AdS/CFT at finite density

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- Properties of "realistic" gauge theories may be accessible by appropriate engineering
- Gauge theories at finite temperature and strong coupling can be studied.
- Density and chemical potentials can be easily introduced.

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- The $U(N_f)$ gauge symmetry on the D7 branes in the bulk becomes a global symmetry of the dual gauge theory.
- Since all fundamental "quarks" transform identically under the $U(1)$ of $U(N_f)$, we may regard this as "baryon number".

Chemical Potential

Introducing a particle charged under the U(1) into the D7 world volume costs some energy

$$\mu = -\int_{\infty}^{\rho_{min}} d\rho E(x,\rho) = A_0(\infty) - A_0(\rho_{min}),$$

We identify μ as the chemical potential corresponding to "baryon number"
Alternately, the AdS/CFT recipe uses an asymptotic expansion

$$A_0(x,z) = \mu - \frac{1}{2}\langle\psi^\dagger\psi\rangle z^2 + \cdots .$$

μ couples to a dimension 3 operator $S_{int} \sim -\mu \int dx^4 \psi^\dagger \psi$
The two definitions agree.

Equations of motion

Since we are considering finite temperature as well as finite density, we will use the AdS-Schwarzschild geometry in Poincare co-ordinates.

$$ds^2 = \frac{U^2}{R^2}\left(f(U)dt^2 + d\vec{x}^2\right) + R^2\left(\frac{dU^2}{f(U)U^2} + d\Omega_5^2\right), \qquad (1)$$

where

$$f(U) = 1 - \left(\frac{U_0}{U}\right)^4.$$

The D7 branes wrap an S^3 inside the S^5.

E.O.M

- Rewrite the metric as

$$ds^2 = \frac{U^2}{R^2}\left(f dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left(d\rho^2 + \rho^2 d\Omega_3^2 + dy^2 + y^2 d\varphi^2\right)$$

  where $\xi^2 \equiv y^2 + \rho^2$
- The D7 brane extends along R^4 and the ρ directions.
- We turn on the gauge field $F_{\rho t}$ and solve the DBI equations for $y(\rho)$

Definitions

- Gauss' law allows us to define the total U(1) charge

$$\frac{\partial \mathcal{L}}{\partial F_{\rho t}} \equiv -Q$$

  which is also the density (thermodynamic conjugate to μ).

- The asymptotic expansion for $y(\rho) = L + \frac{\tilde{c}}{\rho^2} + \cdots$, defines the quark mass and condensate

$$L \sim \frac{m_q}{T}, \quad \tilde{c} \sim \frac{\langle \bar{\psi}\psi \rangle}{m_T^3}.$$

DBI Action and Thermodynamic Potentials

The action for the D7 branes is

$$S = \beta V_3 \tau_7 \int d\rho \rho^3 \omega_+^{3/2} \sqrt{\frac{\omega_-^2}{\omega_+}(1 + y'^2) - \tilde{F}_{\rho t}^2} = \beta\Omega$$

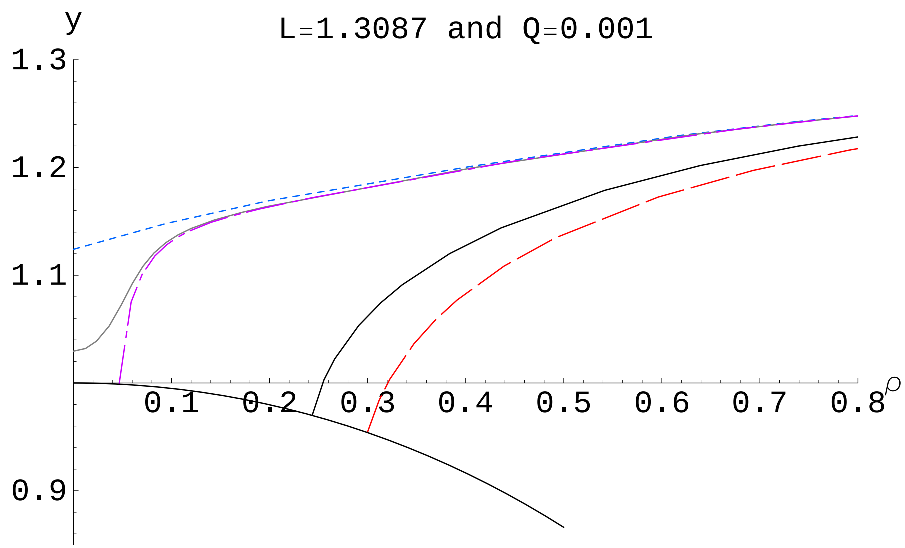
is the <u>grand potential</u> of the dual gauge theory.
Eliminate the gauge field for the charge, we obtain

$$\mathcal{H} = \frac{N_f N_c T^4 \lambda}{32} \sqrt{\frac{\omega_-^2}{\omega_+}(\tilde{Q}^2 + \omega_+^3 \rho^6)} \sqrt{1 + y'^2}, \tag{2}$$

This is the <u>Helmoltz free energy</u> and defines the canonical ensemble.

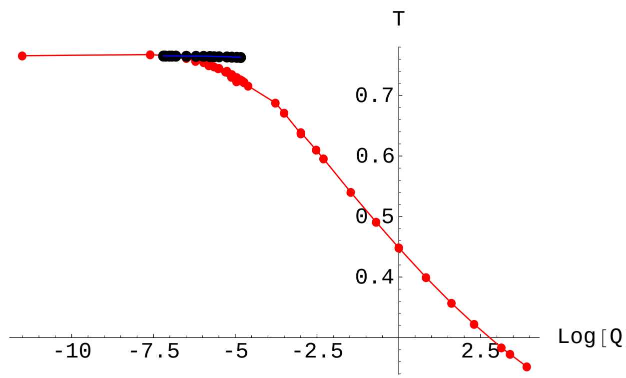
DBI Solutions

The solutions are plotted in the $y - \rho$ plane.



For fixed temperature and quark mass, as we increase density the branes bend more and eventually intersect the black hole.
The least free energy configuration determines the phase of the theory.

Phase transition at finite density



Features:

- First order phase transition between confined and deconfined phases
- First order phase transition b/w deconfined phases for small density
- Second order phase transition point at $Q = 0.0088$

contd

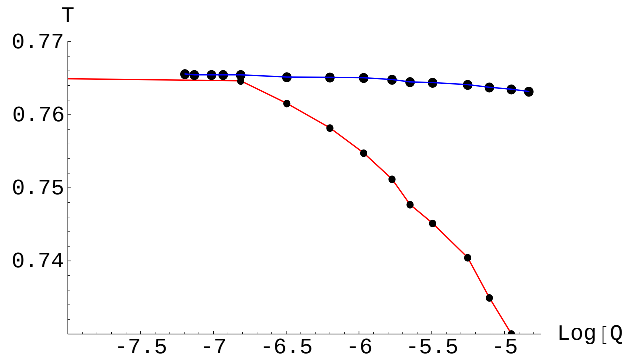


Figure 1: The region of two phase transitions zoomed

Issues and future work

- Charge conservation
- Thermodynamic properties
- Fluctuation spectrum
- Quark potential at finite density
- Back reaction issues