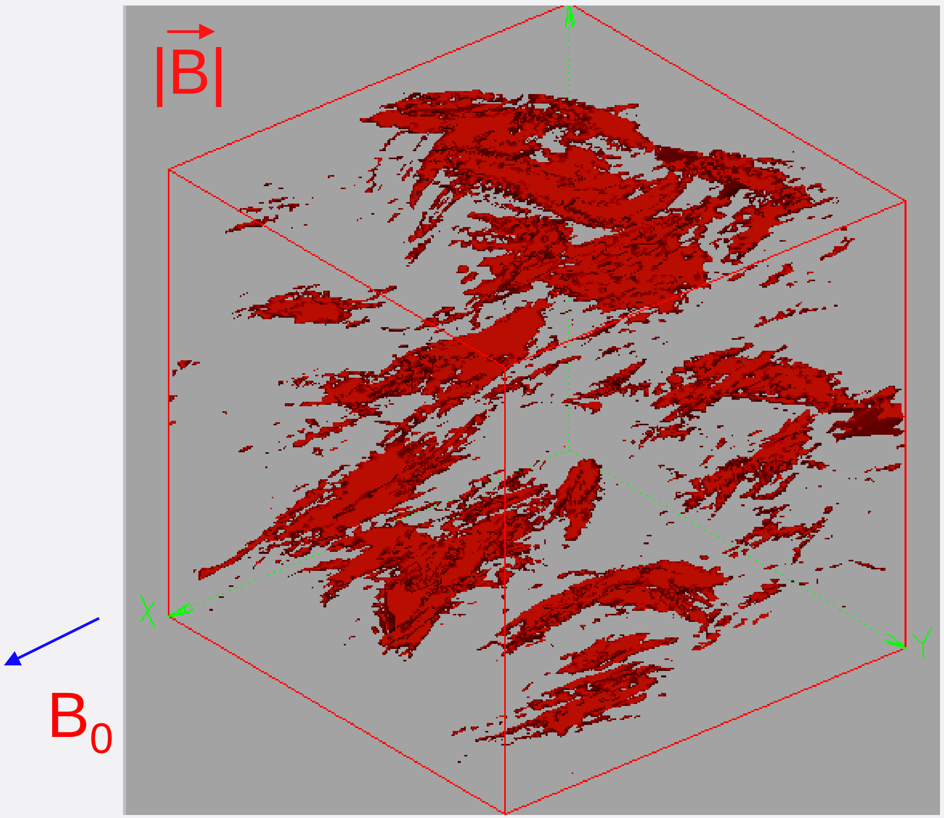
Or, $E(k) \sim k^{-5/3}$

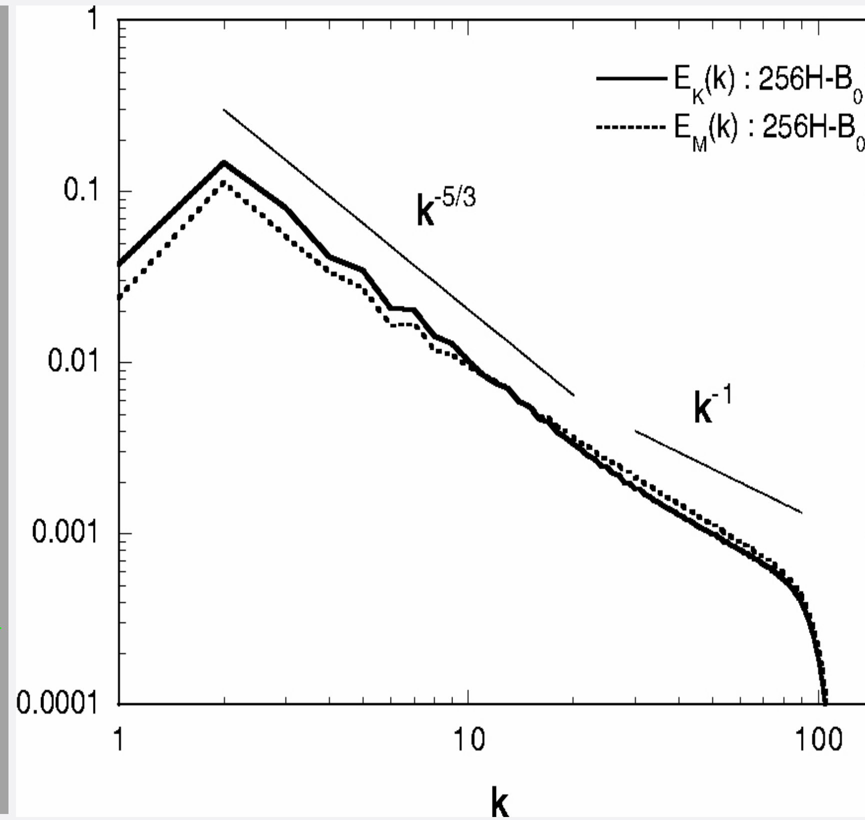
$$l_\parallel \sim l_\perp^{2/3}$$

Numerical test: Cho & Vishniac (2000)



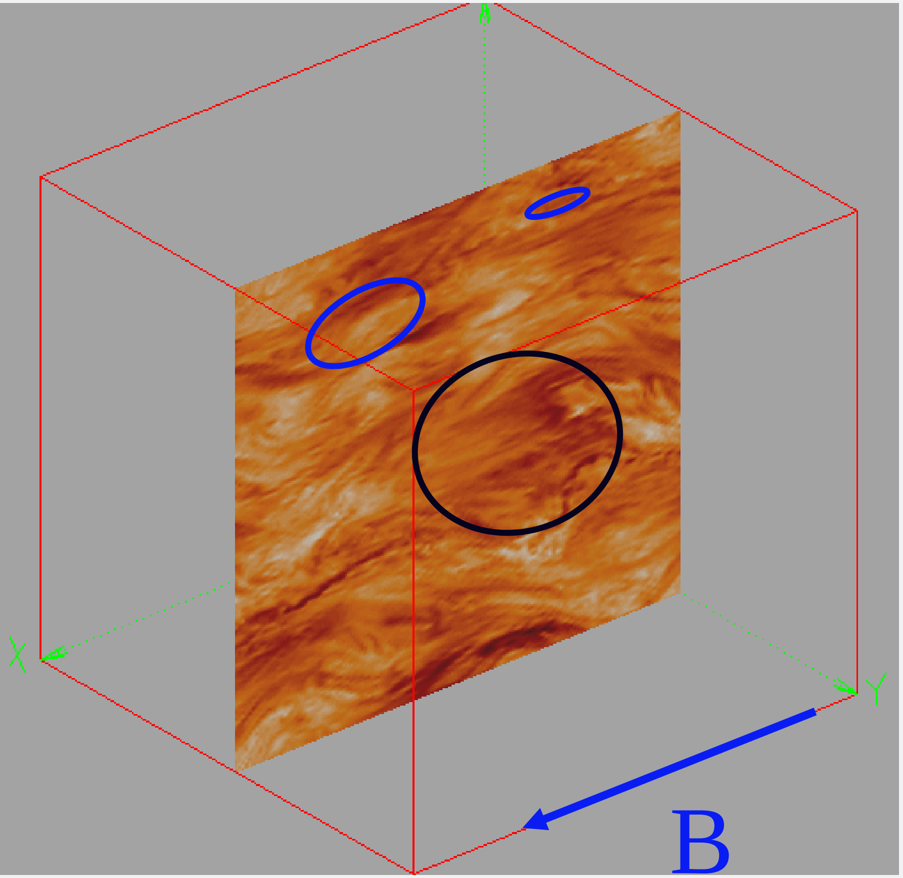
Spectra: Cho & Vishniac (2000)

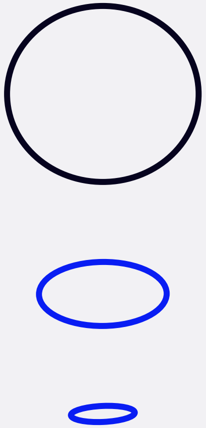




See also Muller & Biskamp (2000); Maron & Goldreich (2001)

Anisotropy

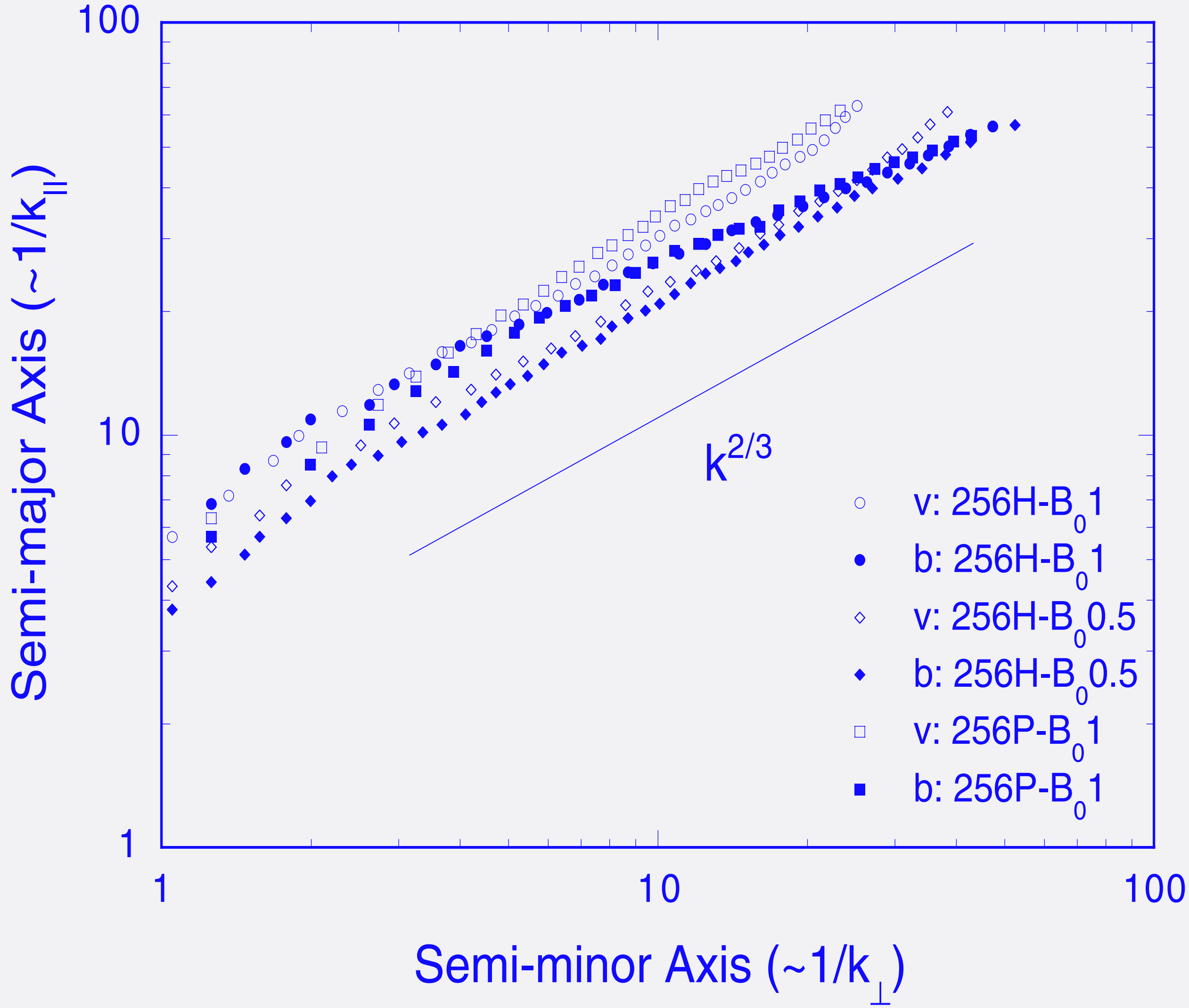




Smaller eddies are more elongated

=> Relation between parallel size and perp size?

Anisotropy: Cho & Vishniac (2000)



* Maron & Goldreich (2001) also obtained a similar result

Let's go back to relativistic FF-MHD

- Force-free ($B^2 >> \rho c^2 \Rightarrow \rho_e E + B \times J = 0$)
- Theory: Thompson & Blaes (1998)

$$\partial_\mu {}^*F^{\mu\nu} = 0 \quad \text{(Maxwell's eq.)},$$
$$\partial_\mu F^{\mu\nu} = -J^\nu \quad \text{(Maxwell's eq.)},$$
$$\partial_\mu T^{\nu\mu}_{(f)} = 0 \quad \text{(energy-momentum eq.)},$$
$$F_{\nu\mu} u^\mu = 0 \quad \text{(perfect conductivity)},$$

$$T^{\mu\nu}_{(f)} = F^\mu_\alpha F^{\alpha\nu} - \frac{1}{4}(F_{\alpha\beta}F^{\alpha\beta})g^{\mu\nu},$$

* c=1, flat space-time

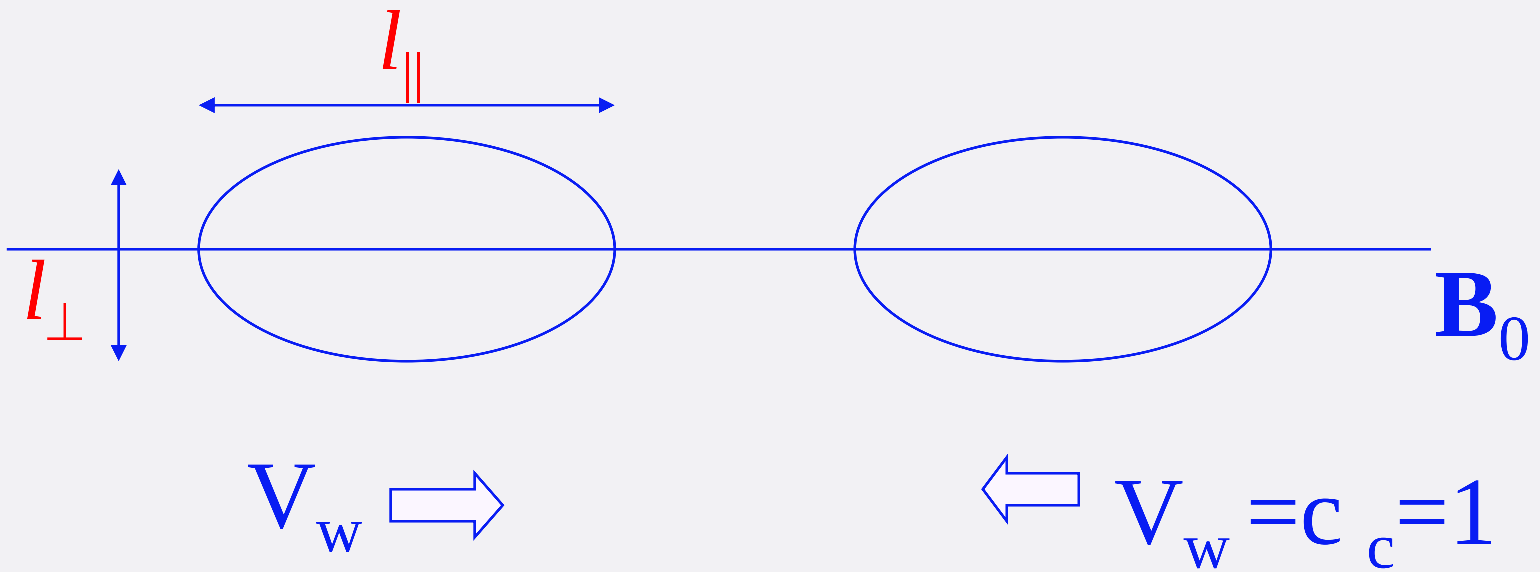
$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x^1} = 0,$$

$$\mathbf{Q} = (S_1, S_2, S_3, B_2, B_3),$$
$$\mathbf{F} = (T_{11}, T_{12}, T_{13}, -E_3, E_2),$$
$$T_{ij} = -(E_i E_j + B_i B_j) + \frac{\delta_{ij}}{2}(E^2 + B^2),$$
$$\mathbf{S} = \mathbf{E} \times \mathbf{B},$$
$$\mathbf{E} = -\frac{1}{B^2}\mathbf{S} \times \mathbf{B},$$

Conserved form!

Relativistic FF-MHD turbulence

Consider two wave packets:



Let's rewrite the equations:

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E},$$
$$\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{B} - \mathbf{J},$$

where

$$\mathbf{J} = \frac{(\mathbf{E} \times \mathbf{B})\nabla \cdot \mathbf{E} + (\mathbf{B} \cdot \nabla \times \mathbf{B} - \mathbf{E} \cdot \nabla \times \mathbf{E})\mathbf{B}}{B^2}$$

$\sim b_l^2/(l_\perp B_0)$

Lyutikov 2003

$$\Delta E \sim (dE/dt)\Delta t \sim (EJ)\Delta t \sim (b_l b_l^2/l_\perp B_0)(l_\parallel/c)$$

energy

$E_l \sim b_l$

$$\Rightarrow \Delta E/E \sim (b_l/l_\perp B_0)(l_\parallel/c) \sim b\, l_\parallel/l_\perp B_0 = \chi!$$

Note: $\chi = t_w/t_{eddy}$

- $t_w \sim l_\parallel/c = l_\parallel$
- $t_{eddy} \sim l_\perp/v_l \sim l_\perp/(cb_l/B_0) = B_0\, l_\perp/b_l$

Simulation

-512^3

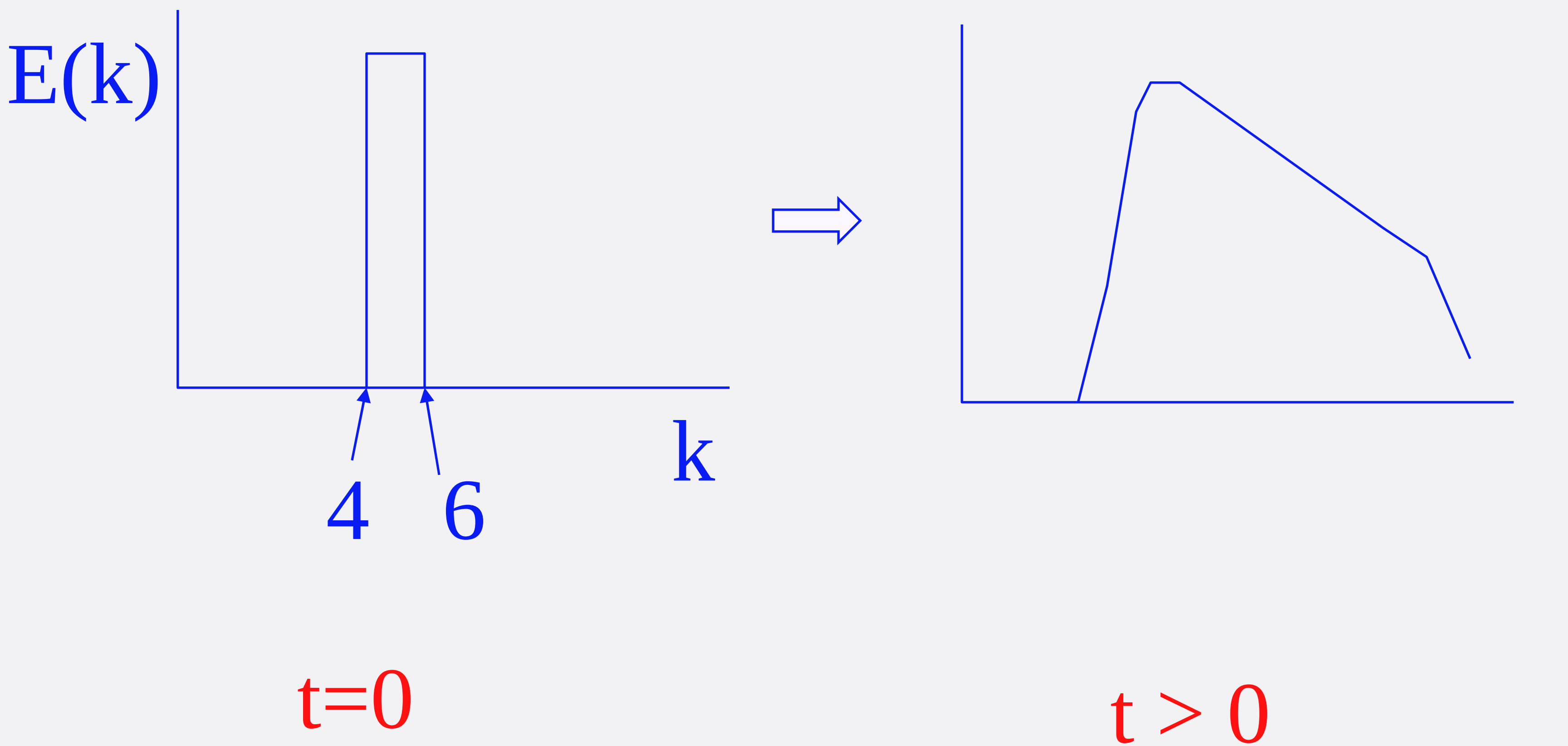
-MUSCL type scheme with HLL flux

-Constrained transport scheme for div B=0 (Toth 2000)

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x^1} = 0,$$

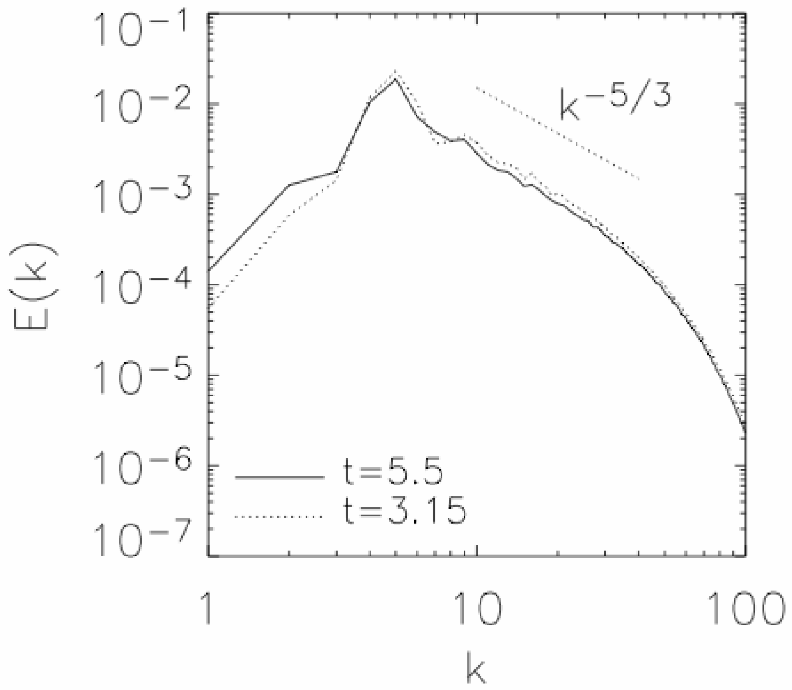
where

$$\mathbf{Q} = (S_1, S_2, S_3, B_2, B_3),$$
$$\mathbf{F} = (T_{11}, T_{12}, T_{13}, -E_3, E_2),$$
$$T_{ij} = -(E_i E_j + B_i B_j) + \frac{\delta_{ij}}{2}(E^2 + B^2),$$
$$\mathbf{S} = \mathbf{E} \times \mathbf{B},$$
$$\mathbf{E} = -\frac{1}{B^2}\mathbf{S} \times \mathbf{B},$$

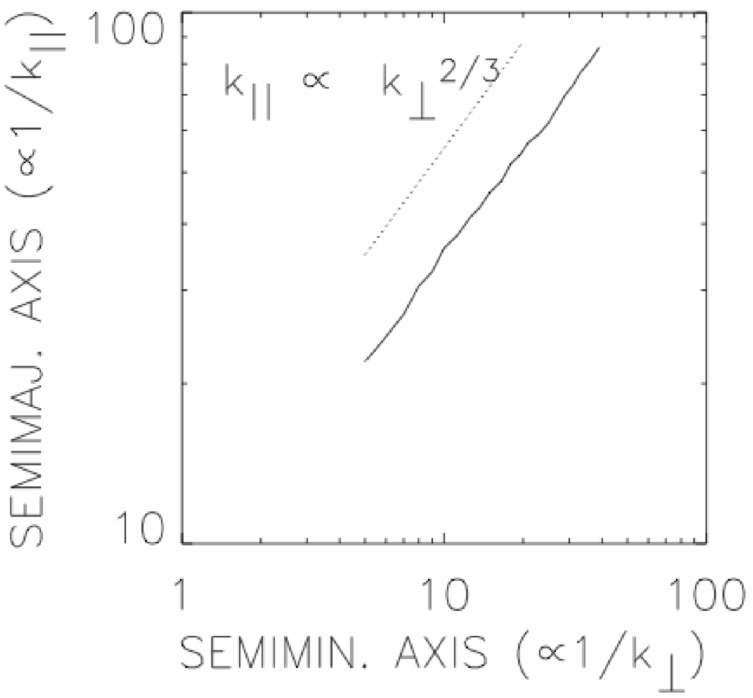

E(k)
k
4
6
t=0
t > 0

Results:

spectrum



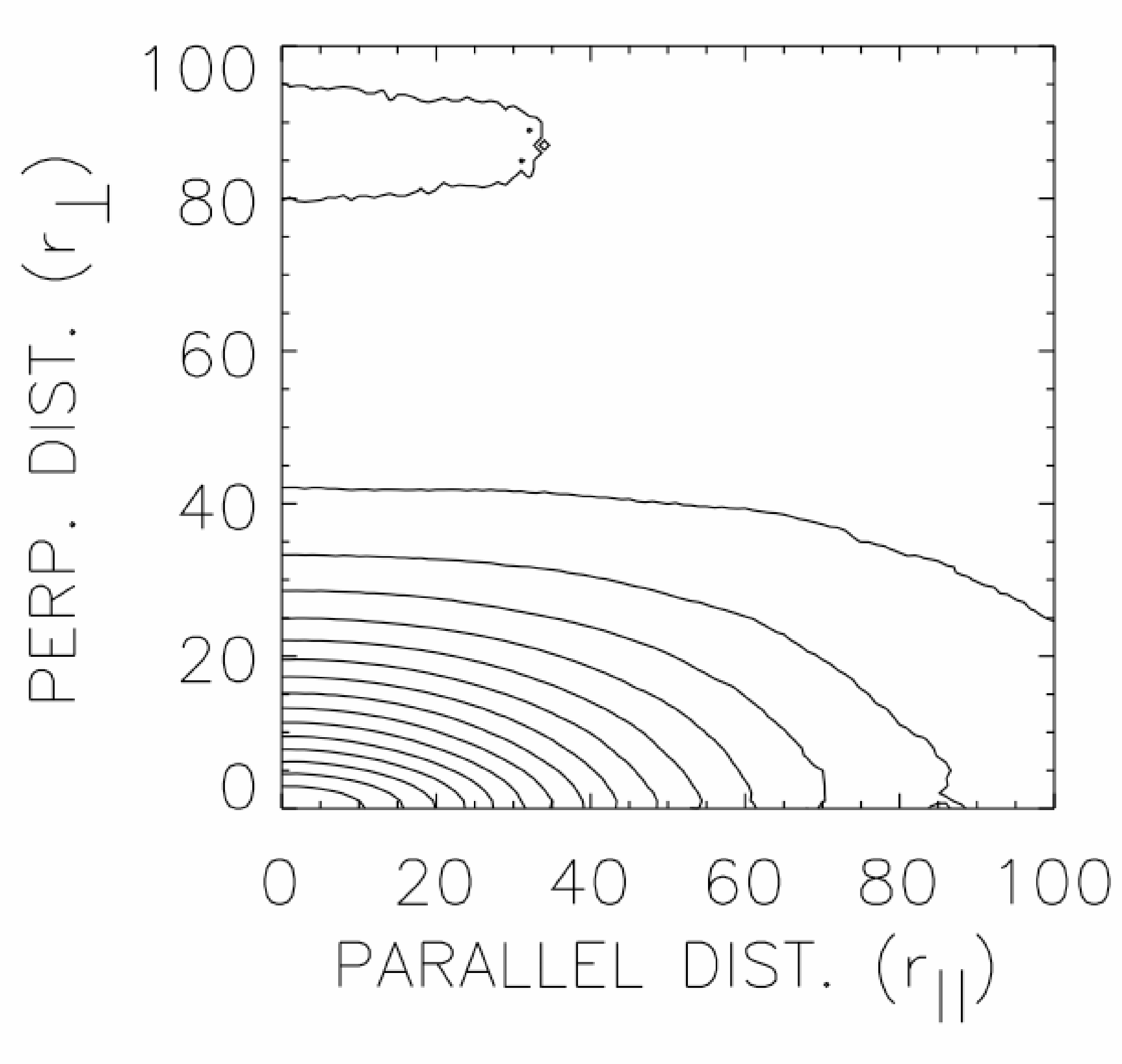
anisotropy



Relativistic MHD ~ classical MHD !

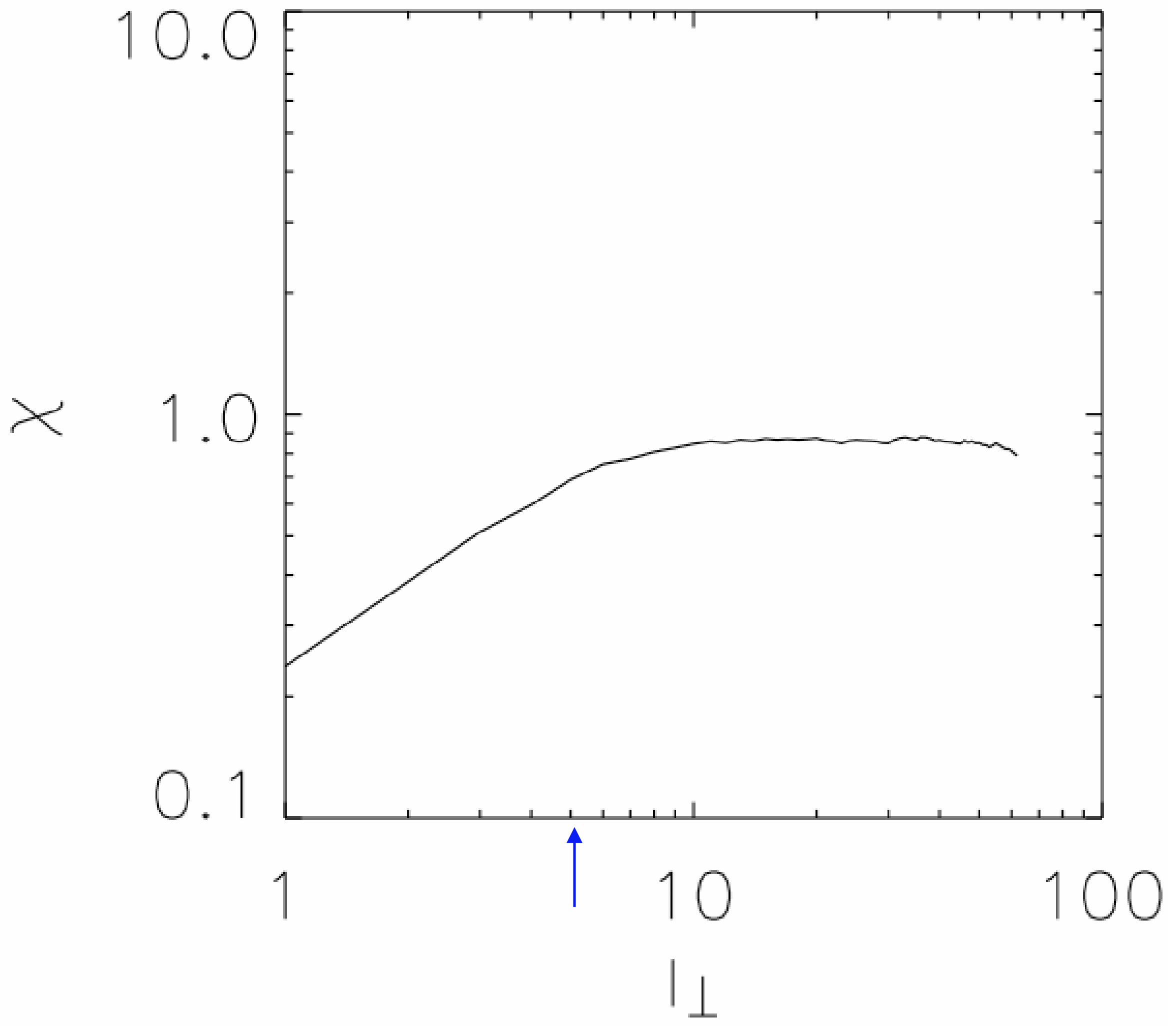
Cho (2005)

Results: eddy shapes



Scale-dependent anisotropy

Results: critical balance



$\chi \sim \Delta E / E$

$\sim t_w / t_{eddy}$

Conclusions

- Relativistic force-free MHD turbulence is similar to its non-relativistic counterpart.
- Kolmogorov spectrum: $E(k) \sim k^{-5/3}$
- Scale-dependent anisotropy: $l_{||} \sim l_{\perp}^{2/3}$
- These results are consistent with theory by Thompson & Blaes (1998).

Critical balance from a different perspective

It's hard to bend field lines
--> Anisotropic eddy shape

$$\frac{l_\perp}{l_\parallel} = \frac{b_l}{B_0}$$

$$\frac{l_\parallel b_l}{B_0 l_\perp} = \frac{t_w}{t_{eddy}} = \chi \sim 1$$