Poynting Flux out of Rotaing Black Hole and Accretion Disk through Force-Free Magnetosphere

Hyun Kyu Lee

Hanyang University

- Introduction
- Force-free magnetosphere
- Basic features of Poynting Flux
- Poynting flux dominated accretion flow
- Poynting flux form Ergosphere

Energy source for AGN, GRB

.....

- Existence of Black Hole
- Gravitational binding energy of accreting material
- Rotational energy of black hole
- Macroscopic magnetic field

Poynting flux along magentic field lines

Magnetosphere

- Maxwell equation

$$F^{\mu\nu}_{;\nu} = 4\pi J^{\mu}$$

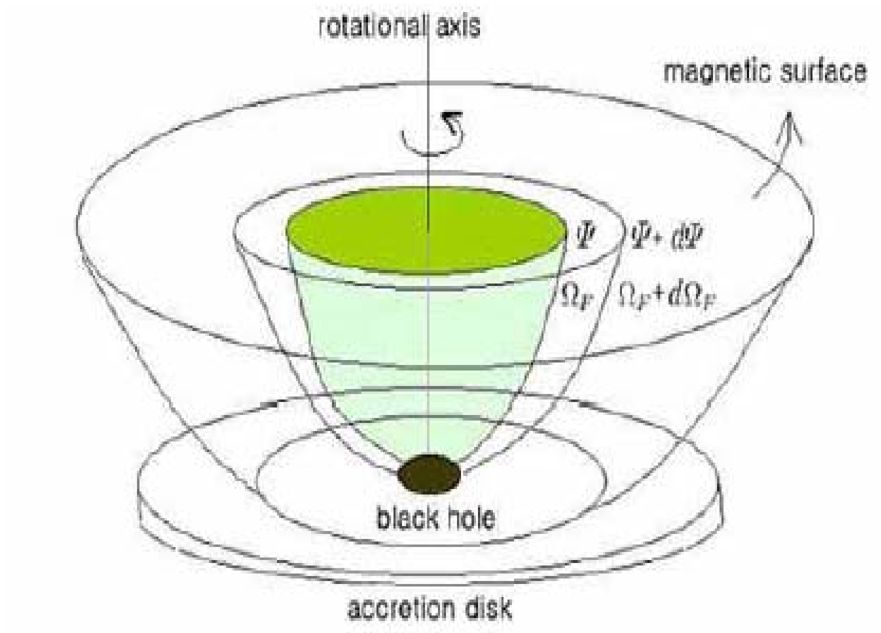
- Force –free magnetosphere

$$F_{\mu\nu} J^{\nu} = 0,$$

E dot B =0, J dot E =0

Axisymmetric and steady state

$$\vec{E} = -\frac{\bar{\omega}}{\alpha}\Omega_F \; e_{\hat{\phi}} \times \vec{B}^p$$



- Rigid rotation of magnetic surface, Ω_F

Basic features of Poynting Flux

Poynting flux :

E X B

Angular momentum
Energy

Effective Surface current

- Surface current density

  J(tot) = J(bulk) + j(surface)

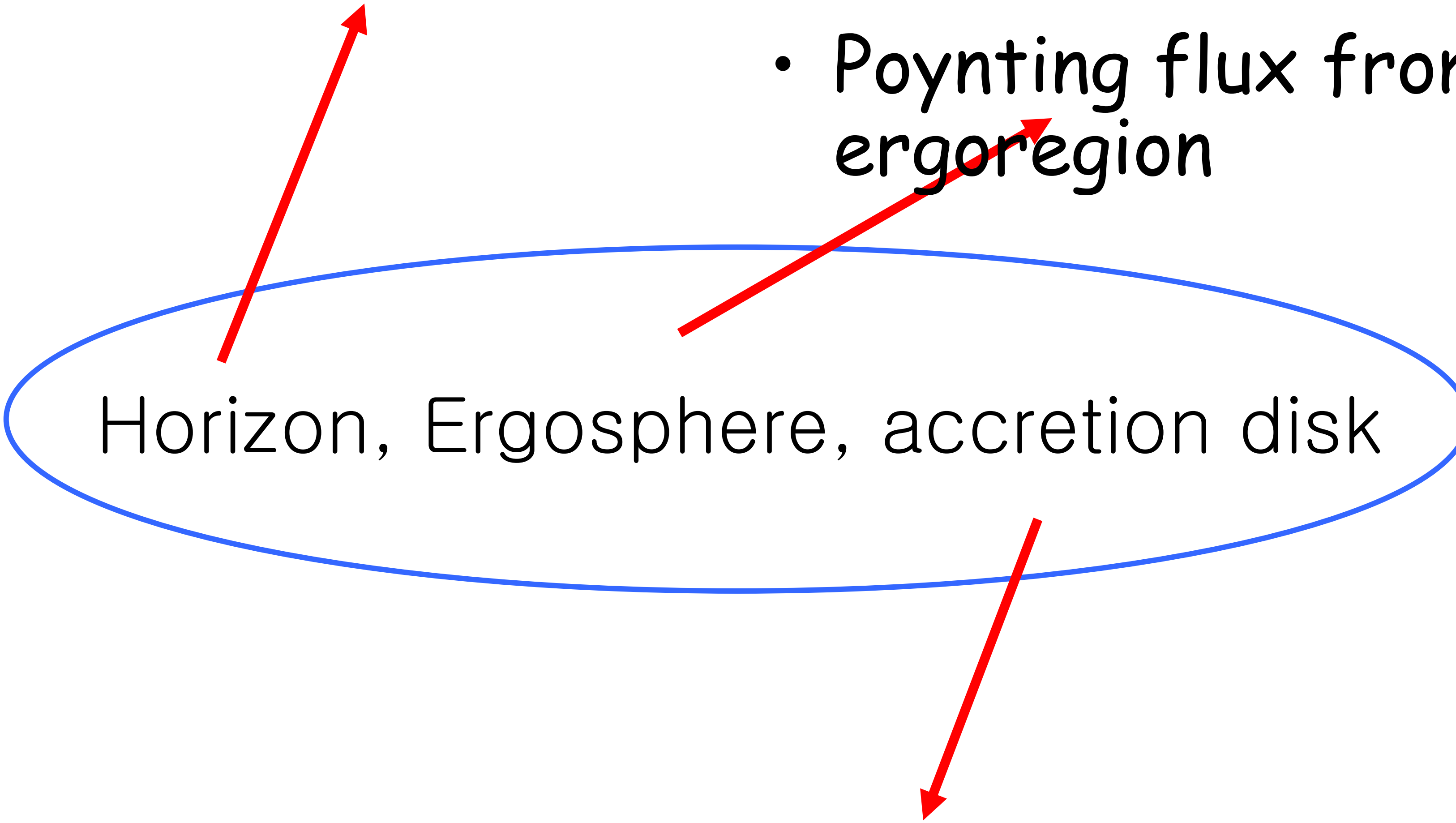
Energy flux:

j dot E

- Manifestation of non force-free nature inside the effective surface for nonvanishing Poynting flux

- Blandford-Znajek process

- Poynting flux from ergoregion

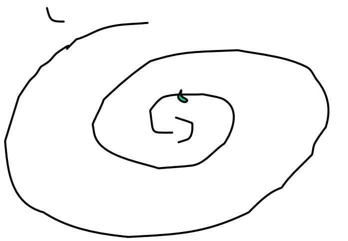


- Poynying flux dominated accretion flow

Poynting flux dominated accretion flow

HKL and J.H.Park, PRD(2004)

- Accretion onto central object



Change of orbital motion

↓

Loss of angular momentum and energy

viscosity, radiation, ...

ordered magnetic field

- Toy Model : Two - dimensional flow

Two dimensional equatorial plane

Non viscous, cool and no radiation

$$T_m^{\mu\nu} = \rho_m u^\mu u^\nu$$

$$u^\theta = 0$$

•1 . What are the field configurations for a stationary accretion flow of a two-dimensional disk with a black hole at the center?

•2. How do the particles of the disk move under those field configurations

• Accretion Equations

• Stress-energy tensor $T^{\mu\nu}$:

$$T^{\mu\nu} = T^{\mu\nu}_{m} + T^{\mu\nu}_{EM} : \quad \text{(Matter part + Electromagnetic field part).}$$

$$T^{\mu\nu}_{m} = \rho_m u^\mu u^\nu, \quad T^{\mu\nu}_{EM} = \frac{1}{4\pi}\left(F^{\mu}{}_{\rho}F^{\nu\rho} - \frac{1}{4}g^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)$$

conservation of the stress-energy tensor

$$T^{\mu\nu}_{;\mu} = 0$$

Accretion equations (stationary and axisymmetric state)

$$(\partial_r u_0)\dot{M}_+ + 2\pi r K^{\hat{r}}(-\alpha E^{\hat{r}} + \beta\varpi B^{\hat{\theta}}) = 0, \quad \text{Energy conservation}$$

$$(\partial_r u_\phi)\dot{M}_+ + 2\pi r K^{\hat{r}}\varpi B^{\hat{\theta}} = 0, \quad \text{Angular momentum conservation}$$

$$\frac{\dot{M}_+}{2\pi r^2} g^{rr}\frac{u^0}{u^r}(\partial_r u_\phi)\left[\Omega_D + \frac{\partial_r u_0}{\partial_r u_\phi}\right] - \frac{\sqrt{\Delta}}{r^2}\left(\sigma_e E^{\hat{r}} - K^{\hat{\phi}}B^{\hat{\theta}}\right) = 0.$$

Angular velocity of the disk, $\Omega_D = u^\phi/u^0$, Mass accretion rate, $\dot{M}_+ = -2\pi r\sigma_m u^r$

surface charge and current density

$$\sigma_e = -\frac{1}{4\pi}(E_+^{\hat{\theta}} - E_-^{\hat{\theta}}),$$

$$K^{\hat{r}} = -\frac{1}{4\pi}(B_+^{\hat{\phi}} - B_-^{\hat{\phi}}), \quad K^{\hat{\phi}} = \frac{1}{4\pi}(B_+^{\hat{r}} - B_-^{\hat{r}}).$$

Multi - component fluid

mass current density

$$j_m{}^{\mu} = \sum_i \rho_m(i)\, u^{\mu}(i) \quad \Longrightarrow \quad j_m{}^{\mu} \equiv \rho_m u^{\mu}$$

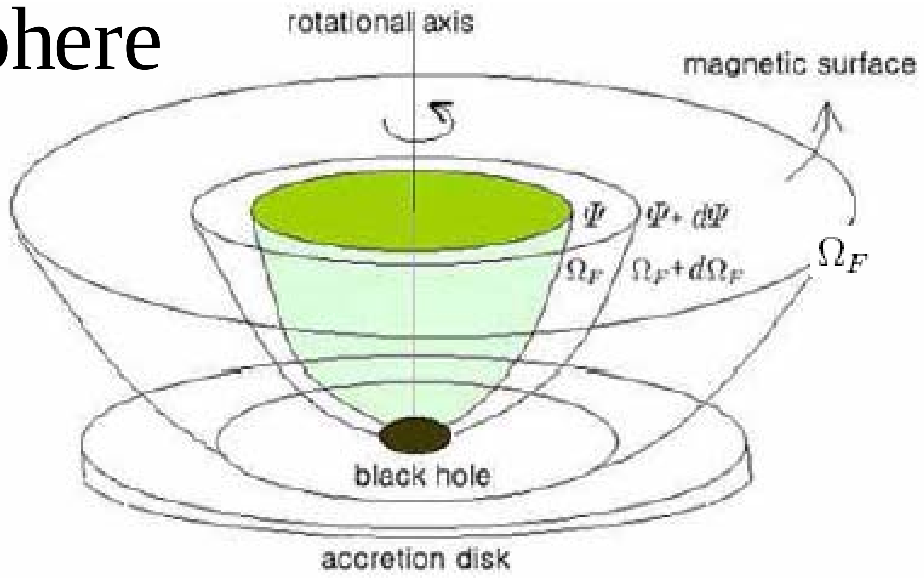
electric charge current density

$$j_e{}^{\mu} = \sum_i \rho_e(i)\, u^{\mu}(i) \quad \not\Leftrightarrow \quad j_e{}^{\mu} \equiv \rho_e u^{\mu}$$

- Force - free magnetosphere

$$F_{\mu\nu} J^{\nu} = 0,$$

$$\rightarrow \quad \vec{E} = -\frac{\bar{\omega}}{\alpha}\Omega_F \; e_{\hat{\phi}} \times \vec{B}^p$$



$$(\partial_r u_0)\dot{M}_+ + r\varpi\Omega_F B^{\hat{\phi}} B^{\hat{\theta}} = 0,$$

$$(\partial_r u_\phi)\dot{M}_+ - r\varpi B^{\hat{\phi}} B^{\hat{\theta}} = 0,$$

$$\frac{\dot{M}_+}{2\pi r^2} g^{rr} \frac{u^0}{u^r} (\partial_r u_\phi) \left[\Omega_D + \frac{\partial_r u_0}{\partial_r u_\phi}\right] - \frac{\sqrt{\Delta}}{2\pi r^2} B^{\hat{r}} B^{\hat{\theta}} \left(\frac{\varpi^2}{\alpha^2}\Omega_F^2 - 1\right) = 0.$$

$$\Rightarrow \quad \frac{\partial_r u_0}{\partial_r u_\phi} = -\Omega_F.$$

- Solutions in Schwarzschild Background

(1) $\Omega_F = I = 0$ (Blandford - Znajak 1977)

$$\Psi_0 = \pi C X, \quad X \equiv r(1 \mp \cos\theta) + 2M(1 \pm \cos\theta)\{1 - \log(1 \pm \cos\theta)\},$$

(2) $\Omega_F \neq 0, I \neq 0$

- $\Psi = \Psi(X)$ (Macdonald 1984)
- $$\frac{d\Psi}{dX} = \frac{\pi C}{(1 + \Omega_F^2 X^2)^{1/2}}.$$

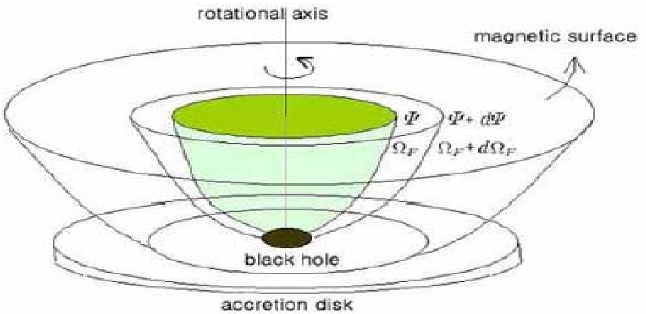
→ equatorial plane

$$B^{\hat{r}} = \frac{\pm C}{2r(1 + \Omega_F^2 X^2)^{1/2}},$$

$$B^{\hat{\theta}} = \frac{-C}{2r(1 + \Omega_F^2 X^2)^{1/2}} \left(1 - \frac{2M}{r}\right)^{1/2},$$

$$B^{\hat{\phi}} = \frac{-2I}{r} \left(1 - \frac{2M}{r}\right)^{-1/2}.$$

• Magnetosphere



Inhomogeneous Maxwell Equation

$$F^{\mu\nu}_{\ ;\nu} = 4\pi J^{\mu}$$ (*Schwarzschild background*)

Grad - Shafranov Equation(Stream Equation)

$$\partial_r\left\{\left(1-\frac{2M}{r}\right)\partial_r\Psi\right\}+\frac{\sin\theta}{r^2}\partial_\theta\left(\frac{1}{\sin\theta}\partial_\theta\Psi\right)-\Omega_F\sin^2\theta\partial_r\left(r^2\Omega_F\partial_r\Psi\right)$$
$$-\frac{\Omega_F}{1-\frac{2M}{r}}\sin\theta\partial_\theta\left(\sin\theta\Omega_F\partial_\theta\Psi\right)=-\frac{16\pi^2 I\frac{dI}{d\Psi}}{\left(1-\frac{2M}{r}\right)}.$$

$$\vec{B}^P=\frac{1}{2\pi\varpi}\nabla\Psi\times e_{\hat{\phi}},$$
$$B^{\hat{\phi}}=-\frac{2I}{\varpi\alpha}.$$

On equatorial plane(disk)

- Grad - Shafranov Equation

$$\frac{4\pi C\Omega_F}{(1+\Omega_F^2 X^2)^{3/2}} \frac{\Delta}{(X-2M)^2} = -16\pi^2 I \frac{dI}{d\Psi},$$

$$\Delta = \frac{d\Omega_F}{dX}(X-3M)(4M^3 - 6M^2X + 5MX^2 - X^3) + \Omega_F\left\{10M^3 - 18M^2X + 8MX^2 - X^3 + 3M^2X(X-2M)^2\Omega_F^2\right\}.$$

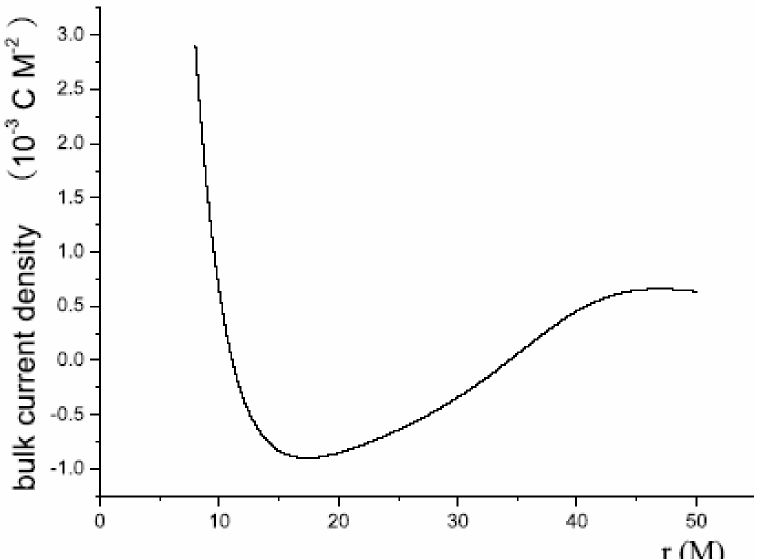
$$\tilde{I} \equiv (1+\Omega_F^2 X^2)^{1/2}\frac{I}{C}$$

- Accretion Equations

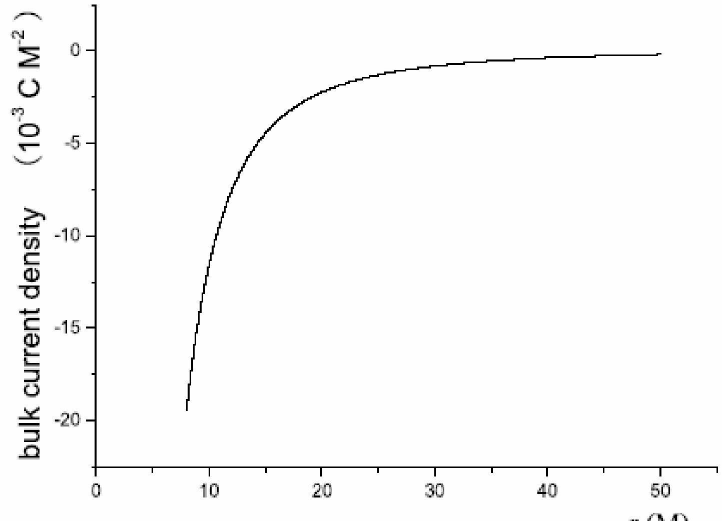
$$(\partial_r u_0)\dot{M}_+ + \frac{C^2\Omega_F}{(1+\Omega_F^2X^2)}\tilde{I} = 0, \quad (\partial_r u_\phi)\dot{M}_+ - \frac{C^2}{(1+\Omega_F^2X^2)}\tilde{I} = 0$$

$$\dot{M}_+ \frac{u^0}{u^r}(\partial_r u_\phi)\left[\frac{u^\phi}{u^0} - \Omega_F\right] + \frac{C^2}{4r(1+\Omega_F^2X^2)}\left(\frac{r^2\Omega_F^2}{1-\frac{2M}{r}} - 1\right) = 0,$$

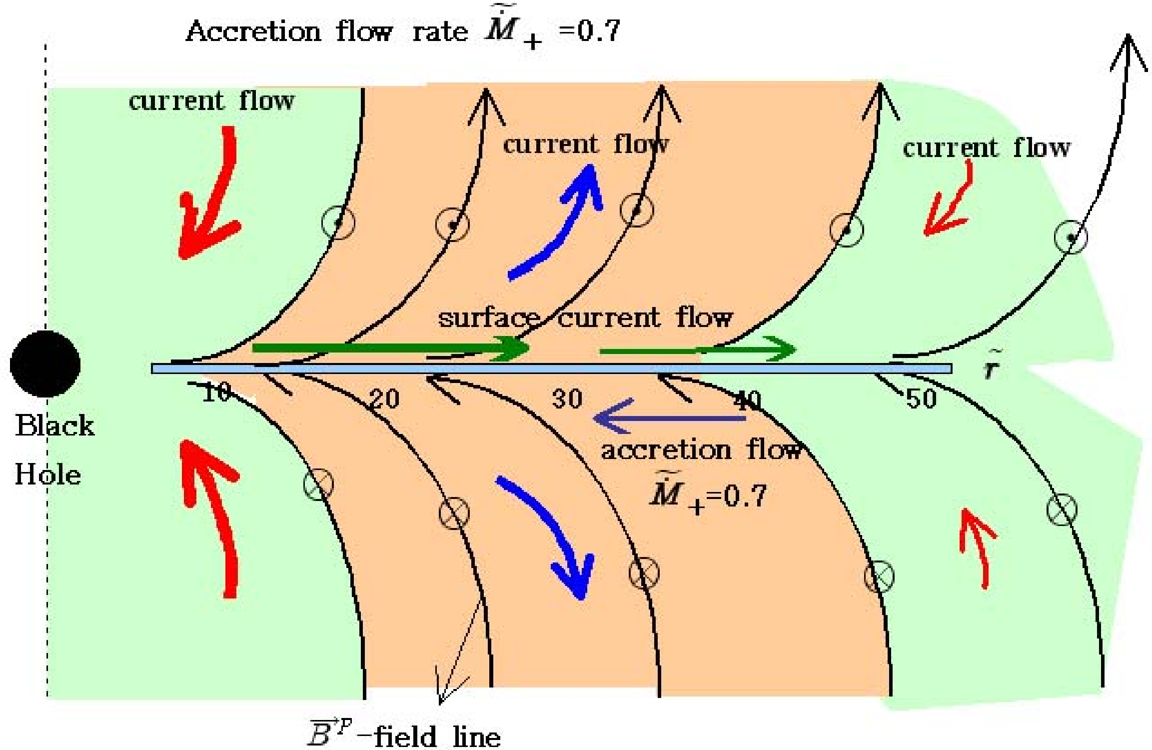
⇒ $u_0\ (u_r),\ u_\phi,\ \Omega_F,\ I$


bulk current density (10^-3 C M^-2)
3.0
2.5
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
0
10
20
30
40
50
r (M)

[a]


bulk current density (10^-3 C M^-2)
0
-5
-10
-15
-20
0
10
20
30
40
50
r (M)

[b]


Accretion flow rate $\widetilde{M}_+$ =0.7
current flow
current flow
current flow
surface current flow
Black
Hole
10
20
30
40
50
$\tilde{r}$
accretion flow
$\widetilde{M}_+$=0.7
$\overrightarrow{B}^p$-field line

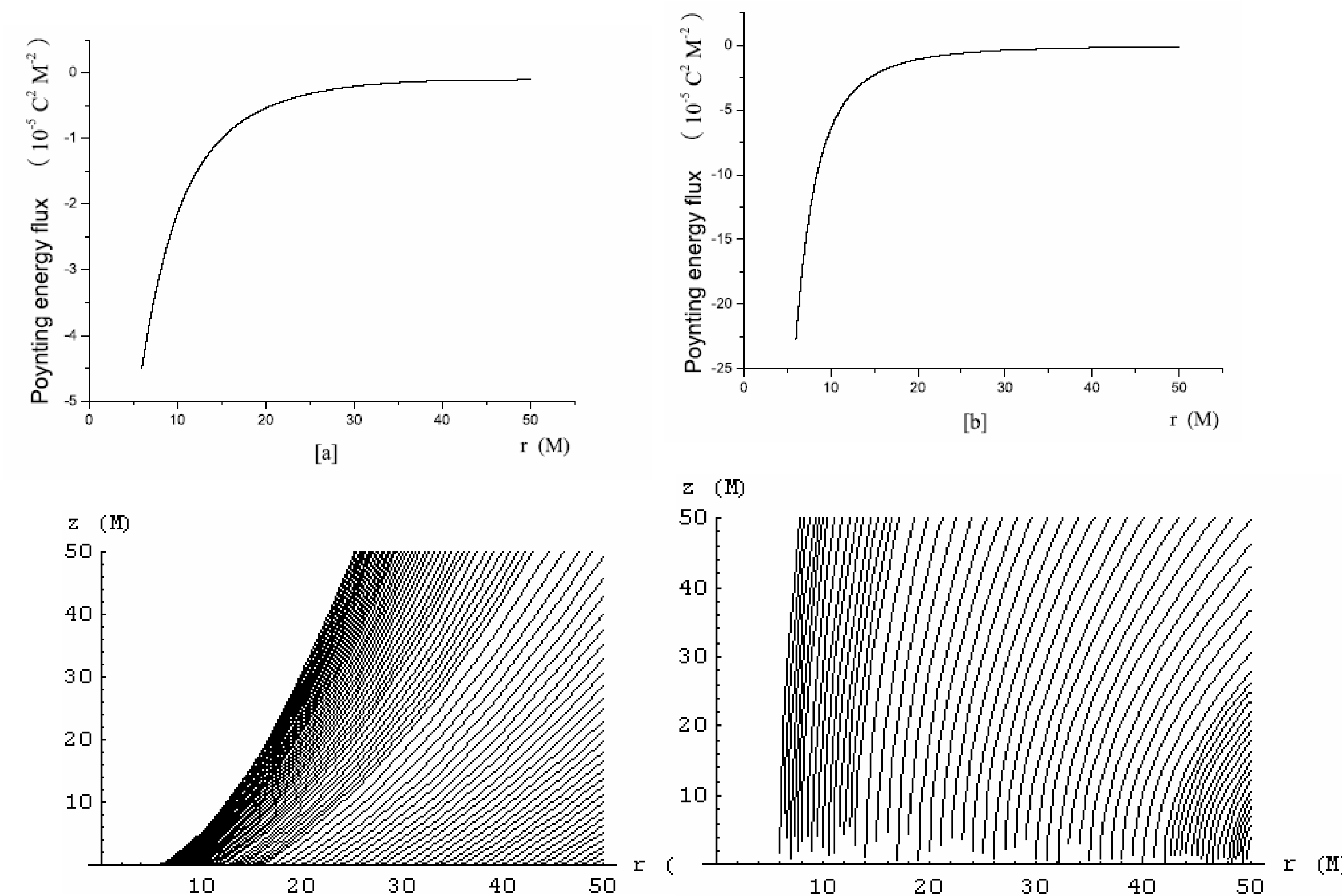


Fig. 6. Poynting energy flux, $\mathcal{E}^{\hat{\theta}}$, in the upper hemisphere.

* $\Omega_F > \Omega_D$

* For smaller accretion rate [a],

  Ω_D changes sign.

* Strength of magnetic field increases

  for smaller radius

  $\Longrightarrow$ Poynting flux enhancement

* Magnetosphere depends on the ratio :

  magnetic - field - strength vs accretion - rate

Poynting flux out of Ergoshpere

HKL and R. Blandford in preperation

- **Consistent condition may not be valid on equatorial plane**

$B^2 - E^2 \geq 0,$ only for $\Omega_-(r) \leq \Omega_F(r) \leq \Omega_+(r)$

$$B^2 - E^2 = -f(\Omega_F, r, \theta = \pi/2)\frac{(B^p)^2}{\alpha^2}$$

$$f(\Omega_F, r, \theta) \equiv \Omega_F^2 g_{\phi\phi} + 2\Omega_F g_{0\phi} + g_{00} = -\left[\frac{\alpha^2 - \varpi^2(\Omega_F + \beta)^2}{\alpha^2}\right]$$

$$\Omega\pm = \Omega_{ZAMO} \pm \frac{\alpha}{\varpi}, \quad \Omega_{ZAMO} = -\beta$$

- **equatorial plane is a non force-free region, for**

$$\Omega(r) \leq \Omega_{-}(r)$$

- **toroidal component of magnetic field can be developed around equatorial plane**

- **Boundary of force-free region above equatorial plane with** $B^{\phi} = B_{+}^{\phi}$

$$-f(\Omega, r, \theta)\frac{(B_{+}^{p})^2}{\alpha^2} + (B_{+}^{\hat{\phi}})^2 = 0$$

- **example**

$$\Omega_F(r) = \frac{1}{2}\Omega_H \left(\frac{r_0 - r}{r_0 - r_H}\right)$$

$$B_+^\theta = -B_0 \frac{r_H}{r}$$

$$B_+^\phi = -B_0 \varpi_H \Omega_F(r)$$

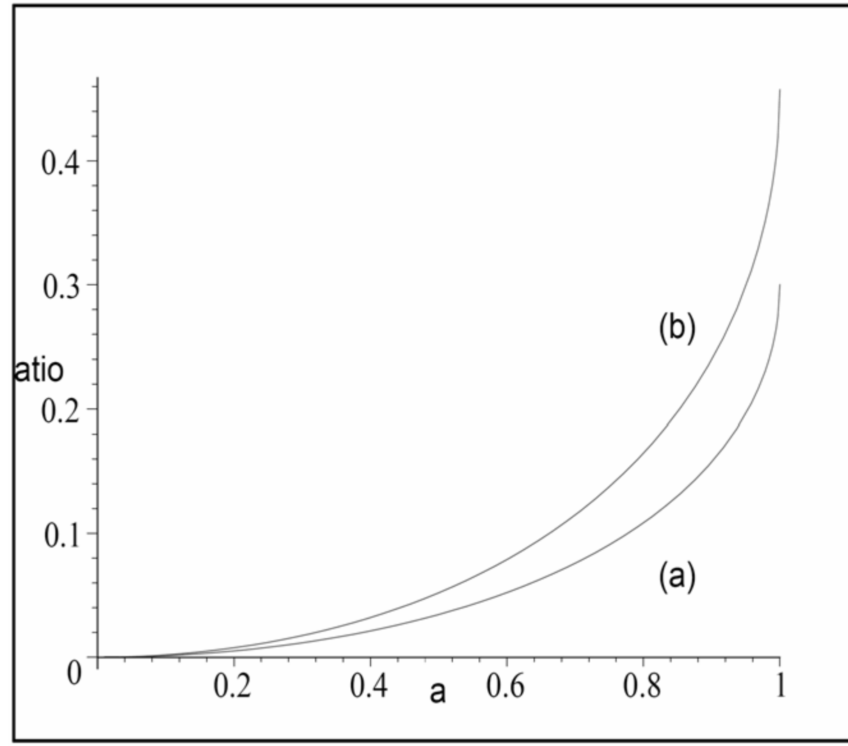


FIG. 2: (a)$P_{ergo}(\tilde{a})/P_{hole}(\tilde{a})$, (b)$P_{ergo}^{L_\phi}(\tilde{a})/P_{hole}^{L_\phi}(\tilde{a})$

Poynting power from ergo sphere can be as much as 30% of Poynting power from horizon.