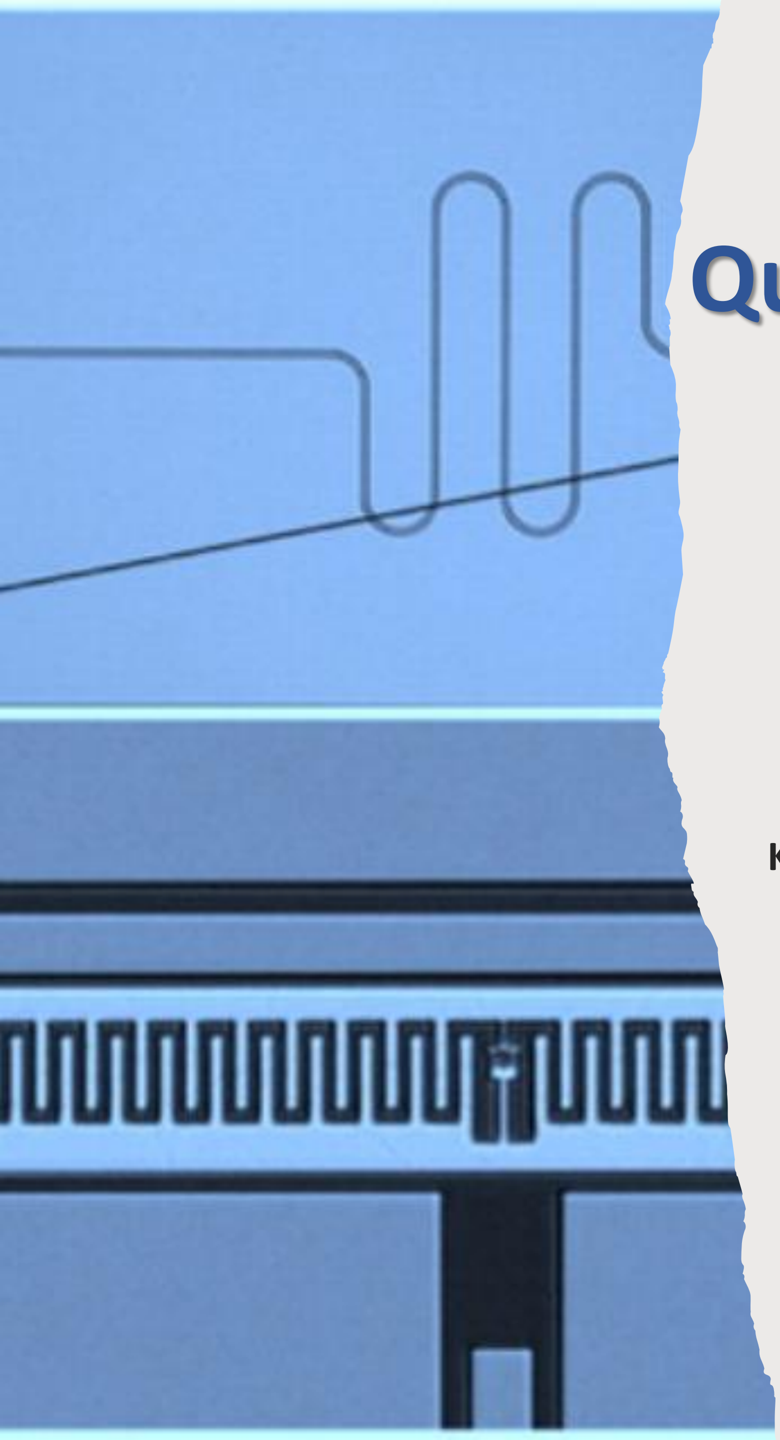

From Cavity to Circuit Quantum Electrodynamics

Dr. Byoung-moo Ann

Senior Research Scientist

Korea Research Institute of Standards and Science (KRISS)

19th May 2023

Outline

Part I - Fundamentals.

Backgrounds : Light-matter interaction.

Introduction to Cavity-QED.

Cavity-QED on circuits : Circuit-QED.

Experimental milestones in circuit QED.

Current trend in circuit QED.

Part II - Methods.

Analytical methods.

- **Rotating frame.**
- **Rotating wave approximation.**
- **Perturbative diagonalization.**

Numerical methods.

- **QuTip**
- **QuCAT** (**Q**uantum **C**ircuit **A**nalysis **T**ool).

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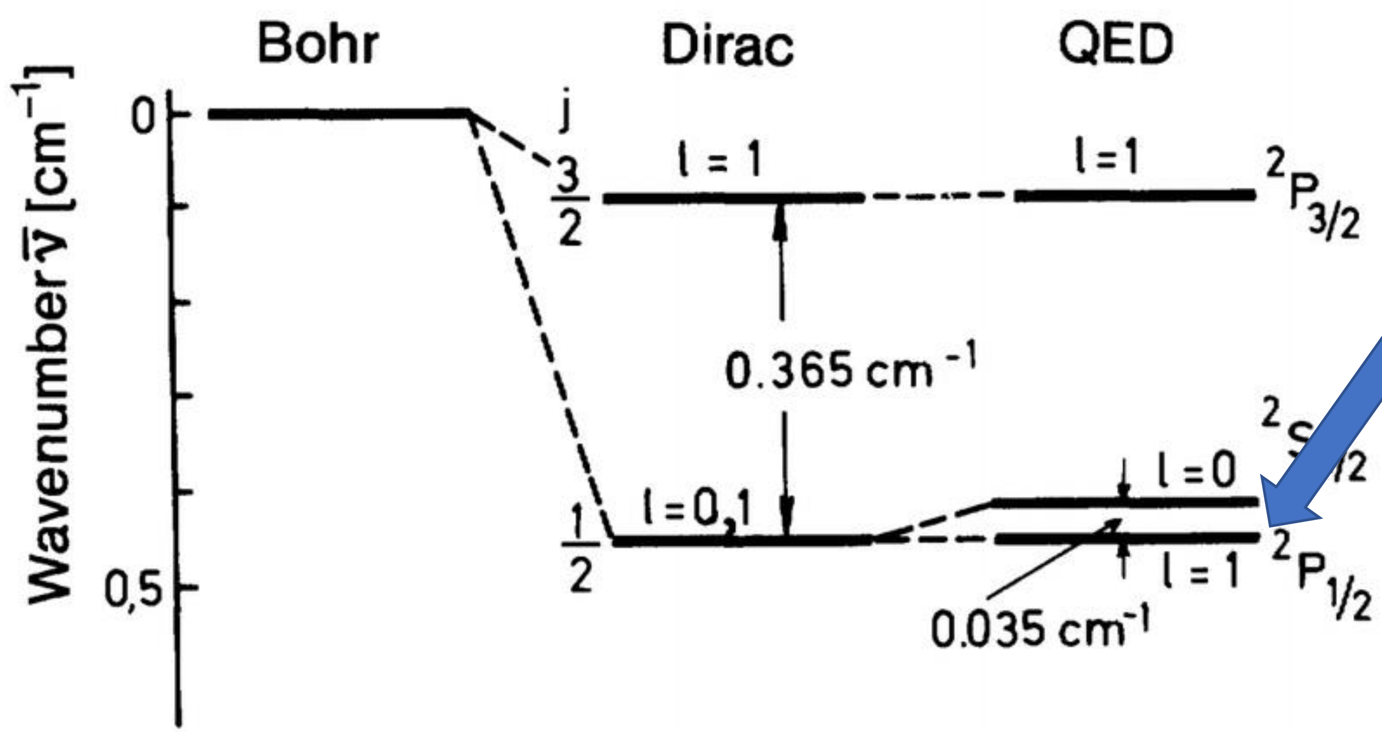
- QuTip
- QuCAT (Quantum Circuit Analysis Tool).

Part I – Quantum ElectroDynamics.

Revealing quantum nature of electromagnetic interaction.



Anomalous frequency shift (Lamb shift).

Feynman, Schwinger, Stueckelberg, Tomonaga, Dyson, …

QED action.

$$S_{\mathrm{QED}} = \int d^4x \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} \left(i\gamma^{\mu} D_{\mu} - m \right) \psi \right]$$

The most successful quantum field theory.

"the jewel of physics"

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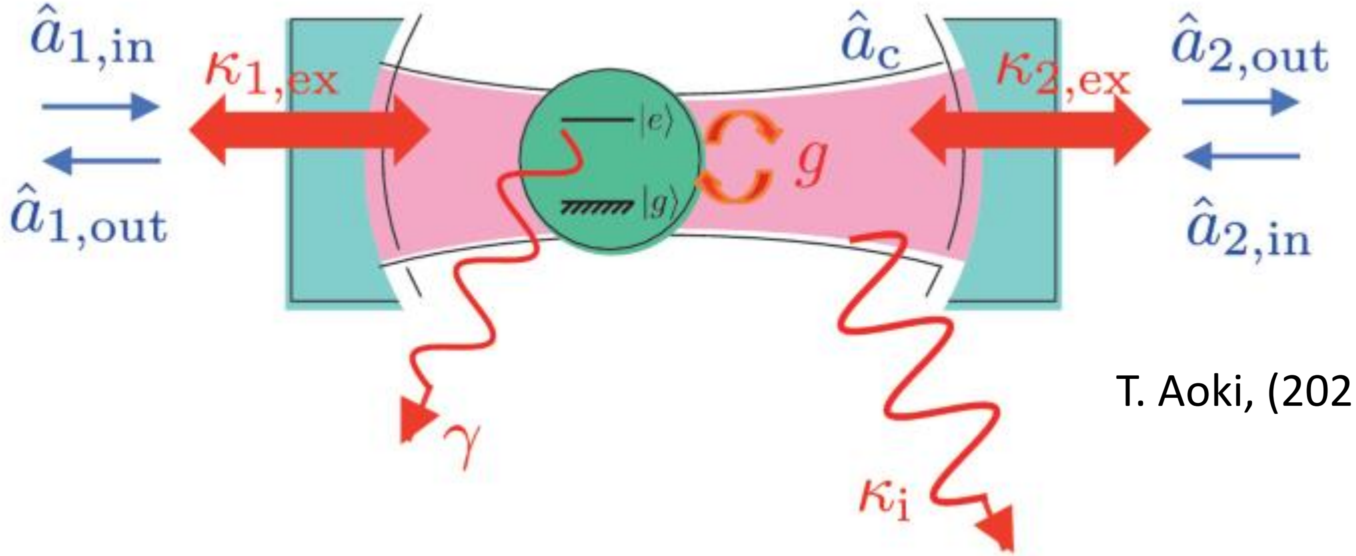
Part I – Cavity QED : The simplest toy-model.

Understanding light-matter interaction at the most fundamental level.



T. Aoki, (2021).

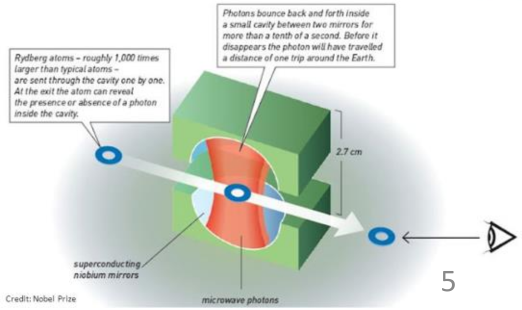
Describe the interaction between 'quantized' matter (atoms) and 'quantized' fields (cavity photons).

The simplest toy-model of QED.

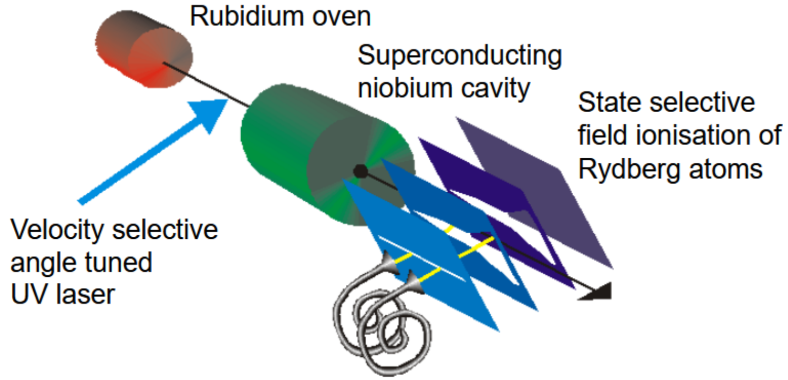
A half of 2012 Nobel prize was shared to works regarding cavity QED.

The Nobel Prize in Physics 2012

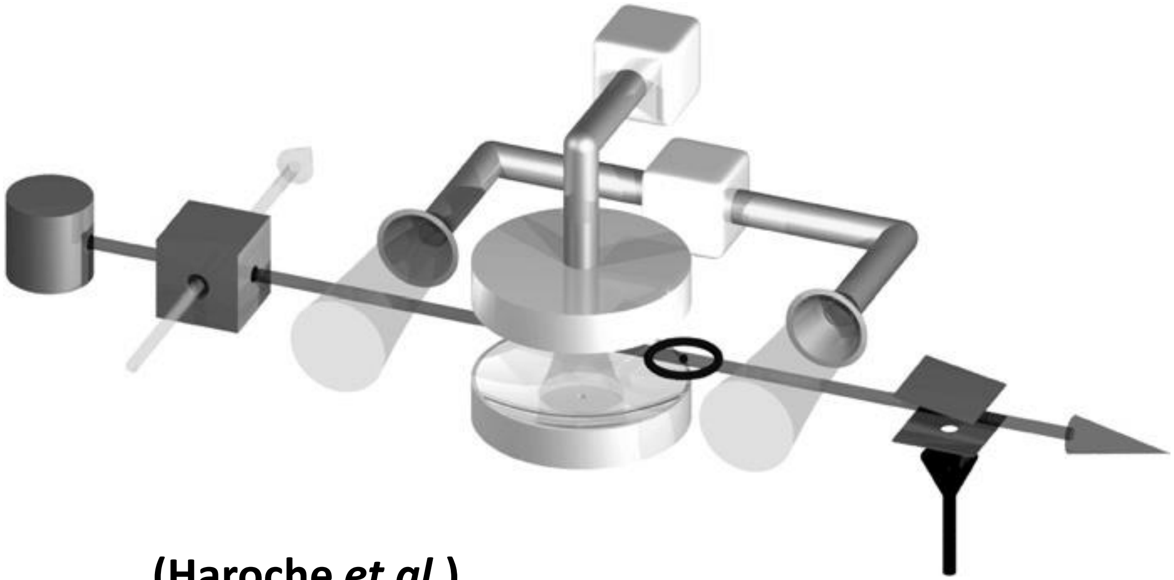
Quantum non-demolition measurement



Part I – Cavity QED in microwave domain.



(Walther *et al.*)



(Haroche *et al.*)

Atom

Atoms in Rydberg states.

Microwave transition frec.

Large dipole strength.

Cavity

Superconducting Fabry-Perot cavity.

$f \sim 100$ GHz.

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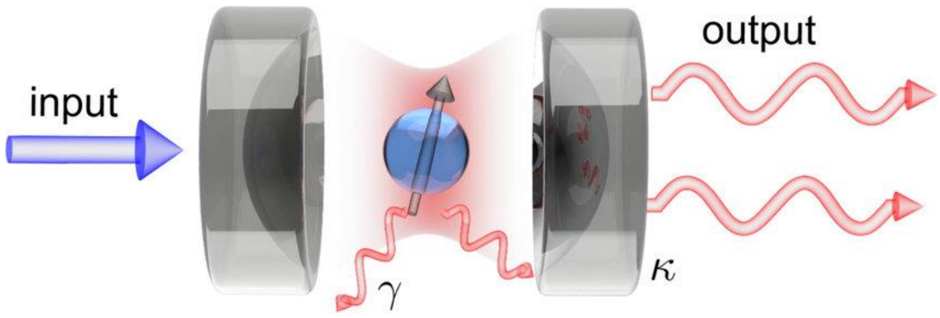
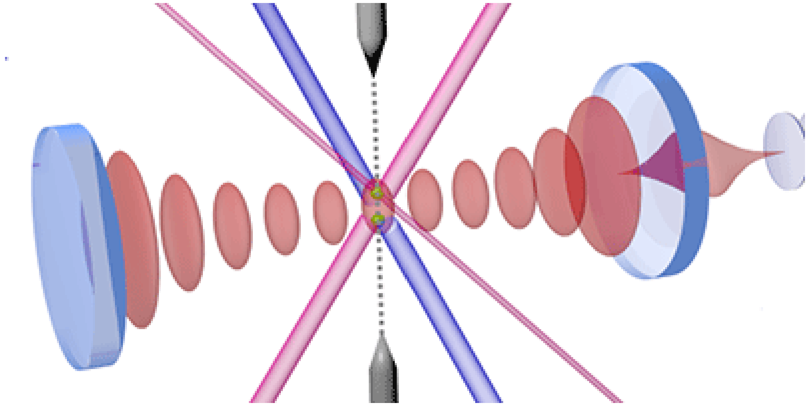
Part I – Cavity QED in optical domain.

Neutral atoms.



(Kimble/Rempe *et al.*)

Ions.

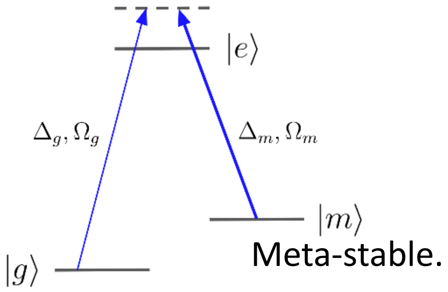


(Blatt *et al.*)

Atom

Various optical transitions.

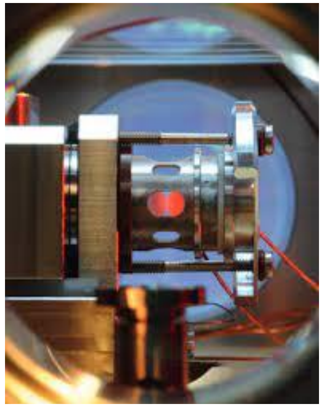
Three-level trick :

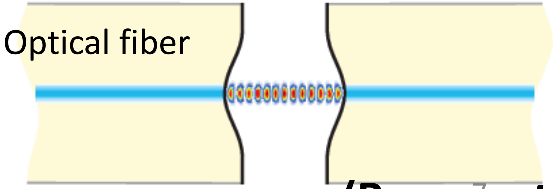


Cavity

Optical Fabry-Perot cavity.

$f \sim 100$ THz.

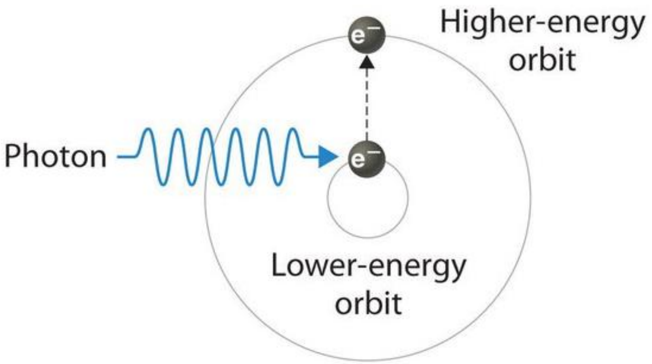




(Rempe *et al.*)

Part I – Light-matter interaction.

An atom under single mode electromagnetic radiation.



EM Field.
(henceforth, called ‘field’).

$$\boxed{H = H_{\mathrm{A}} + H_{\mathrm{AF}} = \hbar\omega_0\sigma^\dagger\sigma - \langle \mathrm{g}|\mathbf{d}|\mathrm{e}\rangle \cdot \mathbf{E}\ (\sigma + \sigma^\dagger)}$$

$$\sigma := |\mathrm{g}\rangle\langle\mathrm{e}|$$

Dipole moment : $\mathbf{d} = -e\mathbf{r}_{\mathrm{e}}$

electron position.

(semi-classical). $\mathbf{E} \sim \cos(\omega t)$ Real number.

(fully quantum). $\mathbf{E} \sim a + a^\dagger$ Field operator.

Photon number state

$$a = \sum_{n=1}^{\infty} |n-1\rangle\langle n|\sqrt{n}$$

mechanical oscillator analogy.

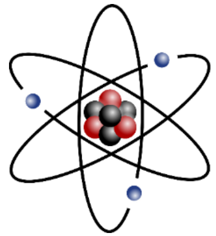
*Reference:

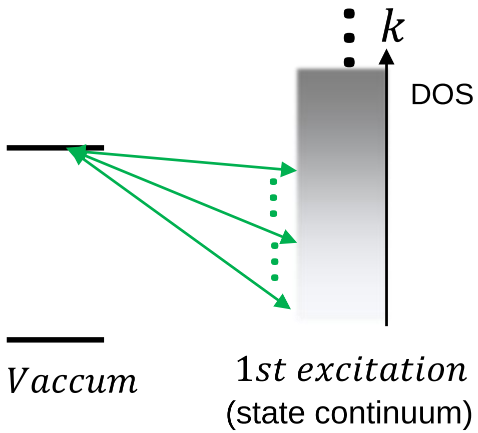
D. Steck, quantum-optics lectures.

https://atomoptics.uoregon.edu/~dsteck/teaching/quantum-optics/quantum-optics-notes.pdf

Part I – How to have quantum fields ?

An atom in a free space.





$$H_{atom} \sim \omega_0 \sigma_z/2$$

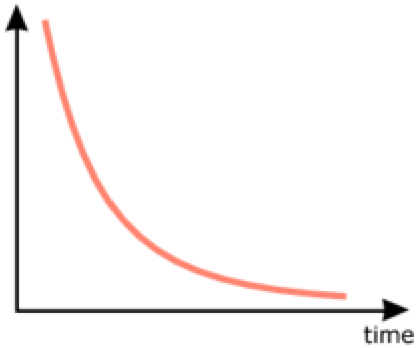
$$H_{em} \sim \int d^3k\ \omega_k N_k$$

$$H_{int} \sim \int d^3k\ D \cdot E_k$$

Free space already provides quantum fields.

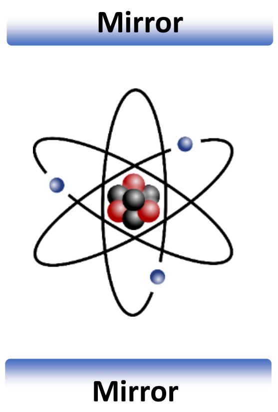
⟶ **Vacuum fluctuation.**

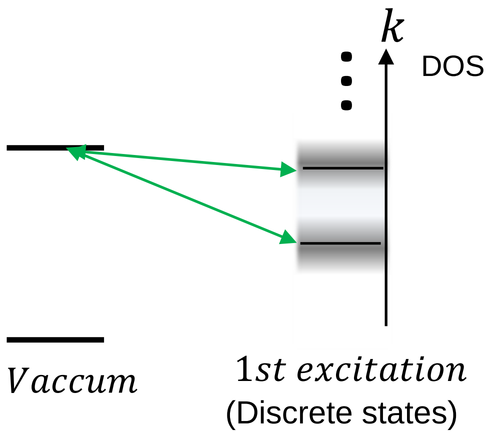
⟶ **Irreversible process only.**
(spontaneous emission, Purcell effect…)



Part I – Necessity of Cavities.

An atom between high-reflectivity mirrors.





$$H_{atom} \sim \omega_0 \sigma_z/2$$

$$H_{em} \sim \Sigma_k \omega_k N_k$$

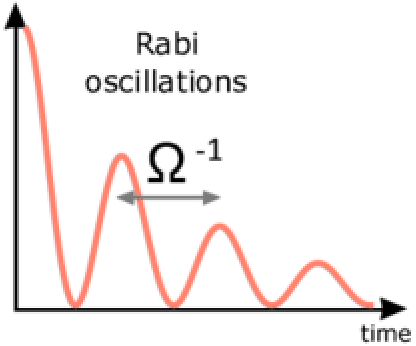
$$H_{int} \sim \sum_k D \cdot E_k$$

Role of cavities :

Amplifying quantum vacuum effects to specific vacuum modes.

Suppress quantum vacuum effects to the other vacuum modes.

$\longrightarrow$ **Coherent process can be realized.**



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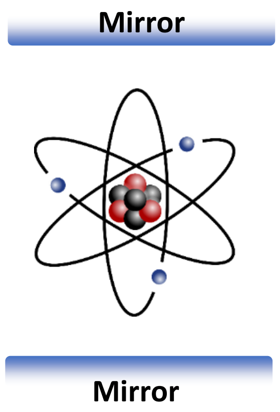
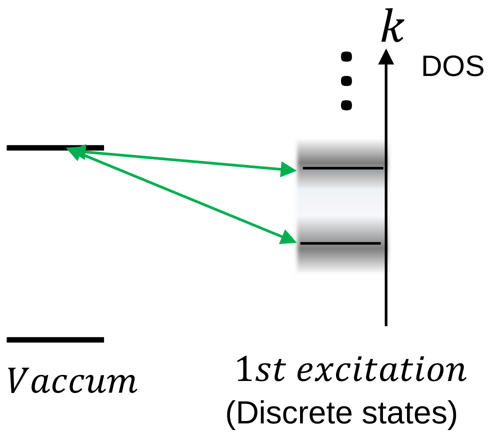
Part I – Necessity of Cavities.

An atom between high-reflectivity mirrors.





$$H_{atom} \sim \omega_0 \sigma_z / 2$$

$$H_{em} \sim \omega N$$

$$H_{int} \sim D \cdot E$$

Couplings to one the cavity modes dominate others.

$\longrightarrow$ **Single-mode cavity-QED : The simplest quantum systems of light-matter interaction.**

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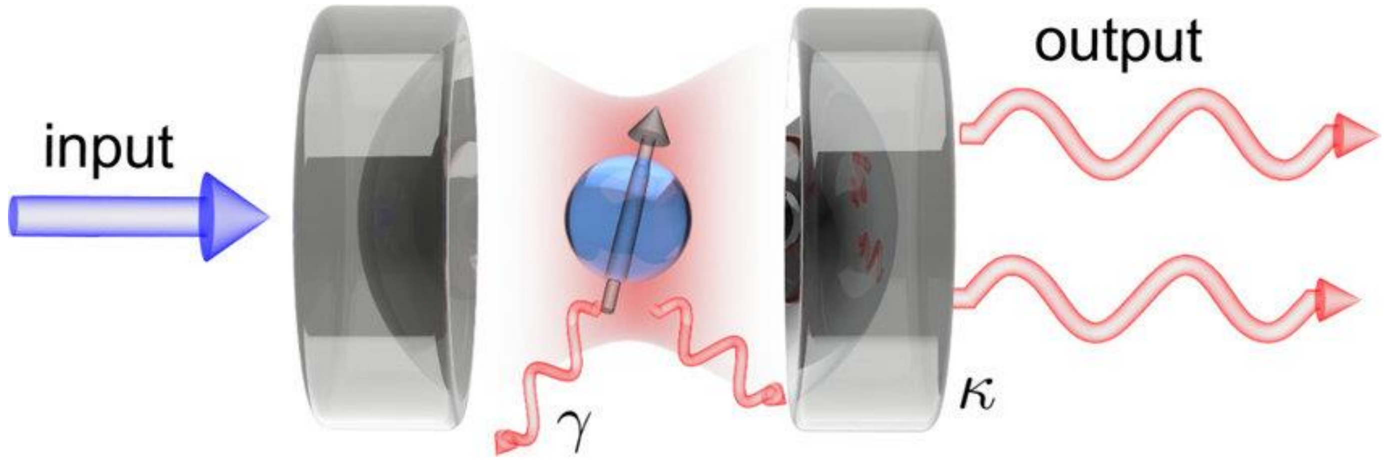
Part I – Cavity QED : Models and key parameters.



Phys. Rev. Research 3, 023079 (2021).

Key parameters :

g	coupling constant.
κ	Resonator decay rate.
γ	Atom decay rate.
$\Delta = \omega_0 - \omega_c$	

Quantum Rabi model : $\hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar\omega_0\frac{\hat{\sigma}_z}{2} + \hbar g\sigma_x(a^\dagger + a)$

Rotating wave approximation.

Jaynes-Cummings model : $\hbar\omega_c\hat{a}^\dagger\hat{a} + \hbar\omega_0\frac{\hat{\sigma}_z}{2} + \hbar g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right)$

+ Dissipation (κ and γ).

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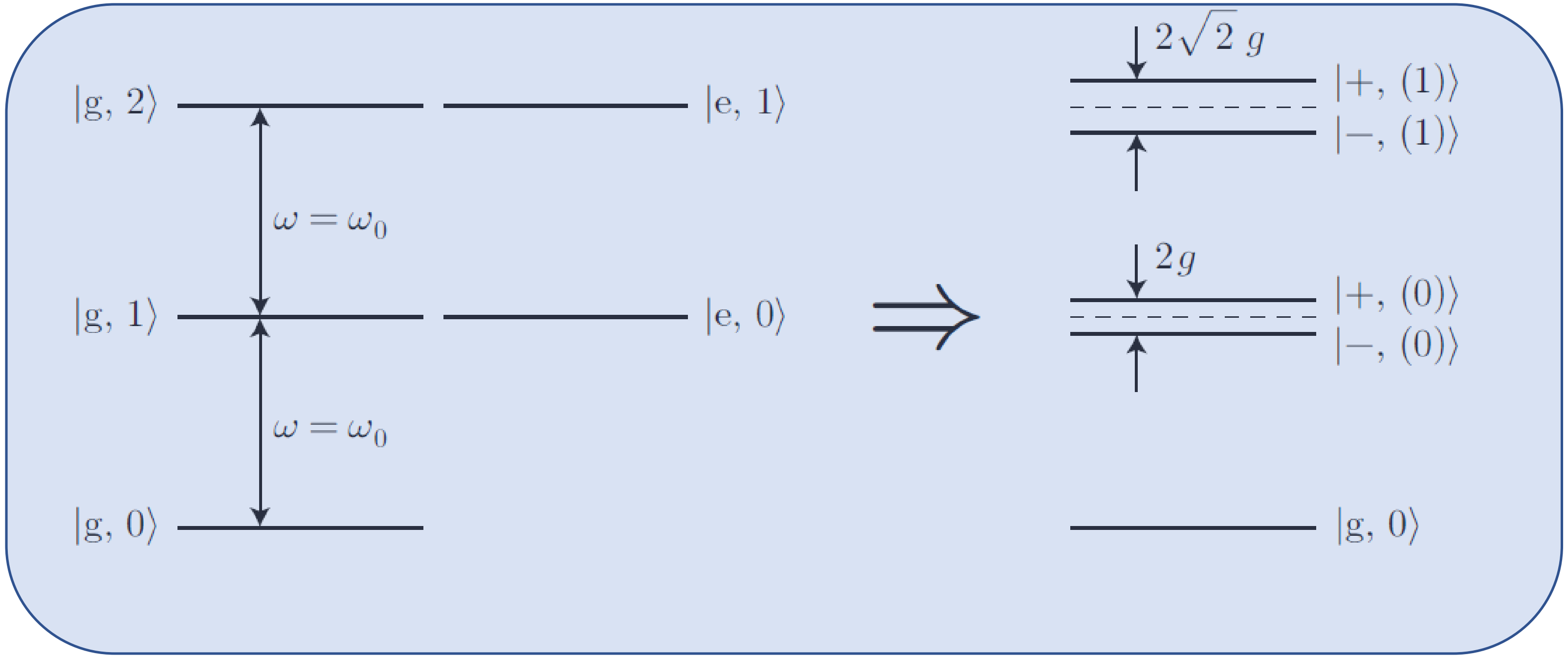
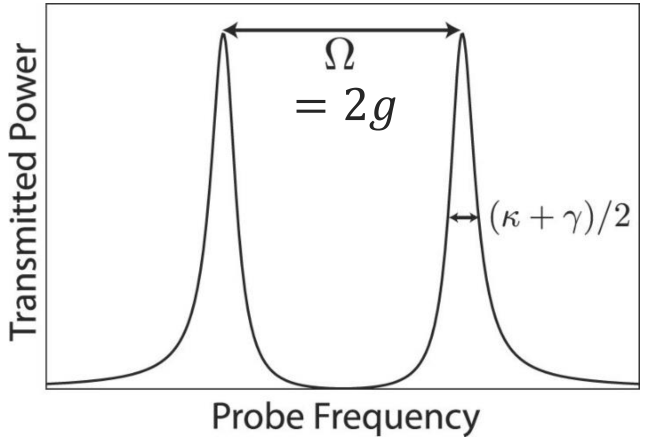
Part I – Cavity QED : Strong resonant regime. $g < \gamma, \kappa \;\; ; \Delta = 0$

Atom and cavity become fully hybridized.

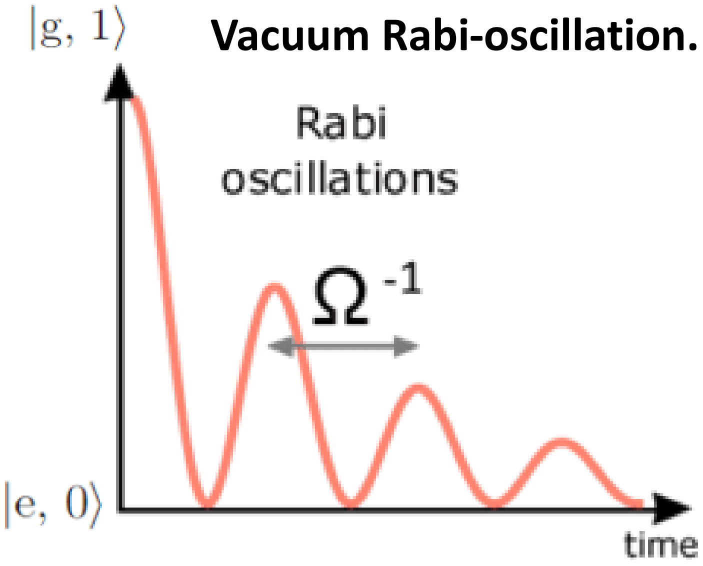
→ Not distinguishable.



Vacuum Rabi-splitting.



Vacuum Rabi-oscillation.



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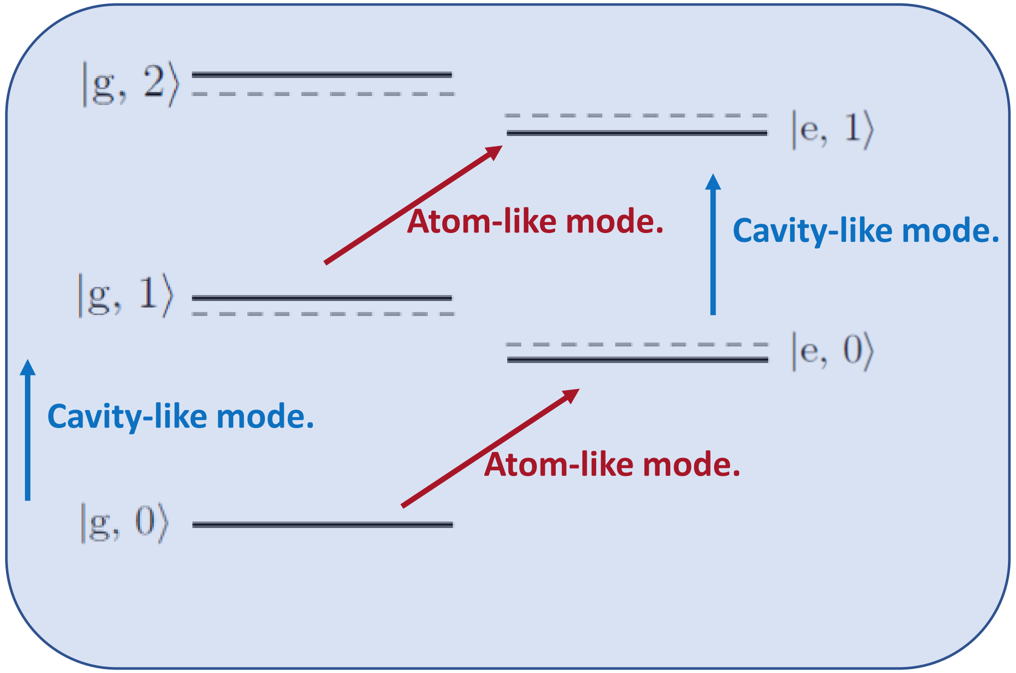
*Reference:

D. Steck, quantum-optics lectures.

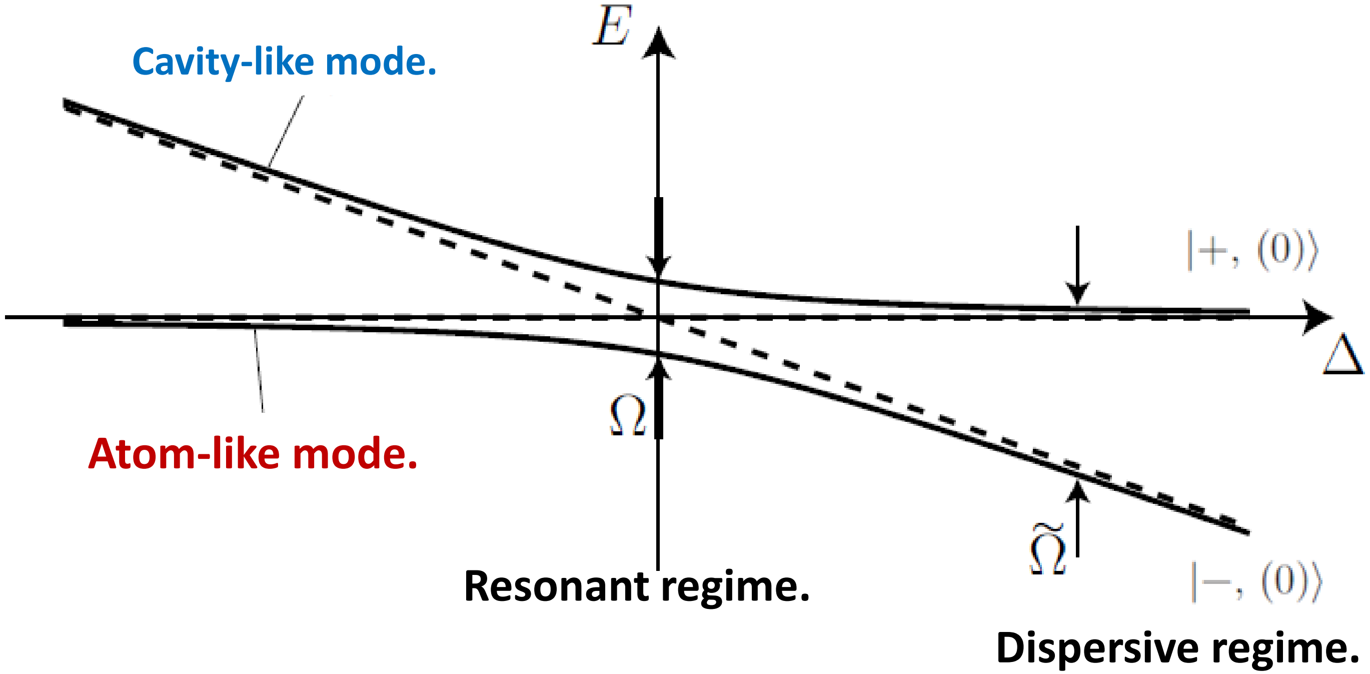
https://atomoptics.uoregon.edu/~dsteck/teaching/quantum-optics/quantum-optics-notes.pdf

Part I – Cavity QED : Dispersive regime. $g \ll \Delta$

Atom and cavity conserve their originalities but undergo renormalization in their transition frequencies.



↓ Sweeping cavity resonant frec across atomic frec...



*Reference:

D. Steck, quantum-optics lectures.

https://atomoptics.uoregon.edu/~dsteck/teaching/quantum-optics/quantum-optics-notes.pdf

Part I – Cavity QED : Milestones – Non-demolition measurement.

In the dispersive regime, atom-cavity interaction can be reduced to $H_I \sim \chi \hat{a}^{+} \hat{a}\, \hat{\sigma}_z .$

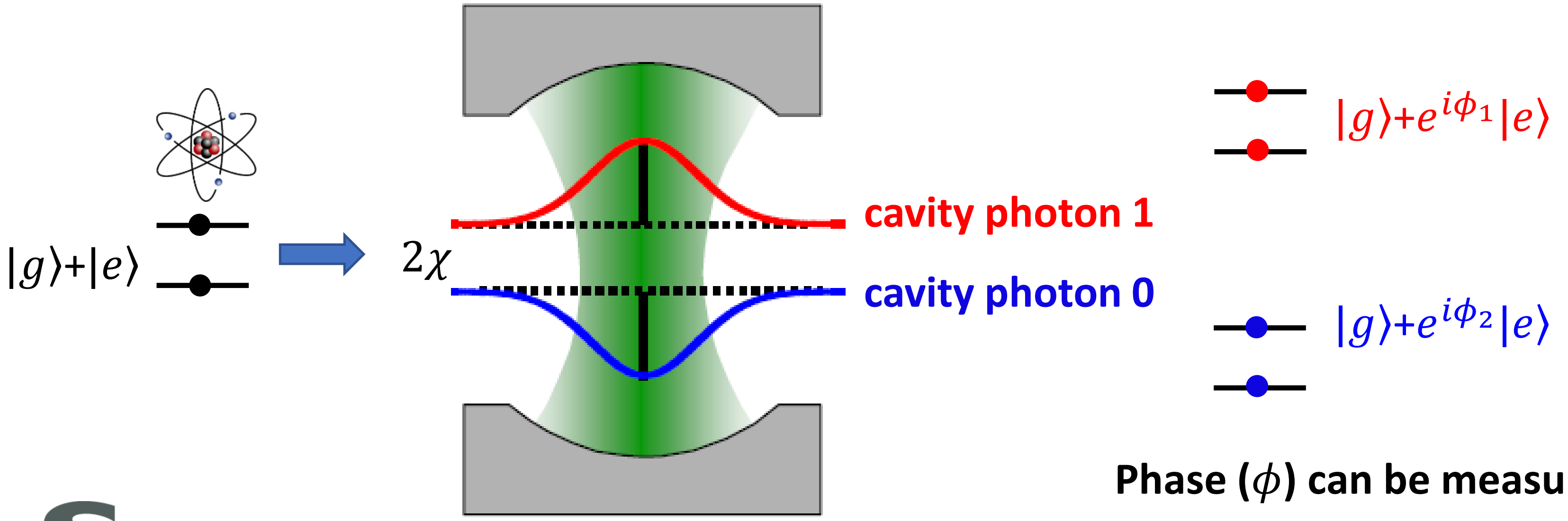
cavity atom

Strong dispersive regime.

$$\gamma, \kappa < \chi \sim \frac{g^2}{\Delta}$$

We can measure the states of atom (cavity) through cavity (atom) **without destroying** (non-demolition measurement).

**But not perfectly non-demolition for large photon numbers (Why?).*



Phase (ϕ) can be measured by Ramsey interferometry.

No energy interchange between atoms and photons.

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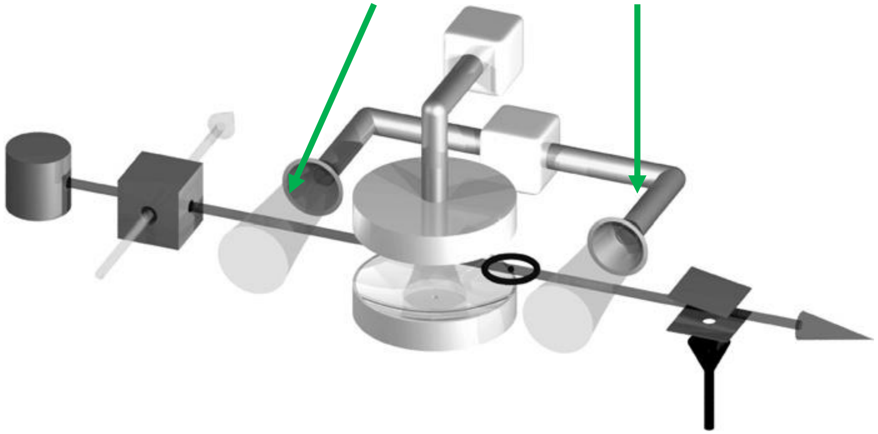
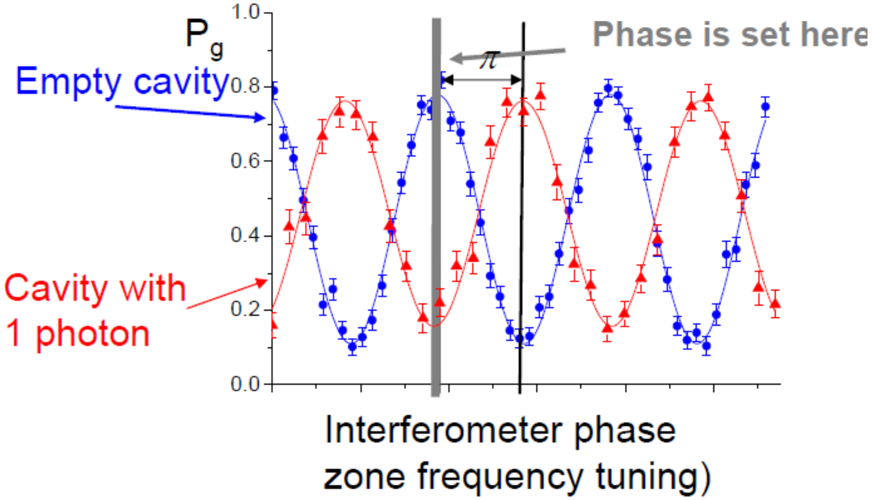
Part I – Cavity QED : Milestones – Non-demolition measurement.

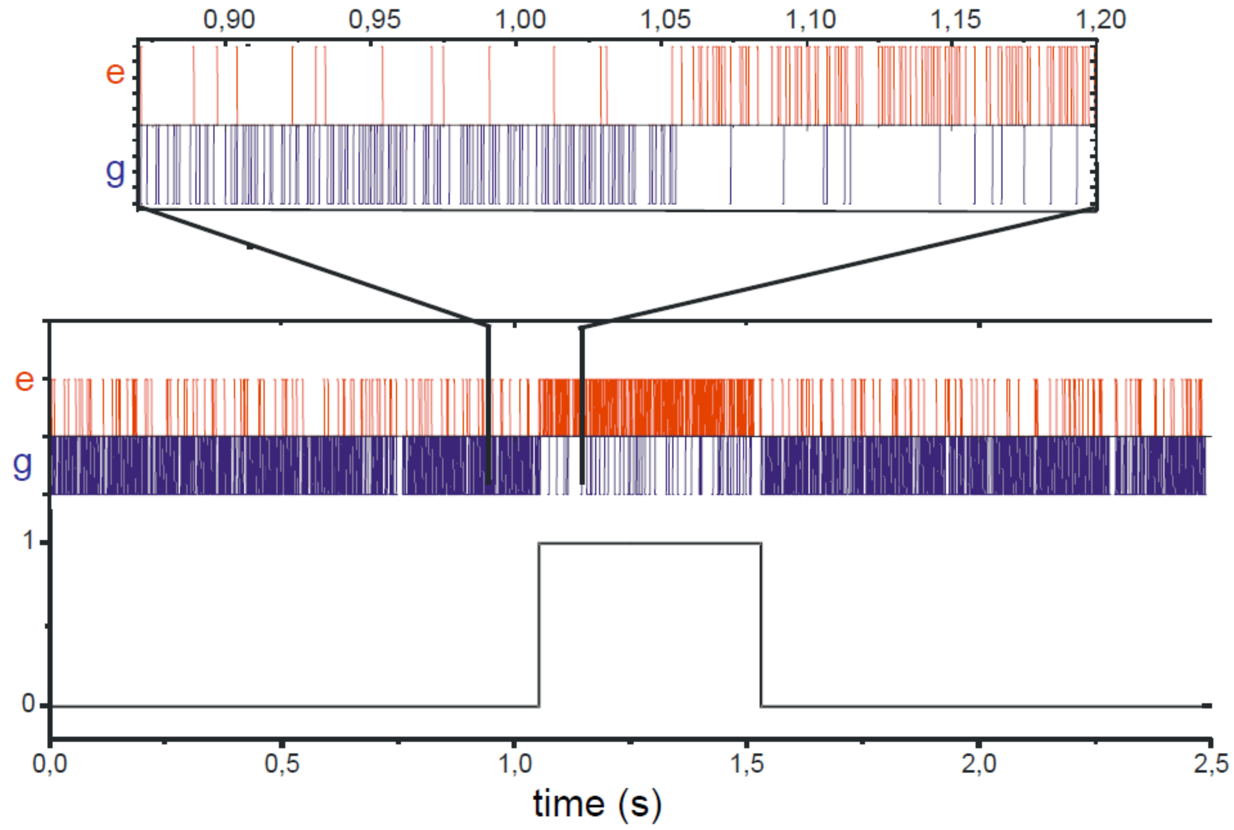
For Ramsey interferometry.



Phase measurement.



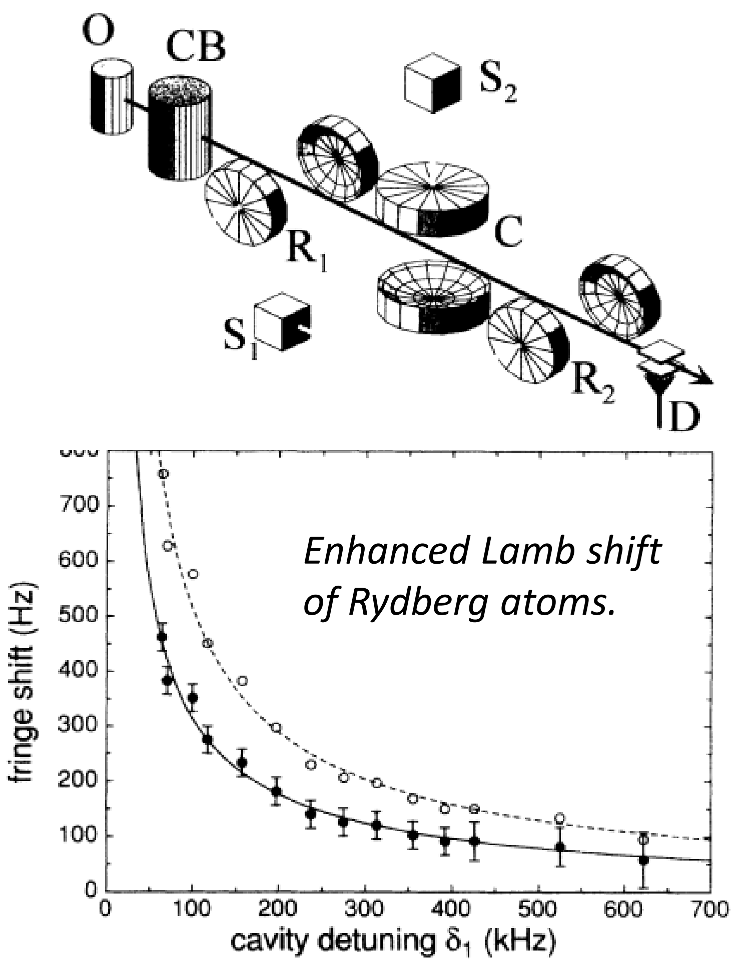
Real-time photon records.



Serge Haroche, FRISNO-Les Houches; February 12 2007.

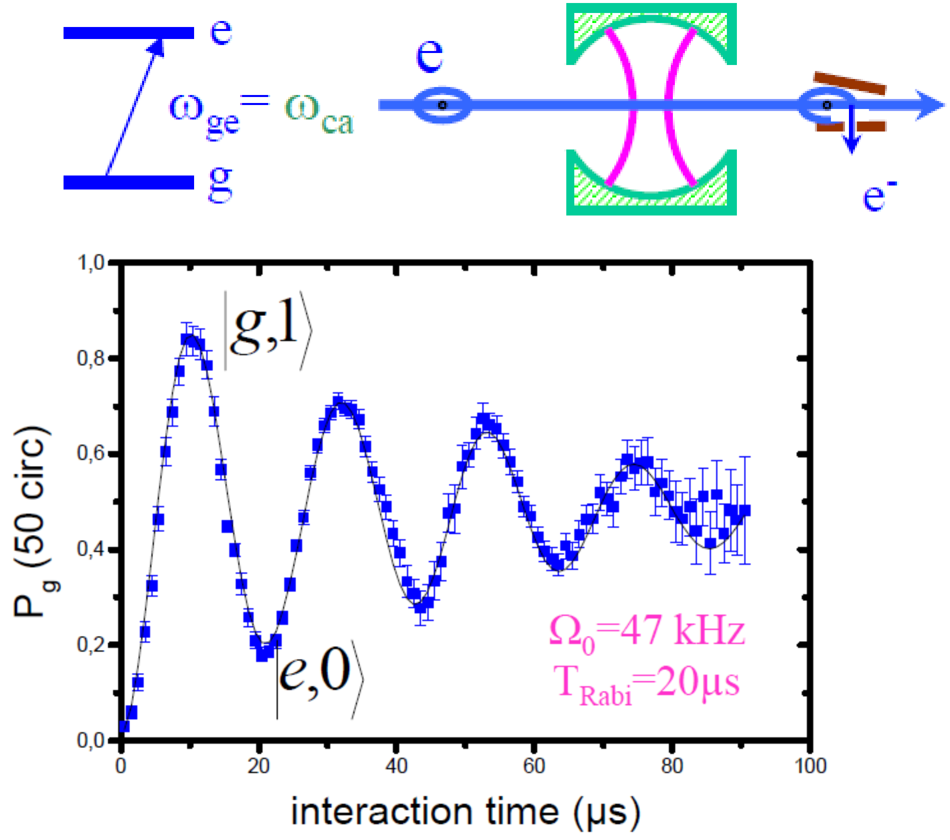
Part I – Cavity QED : Milestones – Quantum vacuum effects.

Single-mode Lamb shift.



Phys. Rev. Lett. ***72****, 3339 (1994).*

Vacuum Rabi oscillation.

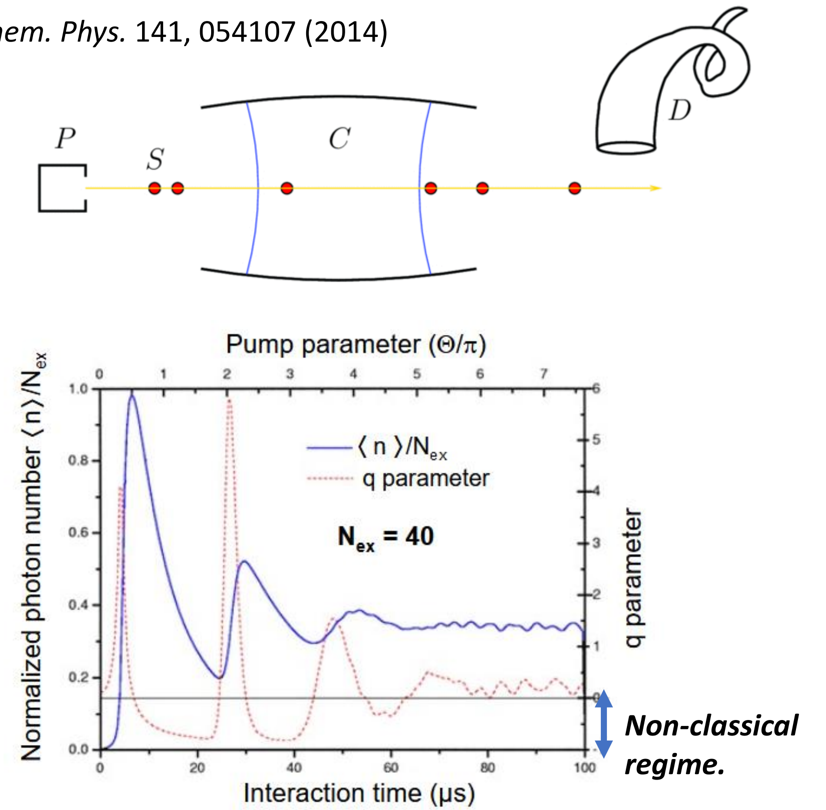


Phys. Rev. Lett. ***76****, 1800 (1996).*

Part I – Cavity QED : Milestones – Nonclassical lasing.

Single-atom maser.

J. Chem. Phys. 141, 054107 (2014)

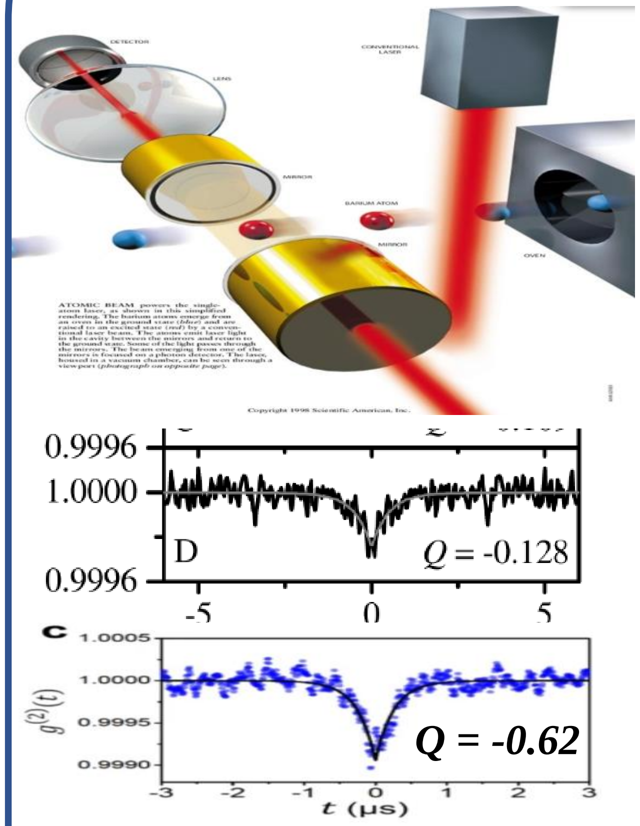


Experiment : Walther, Meschede, Rempe, …

Theory : Meystre, Scully, Zubairy, …

Single/Multi-atom laser.

Atom stream : K. An



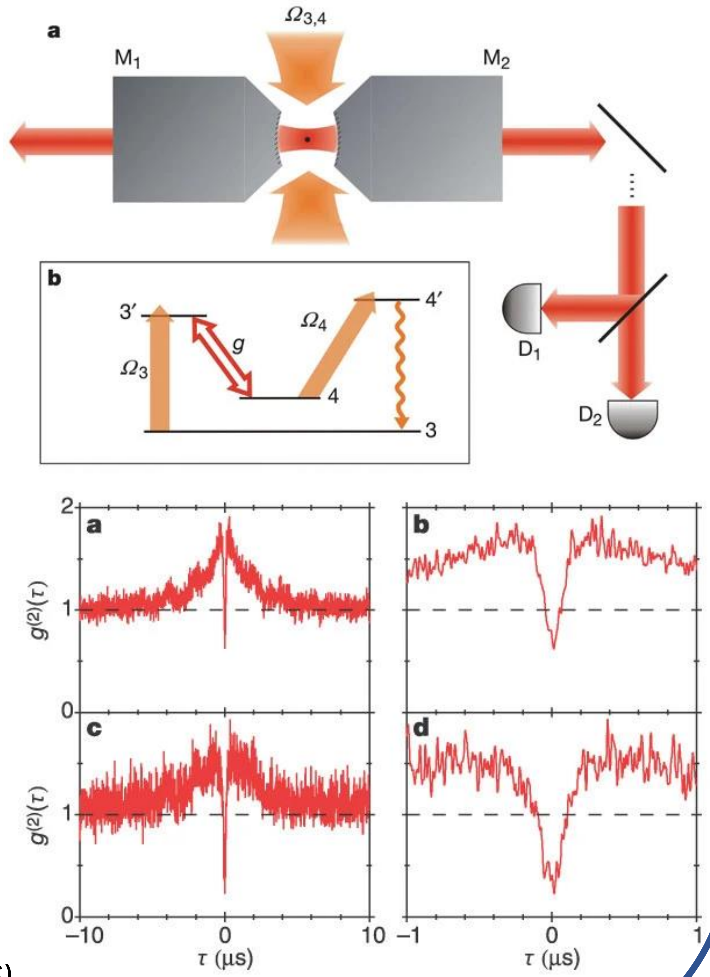
Phys. Rev. Lett. ***73*** *3375 (1994).*

Sci. Am. ***279*** *1 (1998).*

Phys. Rev. Lett. ***96*** *093603 (2006).*

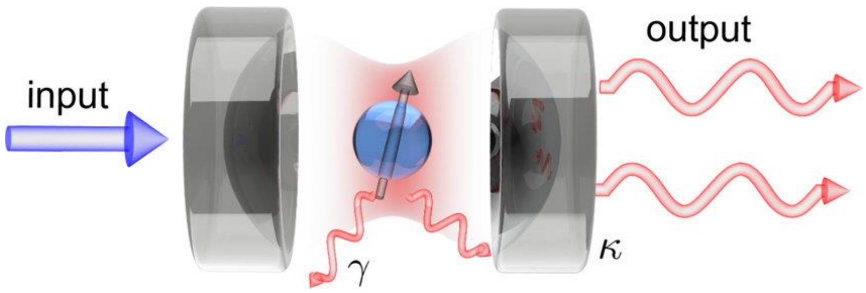
Sci. Rep. ***9*** *17110 (2019).*

Trapped atom : J. Kimble



Nature ***425*** *268 (2003).*

Part I – Cavity QED : Challenges.



Strong nonlinearity + metastable states.

Atomic parameters are already known.

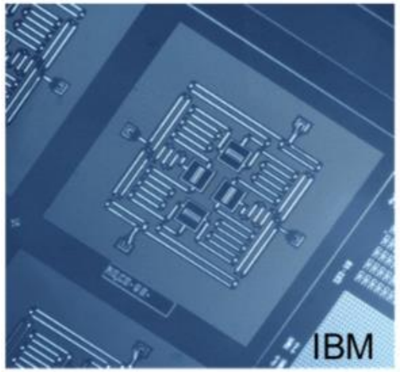
No flexibility in atomic parameters.

Technical overheads:

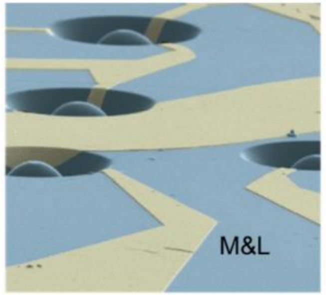
Limited atom-cavity coupling strengths.
Mechanical stability, laser coherence, cooling, trapping...

Alternative systems ?

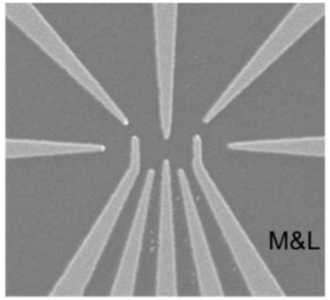
What about using ‘artificial atoms’ defined on solid state platforms?



Superconducting Qubits



Engineered Defects



Quantum Dots

(M.B. Ritter, IEDM 2018 short course).

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QuTip

QuCAT (Quantum Circuit Analysis Tool).

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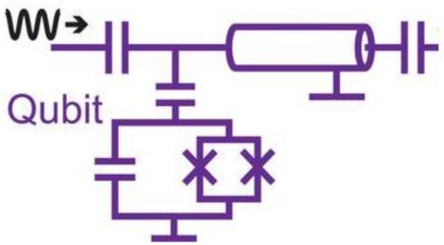
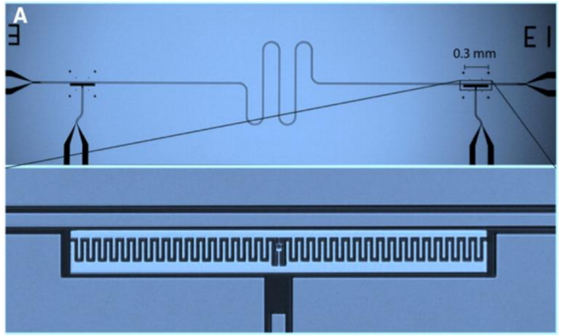
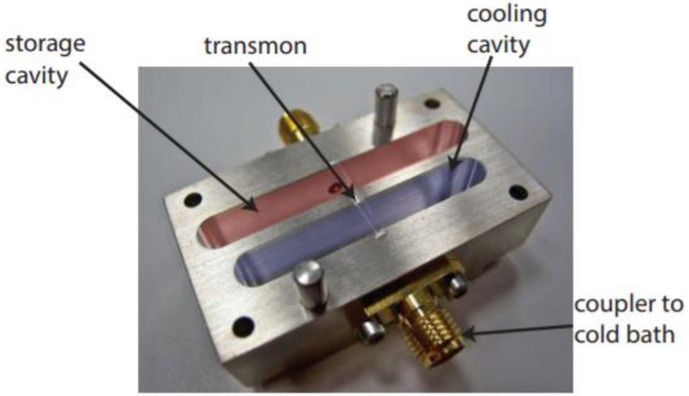
Part I – Introduction to circuit QED.

Atom → **Super Conducting Qubits** + **Cavity** → **SC Resonators.**





2D



3D

Requirements :

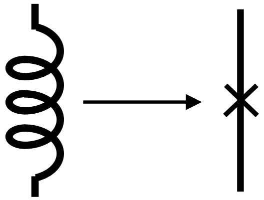
Operation temperature → 10 mK.

Operation frequency → 4-8 GHz.

Lumped + Distributed circuit elements.
ex) transmission lines.

Superconductivity:
Reduce degree of freedoms to unity.

Josephson junction.



$L \rightarrow L_J$

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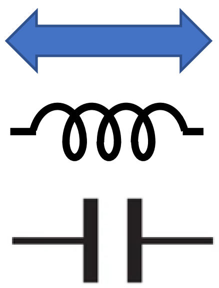
Part I – Ingredients of circuit QED : Linear elements.

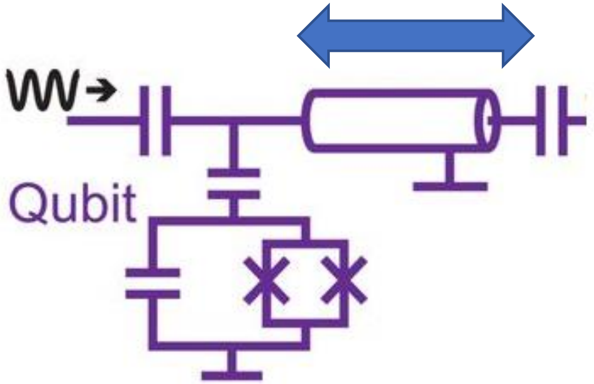
Lumped elements.

$\lambda \gg d$



Distributed elements.

$\lambda \lesssim d$



λ Wavelengths

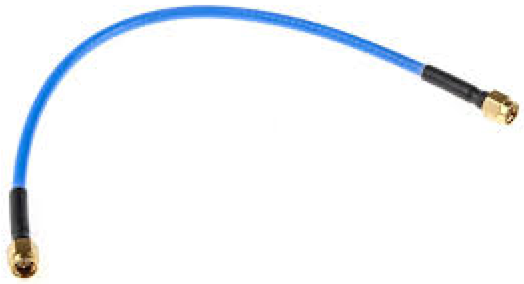
d Sizes of elements.

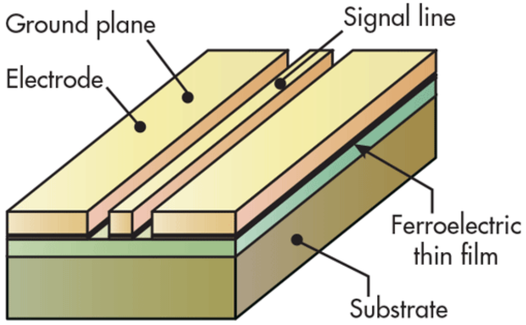
**Reading list :*

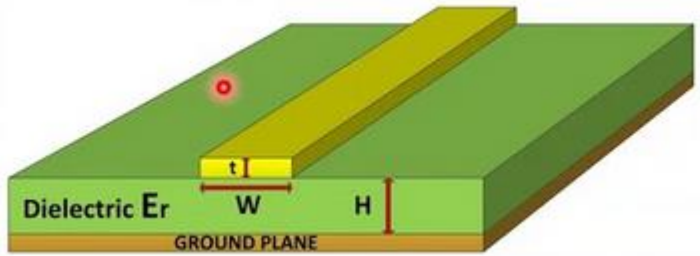
Microwave Engineering, David M. Pozar.

Part I – Ingredients of circuit QED : Linear elements - transmission lines.

Types of transmission lines.



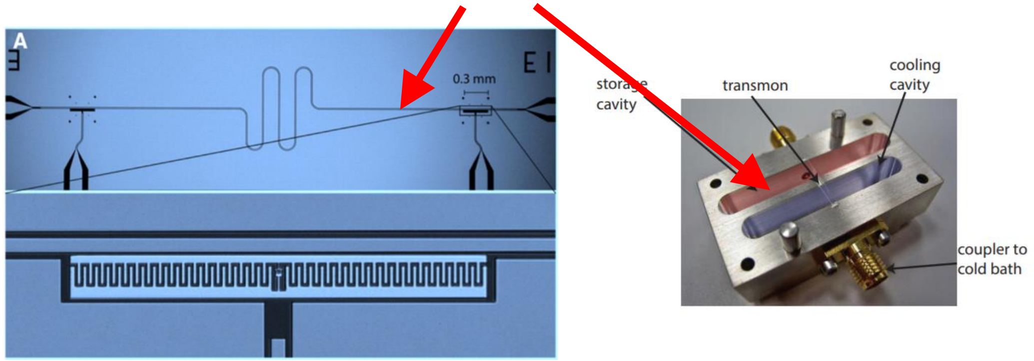




Symbol

Center pin
(signal)

In Circuit QED device



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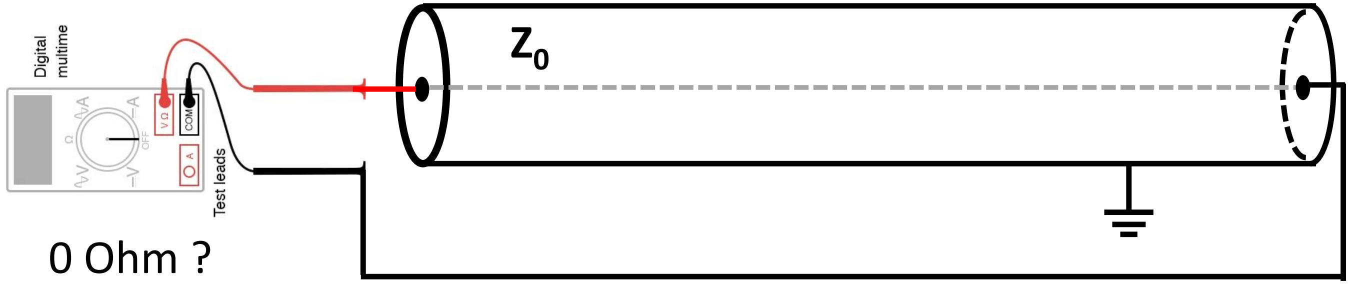
Part I – Ingredients of circuit QED : Linear elements - transmission lines.

50 Ohm 75 Ohm

Characteristic impedance Z_0 of TL.

What do these values mean ?

inf Ohm ?



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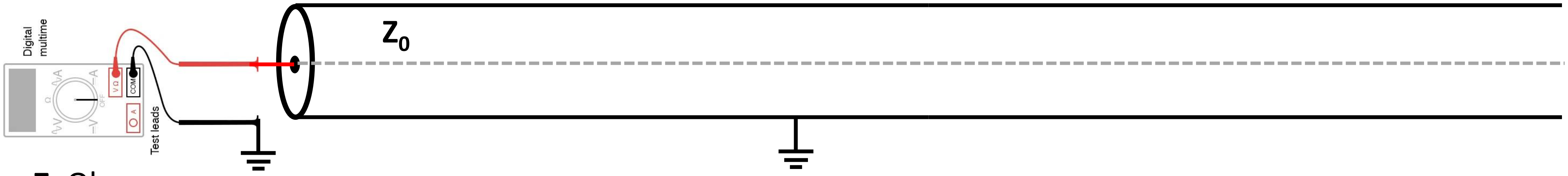
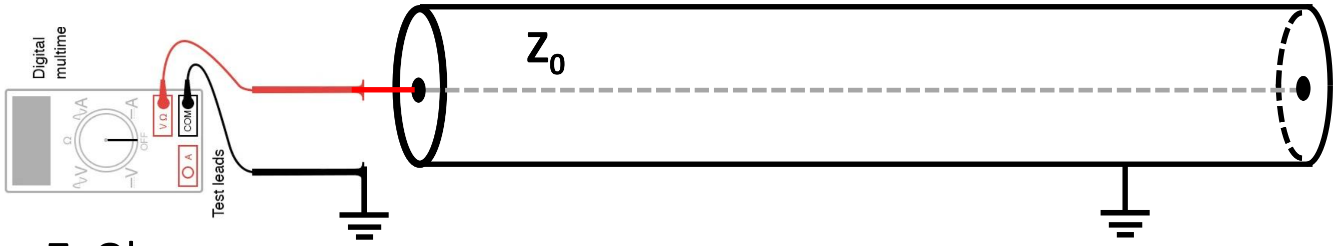
Part I – Ingredients of circuit QED : Linear elements - transmission lines.

If you have an infinitely long transmission line...



Z_0 Ohm.

If you immediately readout the Ohm meter before the probe signal propagates...



Z_0 Ohm.

But... these are imaginary situations.

How can we measure Z_0 in practice?

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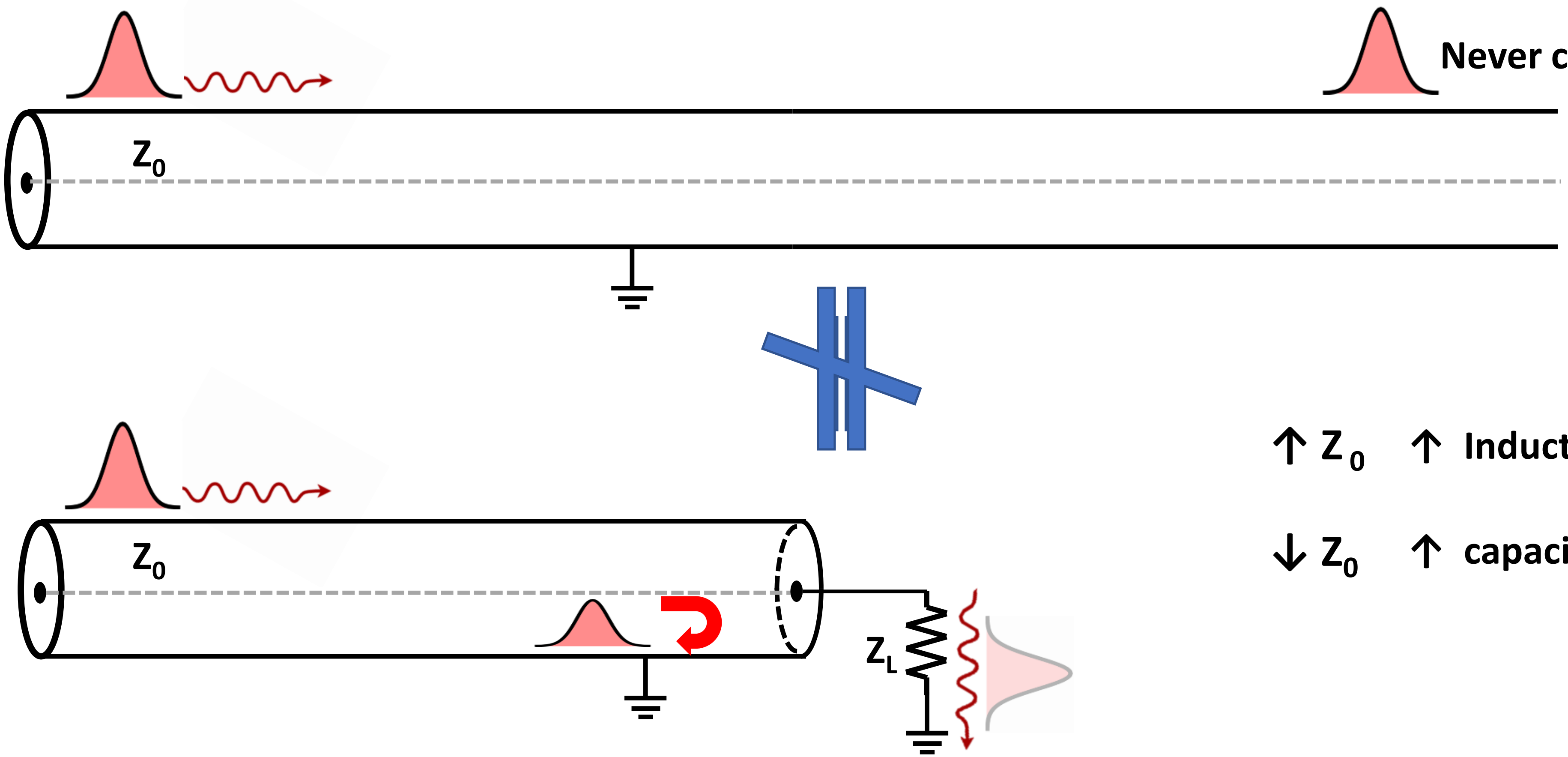
Part I – Ingredients of circuit QED : Linear elements - transmission lines.



$\uparrow Z_0$ $\uparrow$ Inductive energy.

$\downarrow Z_0$ $\uparrow$ capacitive energy.

The pulse is totally dissipated when $Z_0 = Z_L$ (Impedance matching).

Otherwise, not fully dissipated and the remaining part of the pulse will be reflected back.
(Impedance mismatching).

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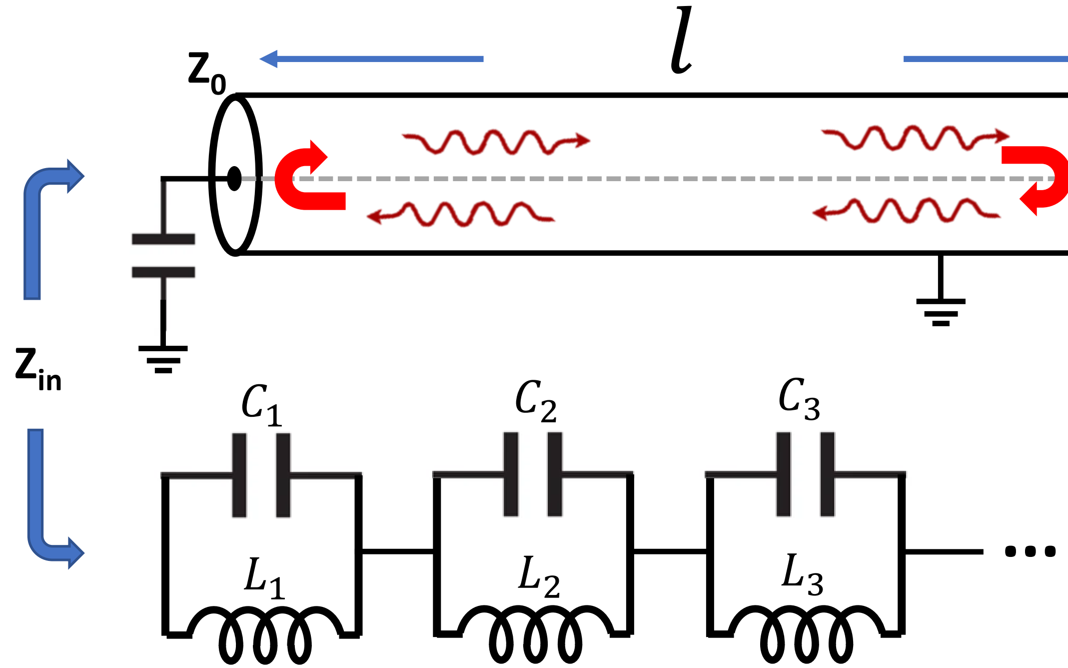
Part I – Ingredients of circuit QED : Linear elements – TL resonators.

Transmission lines + boundary conditions ➡ Tansmission-Line (TL) resonators.



Foster decomposition.

Analogy

Musical instrument.

LC resonators corresponds to modes of TL resonators.

Normally <u>the lowest mode</u> LC resonator is mainly used.

- **Quantitative meaning of Z_0 :**

  Z_0 determines the ratio between L_n and C_n.

  Z_0 together with l determines the resonator frec $\omega_n = \frac{1}{\sqrt{L_n C_n}}$.

  ➡ Z_0 and l fully characterise TL resonators.

- **Downsides :**

  Difficult to design very high or low Z_0 TL.
  Undesired higher modes.

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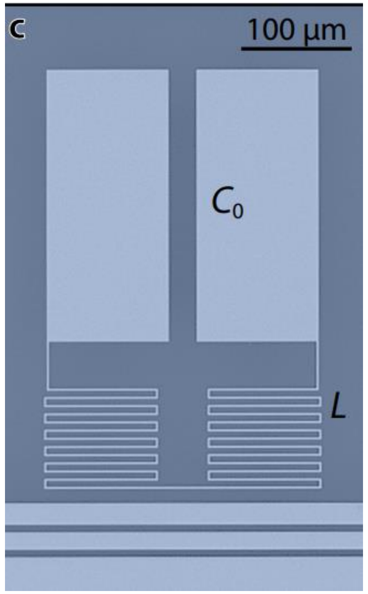
**Reading list :*

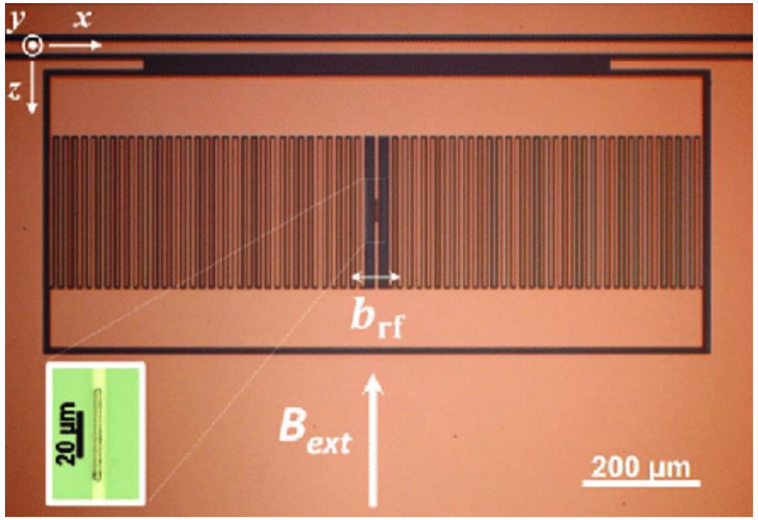
Chapter 6, Microwave Engineering, David M. Pozar.

Part I – Ingredients of circuit QED : Linear elements – LE resonators.

Resonators without TL : Lumped-Element (LE) resonators (useful when you need very high or low impedance).



High-impedance resonator.



Low-impedance resonator.

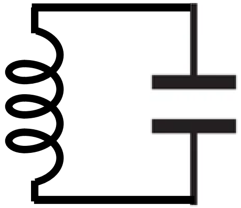
Directly fabricating inductors and capacitance.

$$\lambda \gg d$$

- **Downside :**

  Overheads when you need multi-mode resonators.

Part I – Circuit QED : Circuits ↔ Mechanical systems.



Canonical coordinate ?
Conjugate momentum ?

Flux (Φ) and **charge(Q)** variable are preferred.

$$\frac{dV}{dt} = \Phi, \quad \frac{dQ}{dt} = I.$$

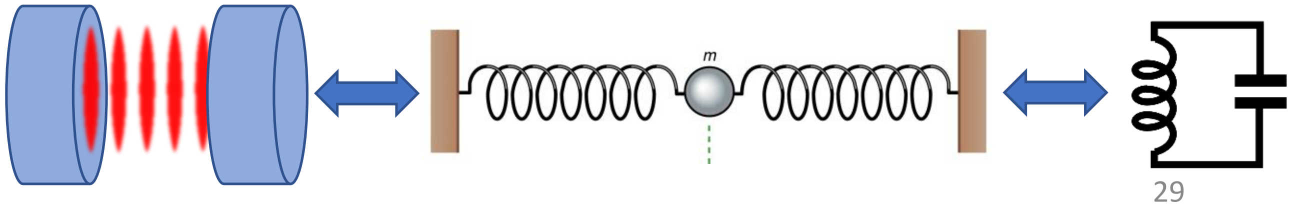
$$H = \frac{1}{2C}Q^2 + \frac{1}{2L}\Phi^2$$

A 'flux particle' moving on a harmonic potential.

People also prefer to use $E_L \sim 1/L$ **Inductive energy**, $E_C \sim 1/C$ **Charge energy**

Canonical quantization.

Mechanical oscillators.	LC resonator.
$\hat{X};\ \hat{P}$	$\hat{\Phi};\ \hat{Q}$
$[\hat{X},\hat{P}] = i\hbar$	$[\hat{\Phi},\hat{Q}] = i\hbar$
$\omega_o = \sqrt{k/m}$	$\omega_o = 1/\sqrt{LC}$
$Z_o = 1/\sqrt{km}$	$Z_o = \sqrt{L/C}$
$X_{\mathrm{zps}} = \sqrt{\hbar Z_o/2};$ $P_{\mathrm{zps}} = \sqrt{\hbar/2Z_o}$ $\Rightarrow X_{\mathrm{zps}}P_{\mathrm{zps}} = \hbar/2$	$\Phi_{\mathrm{zps}} = \sqrt{\hbar Z_o/2};$ $Q_{\mathrm{zps}} = \sqrt{\hbar/2Z_o}$ $\Rightarrow \Phi_{\mathrm{zps}}Q_{\mathrm{zps}} = \hbar/2$
$\hat{a} = \frac{1}{2}\left(\frac{\hat{X}}{X_{\mathrm{zps}}} + i\frac{\hat{P}}{P_{\mathrm{zps}}}\right)$	$\hat{a} = \frac{1}{2}\left(\frac{\hat{\Phi}}{\Phi_{\mathrm{zps}}} + i\frac{\hat{Q}}{Q_{\mathrm{zps}}}\right)$
$\hat{X} = X_{\mathrm{zps}}\left(\hat{a} + \hat{a}^\dagger\right)$	$\hat{\Phi} = \Phi_{\mathrm{zps}}\left(\hat{a} + \hat{a}^\dagger\right)$
$\hat{P} = -iP_{\mathrm{zps}}\left(\hat{a} - \hat{a}^\dagger\right)$	$\hat{Q} = -iQ_{\mathrm{zps}}\left(\hat{a} - \hat{a}^\dagger\right)$
$\left[\hat{a},\hat{a}^\dagger\right] = 1$	$\left[\hat{a},\hat{a}^\dagger\right] = 1$



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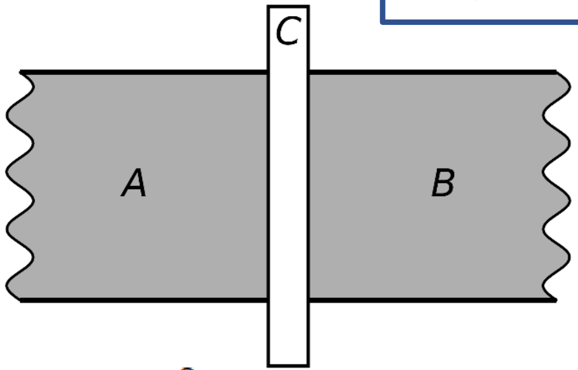
Part I – Ingredients of circuit QED : Nonlinear elements – Josephson junction

Definition.

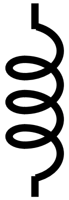


$$V = \frac{\Phi}{2\pi}\frac{d\phi}{dt}.$$

$$I = I_0 \sin\phi.$$ → SC phase. (periodic)

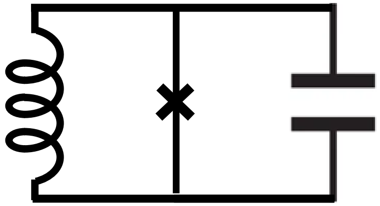
$$H_J = \int V I\, dt = -E_J \cos(2\pi \frac{\Phi}{\Phi_0}) = -E_J \cos\phi,$$

↓ Josephson energy.



$$H_L = E_L \left(\frac{2\pi^2}{\Phi_0^2}\right)\Phi^2 \qquad H_J = -E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

JJ circuit



Canonical variable

: $\phi, n.$

$$H = \underbrace{4E_c n^2}_{\text{CP \# (momentum).}} - E_J \underbrace{\cos(\phi)}_{\text{SC phase. (coordinate).}} + \frac{E_L}{2}\phi^2$$

A phase particle moving on a nonlinear potential.

If $E_L = 0$,

Circuit analogy of a pendulum.

ϕ

Nonlinearity from JJ is the key to have qubit states.

Part I - SuperConducting Qubits.

Superconducting Qubit Evolutionary Phylogeny

RF-SQUID

COOPER-PAIR-BOX

(S. Girvin,
Circuit QED: Superconducting Qubits Coupled to Microwave Photons).

Old-fashion classification.

Logic

"Phase"

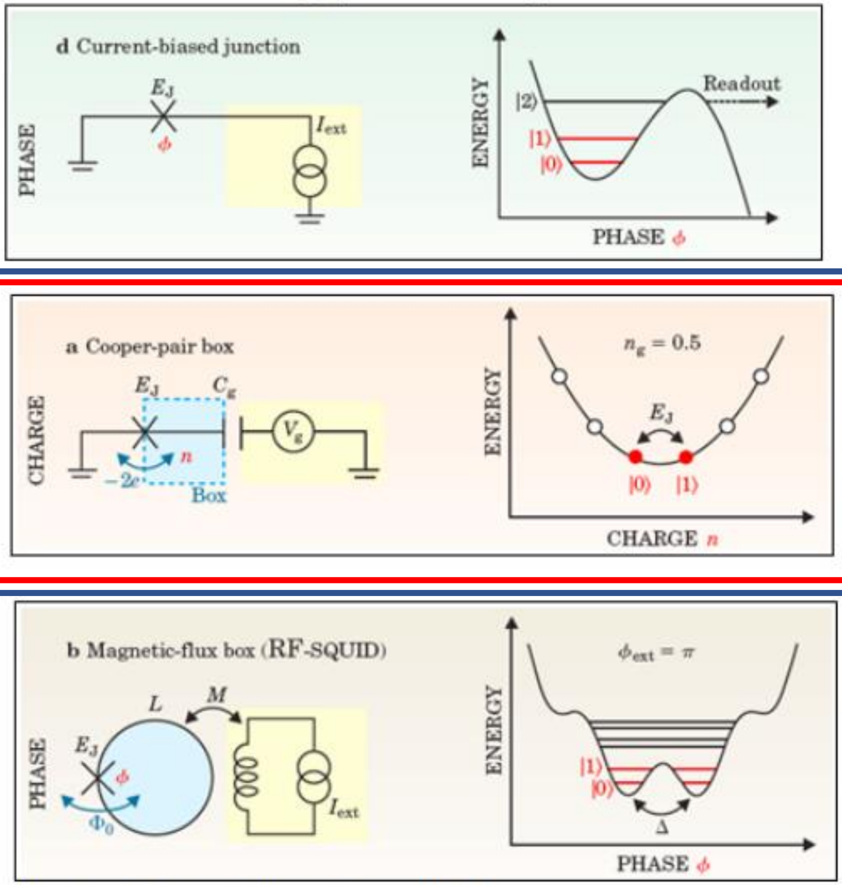
Non-linear oscillator
Excited |1>
vs
Ground state |0>

"Charge"

Island
Charged
vs.
Not charged

"Flux"

Current circulation
Left
vs
Right



You & Nori. Physics Today, November (2005)

Part I - SuperConducting Qubits.

Superconducting Qubit Evolutionary Phylogeny

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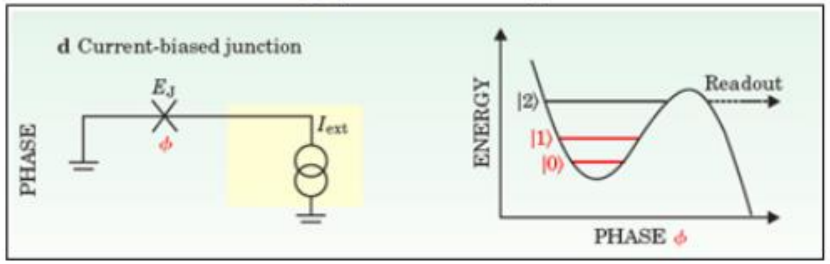
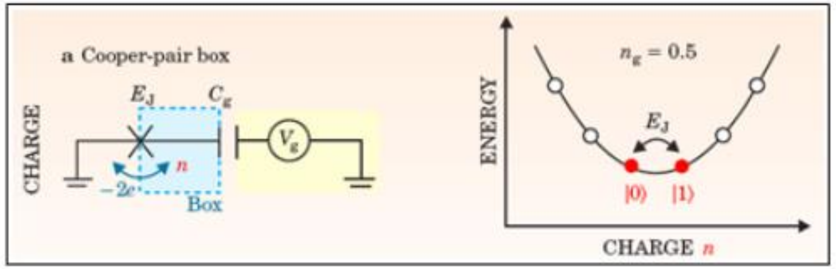


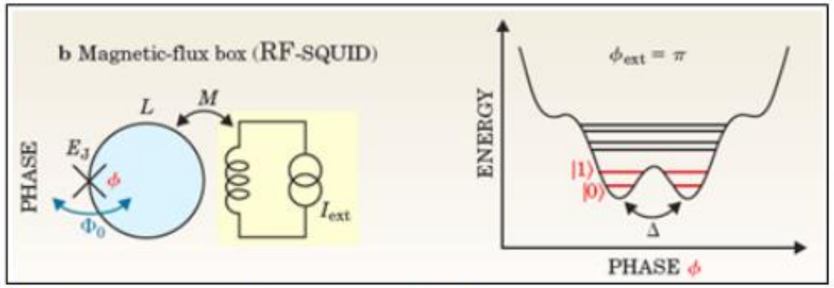
<u>Non-linear oscillator</u>
Excited |1>
vs
Ground state |0>

"Charge"



<u>Island</u>
Charged
vs.
Not charged

"Flux"

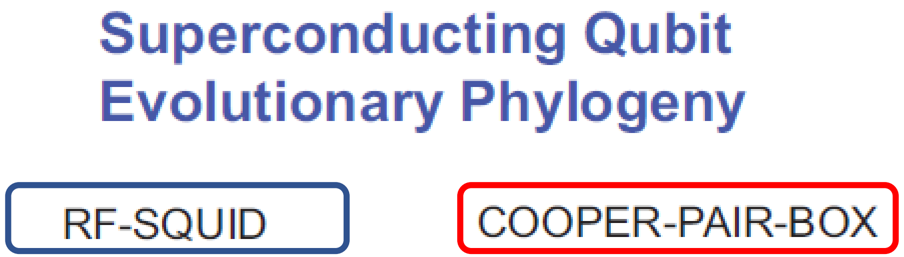


<u>Current circulation</u>
Left
vs
Right

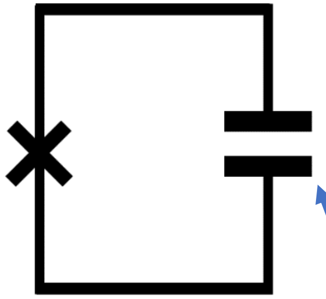
You & Nori, Physics Today, November (2005)

(S. Girvin,
Circuit QED: Superconducting Qubits Coupled to Microwave Photons).

Part I - SuperConducting Qubits.



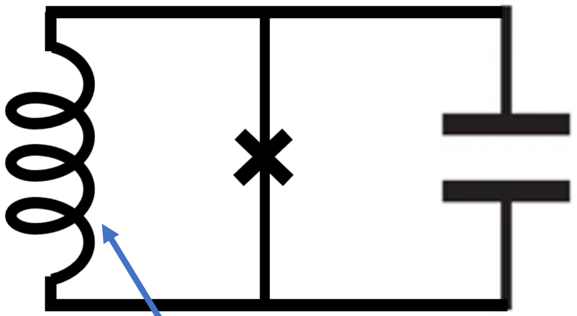
Transmon



JJ with a big shunt capacitance.

Currently sitting on the throne.

Fluxonium



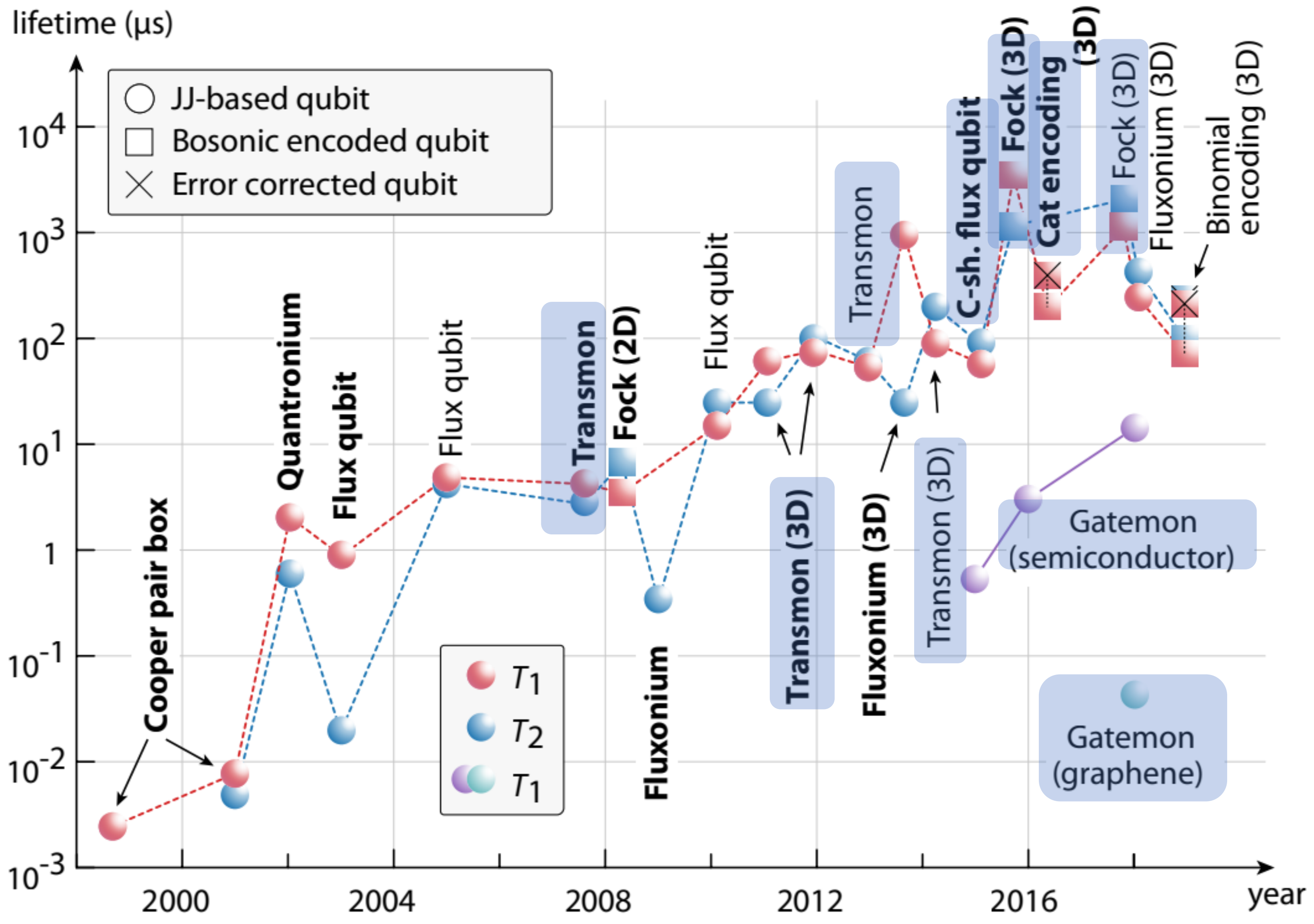
Super-inductor.

Strong challenger.

But many other qubit designs are still being devised and demonstrated.

(S. Girvin,
Circuit QED: Superconducting Qubits Coupled to Microwave Photons).

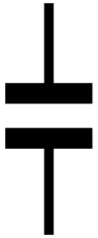
Part I - SCQ Zoo.



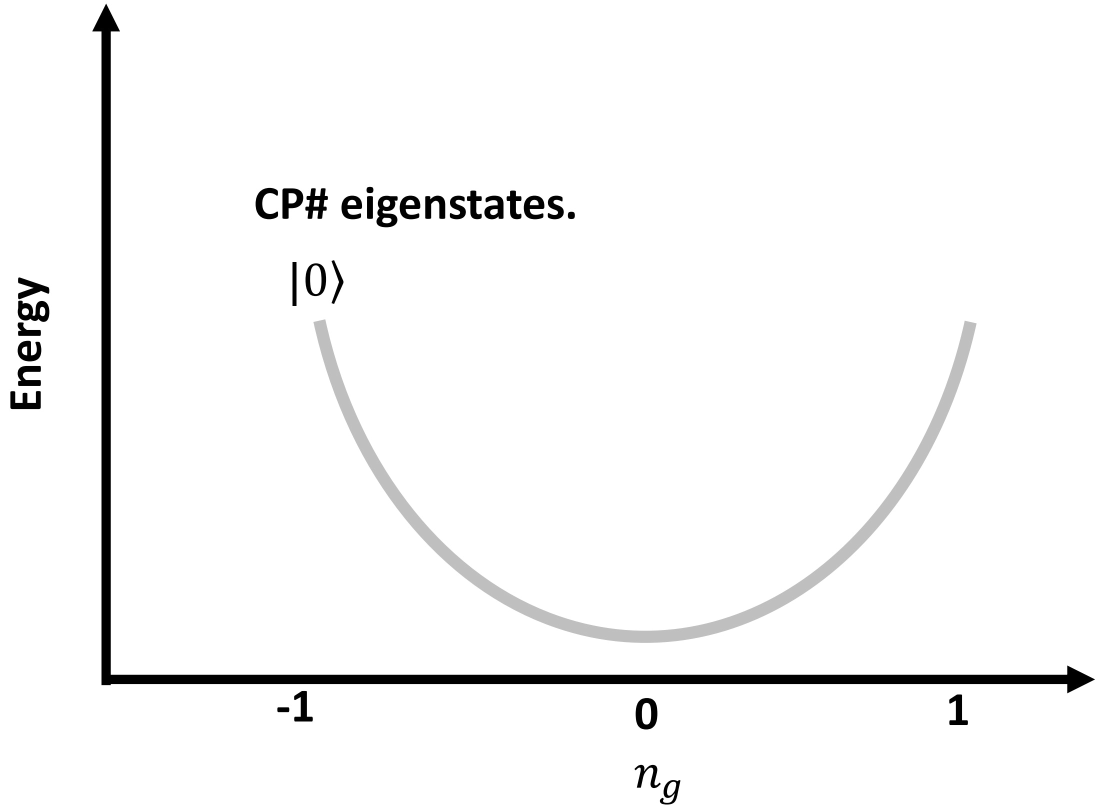
Transmon (-related systems).

(S. Girvin,
Circuit QED: Superconducting Qubits Coupled to Microwave Photons).

Part I – SCQ : From capacitance to transmon.



$$\hat{H} = 4E_c(\hat{n} - n_g)^2$$



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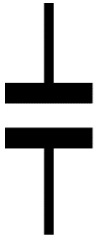

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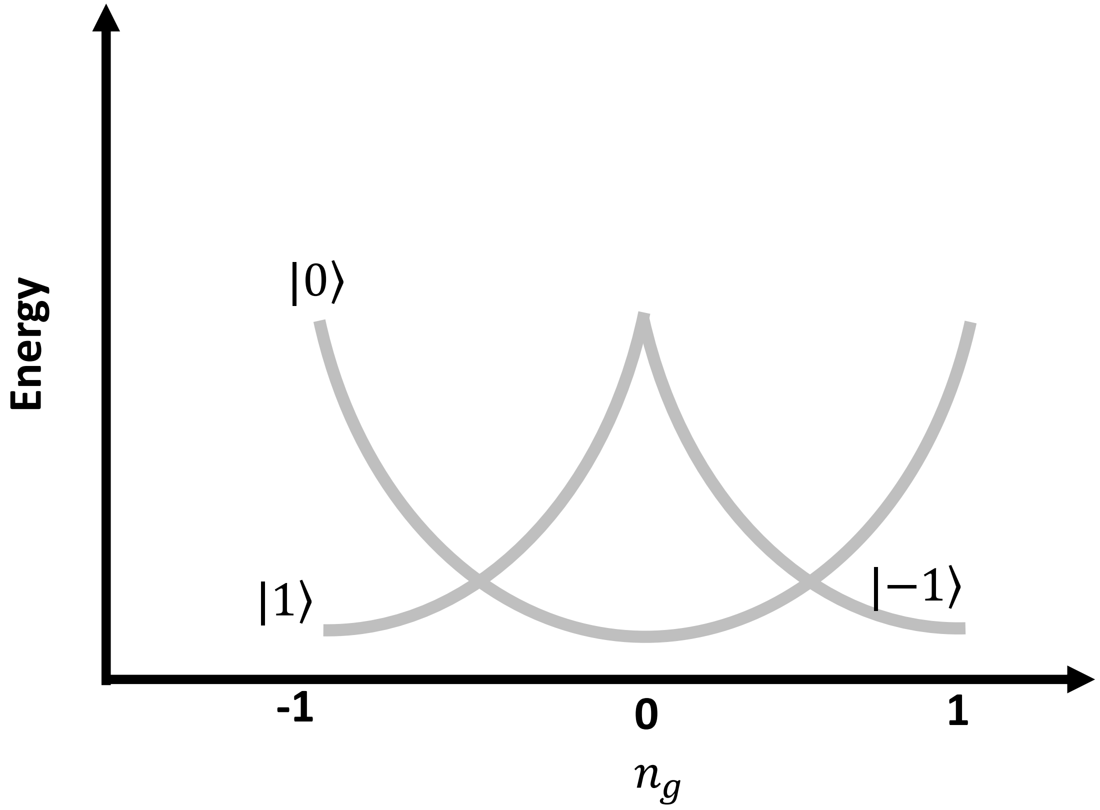

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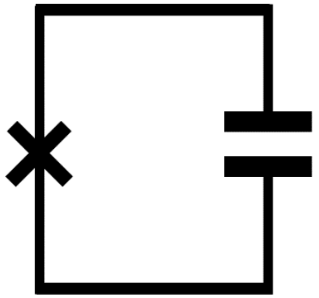
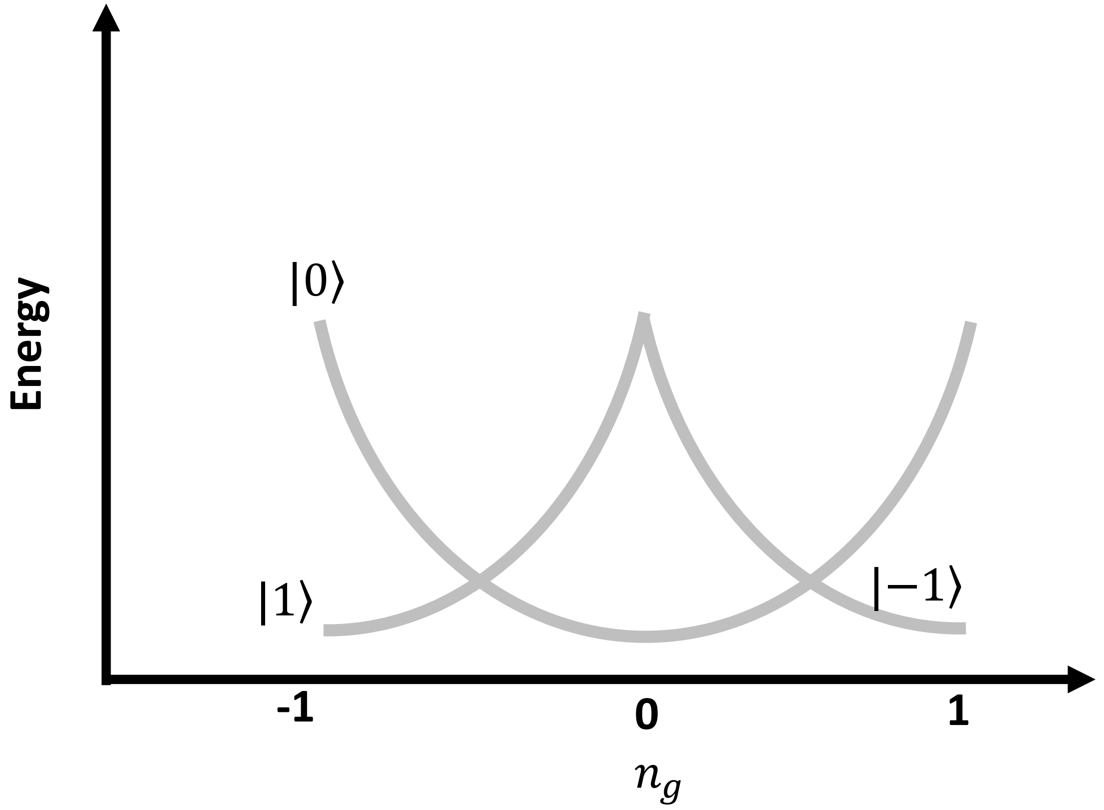


$$\hat{H} = 4E_c(\hat{n} - n_g)^2 - \underline{E_J\cos(\hat{\phi})}$$

$$= \frac{E_J}{2}\left[|n\rangle\langle n+1| + |n+1\rangle\langle n|\right]$$



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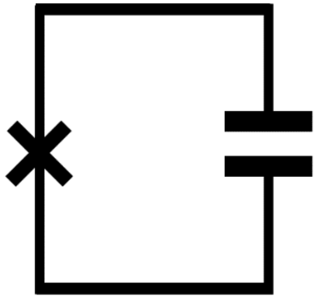

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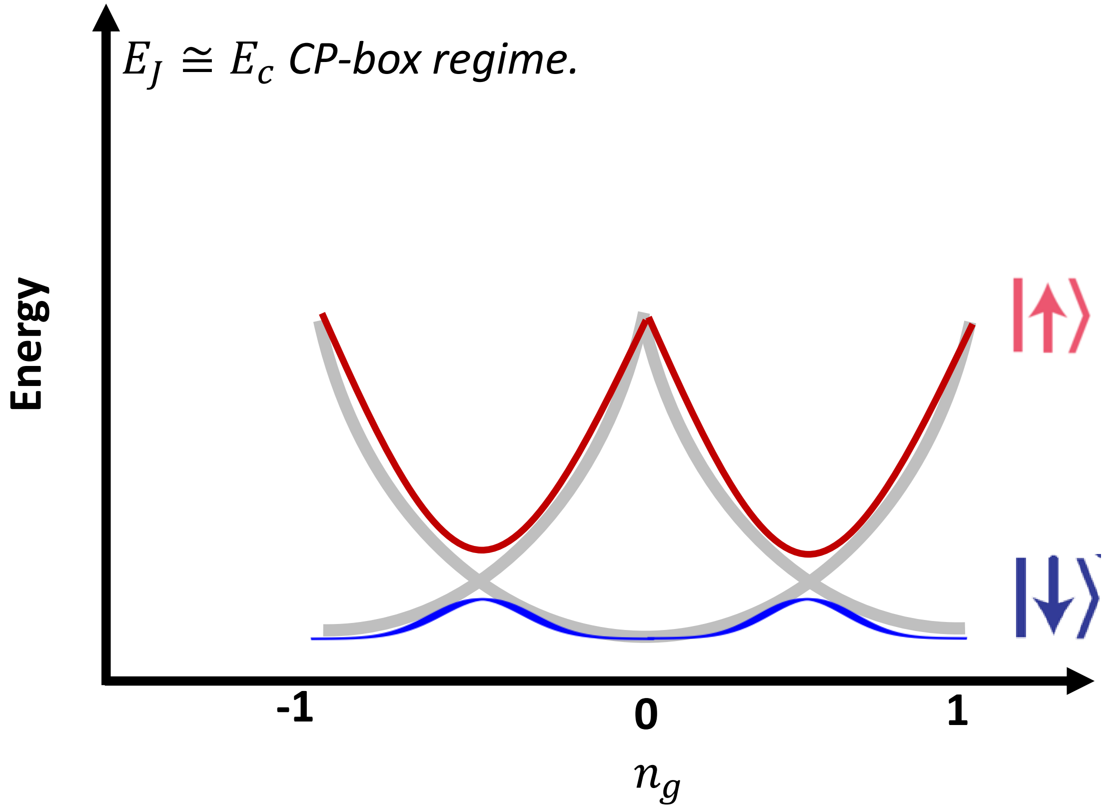


half Integer n_g

Insensitive to charge noise (charge sweep spot).

Lose single-photon interface to others.

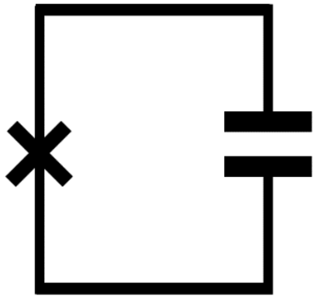
Integer n_g

Have single-photon interface to others.

Sensitive to charge noise.

KRISS

Part I – SCQ : From capacitance to transmon.



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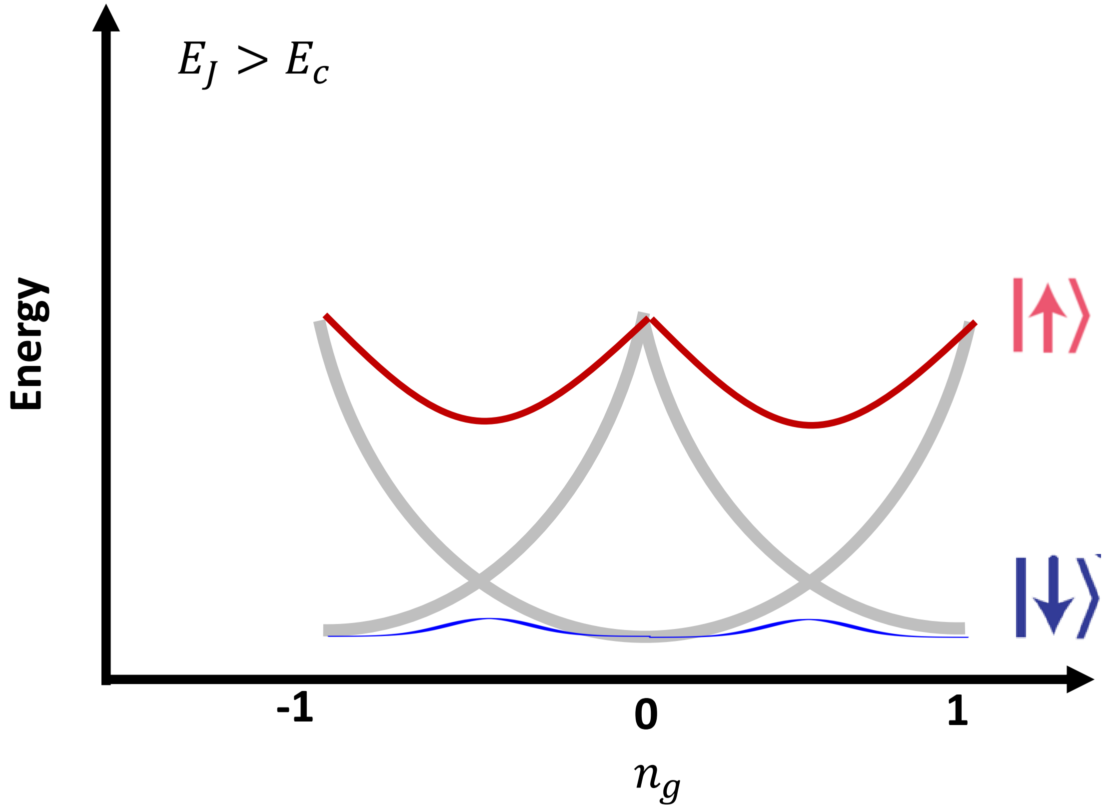

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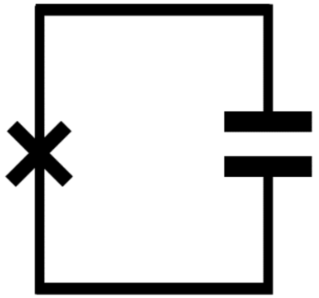
Part I – SCQ : From capacitance to transmon.



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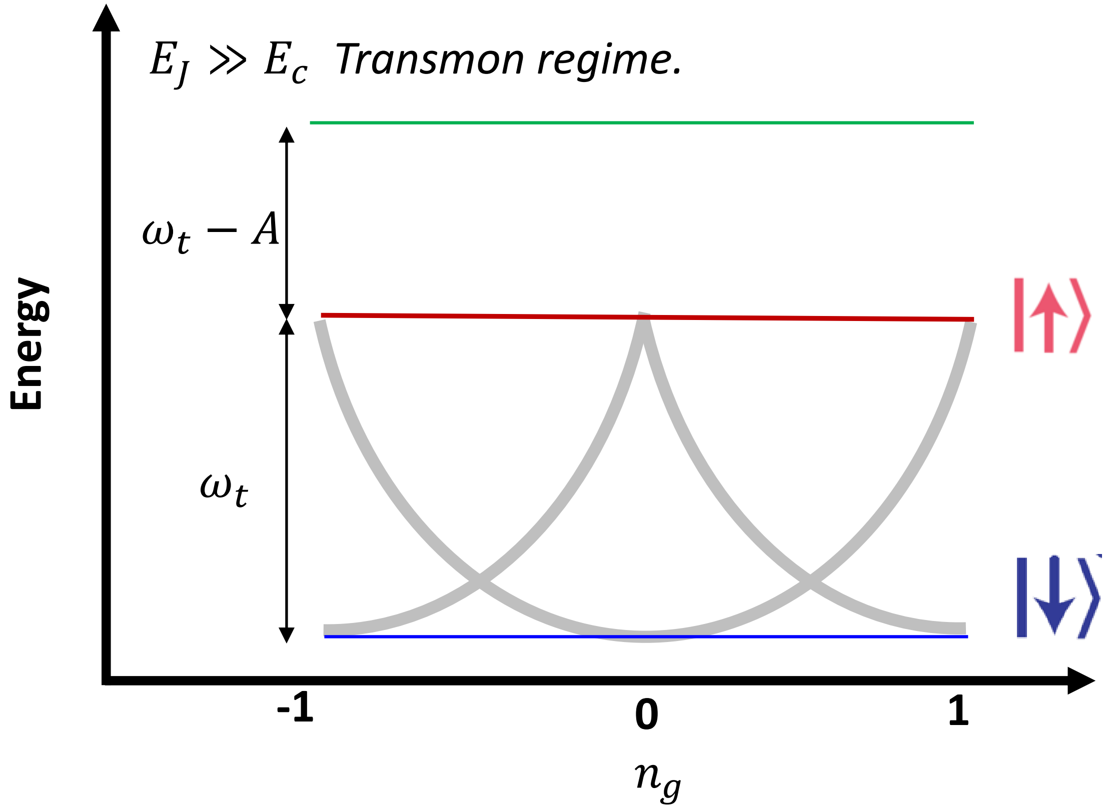
$$= \frac{E_J}{2}[|n\rangle\langle n+1| + |n+1\rangle\langle n|]$$



$\omega_t \sim \sqrt{8E_J E_C}$

$A \sim E_C$

Get rid of charge noise !

Weak anharmonicity ...

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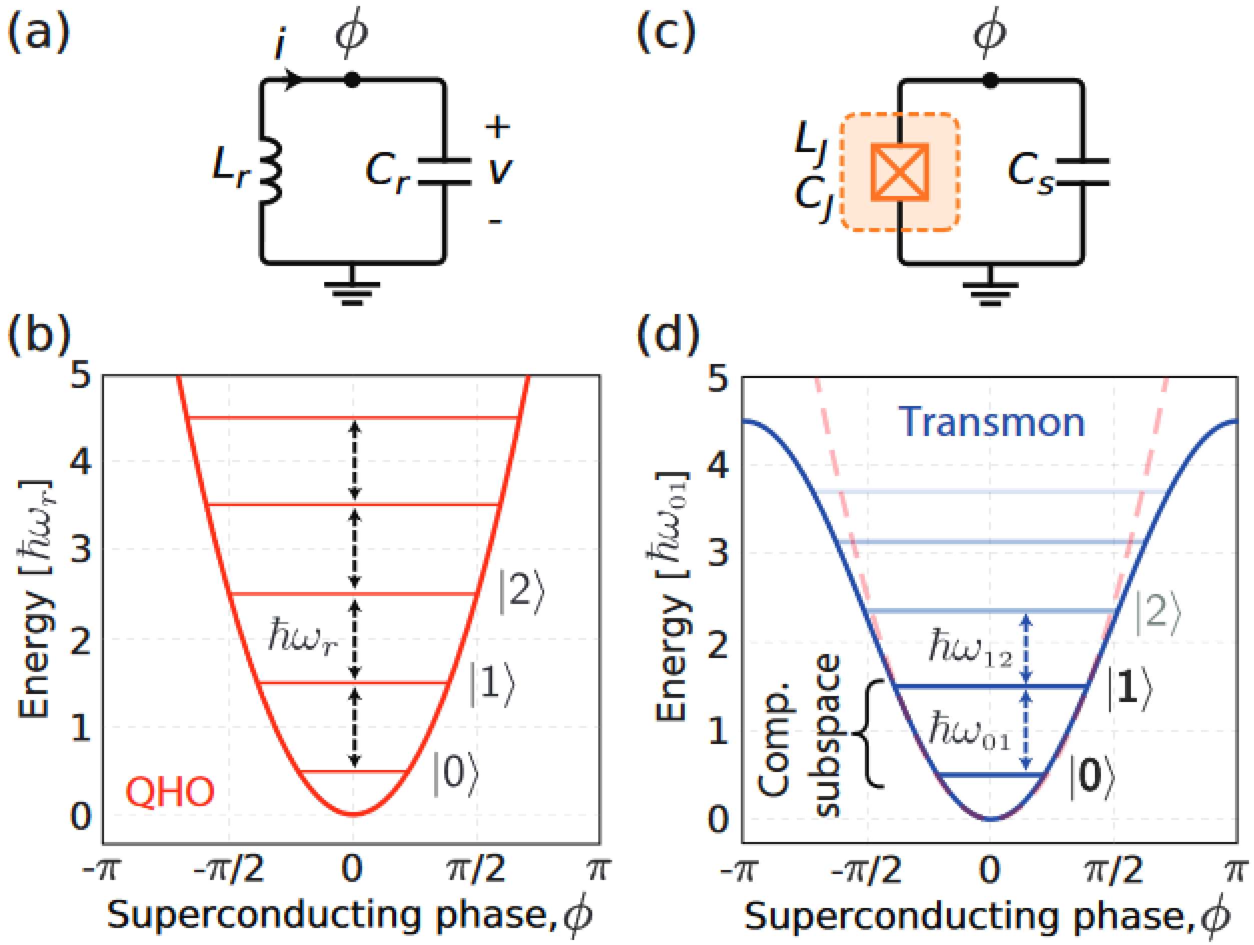
Part I – SCQ : From LC resonator to transmon.



Applied Physics Reviews ***6****, 021318 (2019).*

Transmons as weakly anharmonic resonators.

→ Charge and phase ≠ good quantum number.

→ Oscillator excitation = good quantum number.

$$\hat{n} = -\frac{i}{2}\left(\frac{E_J}{2E_C}\right)^{\frac{1}{4}}(\hat{a}-\hat{a}^\dagger)$$

$$\hat{\phi} = \left(\frac{2E_C}{E_J}\right)^{\frac{1}{4}}(\hat{a}+\hat{a}^\dagger).$$

$$\hat{H}_\mathrm{t} = \hbar\overline{\omega}_\mathrm{t}\hat{a}^\dagger\hat{a} - \frac{E_C}{12}(\hat{a}+\hat{a}^\dagger)^4.$$

Duffing oscillator model.

$$\hat{H}_\mathrm{t}^\mathrm{low} \approx \hbar\omega_\mathrm{t}\hat{a}^\dagger\hat{a} - \frac{A_\mathrm{t}}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}.$$

Kerr resonator model.

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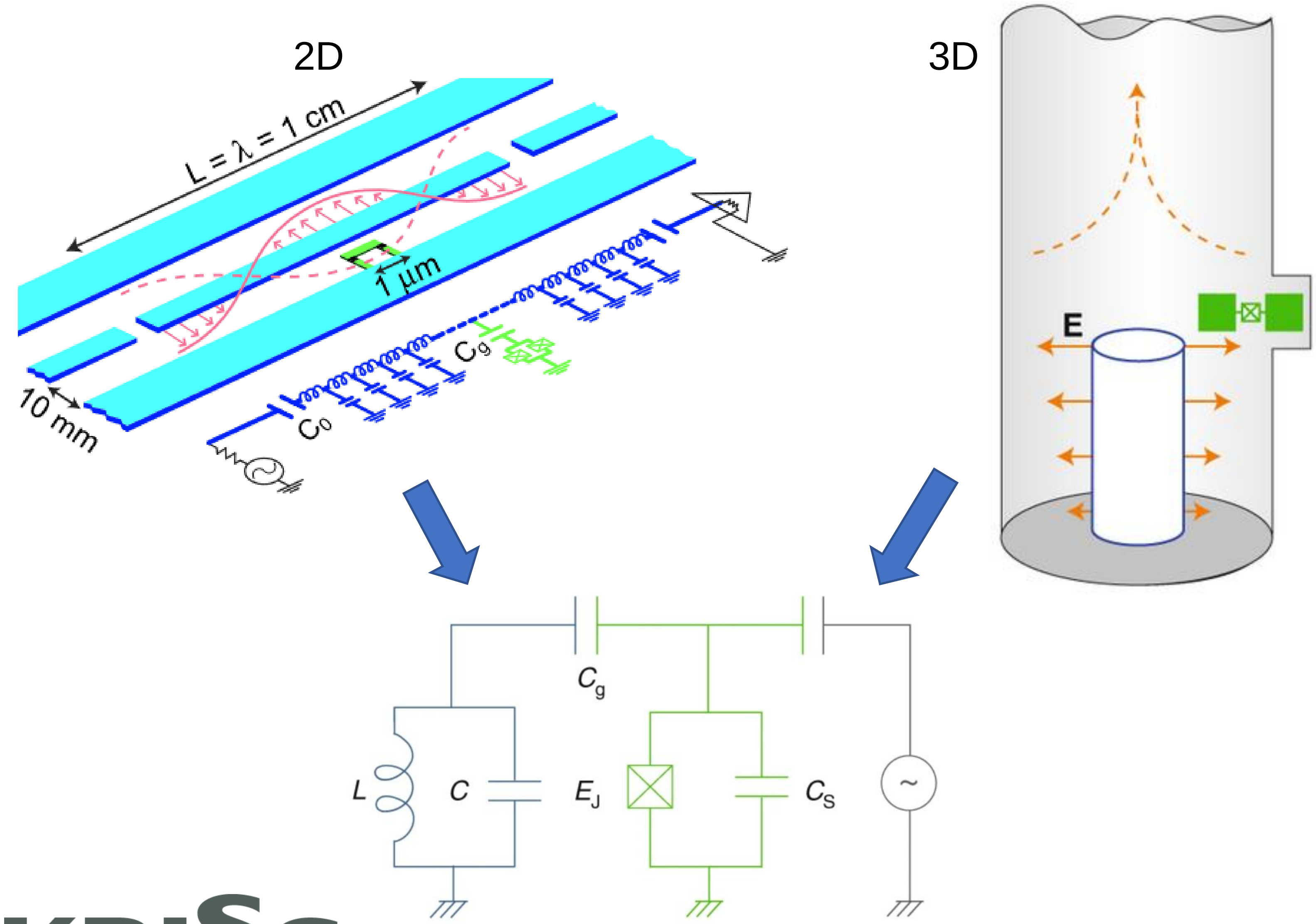
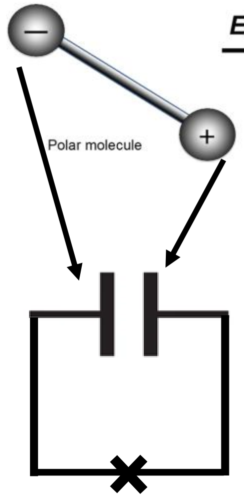
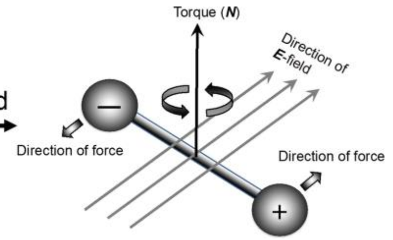
Part I – Circuit QED : SCQs + Resonators.



Classical dipole analogy.





Horikoshi, S., Serpone, N. (2022).

Nat. Phys. ***16*** *247 (2020).*

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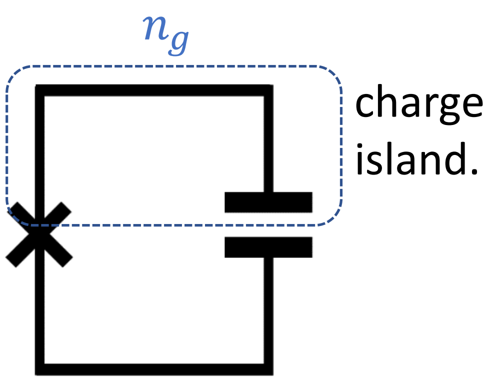
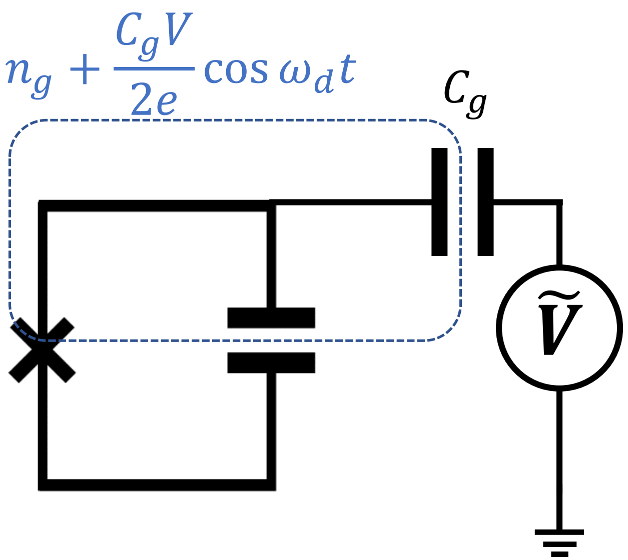
Part I – Circuit QED : SCQs + Resonators.

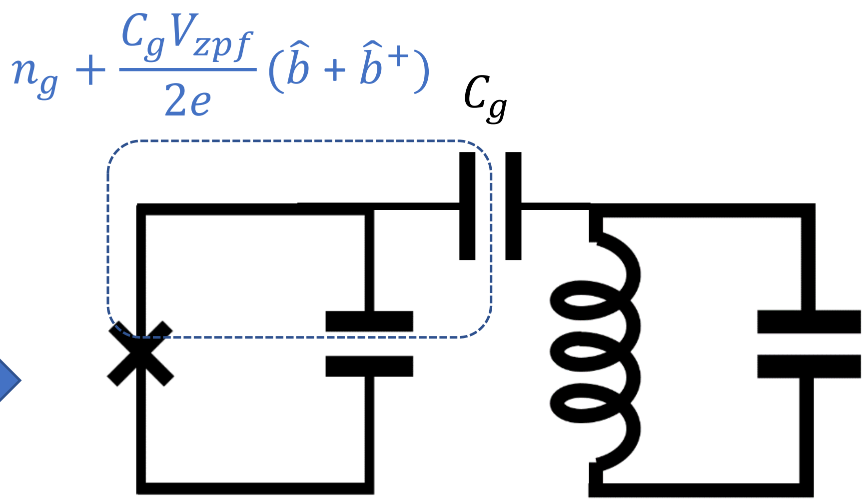


$$\hat{H} = 4E_c(\hat{n} - n_g)^2$$
$$-E_J\cos(\hat{\phi})$$



$$\hat{H} = 4E_c(\hat{n} - n_g + \frac{C_g V}{2e}\cos\omega_d t)^2$$
$$-E_J\cos(\hat{\phi})$$

Driving charge island with classical fields.



$$\hat{H} = 4E_c(\hat{n} - n_g + \frac{C_g V_{zpf}}{2e}[\hat{b} + \hat{b}^+])^2$$
$$-E_J\cos(\hat{\phi})$$

Driving charge island with quantum fields
(LC circuit vacuum fluctuation).

$\hat{n}[\hat{b} + \hat{b}^+] \subset (\ldots)^2$

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Part I – Circuit QED : SCQs + Resonators.

cQED Hamiltonian model :

$$\hat{H}_{\mathrm{cQED}} = \underbrace{4E_C(\hat{n}-n_g)^2 + E_J\cos\hat{\phi}}_{\text{SCQ}} - \underbrace{g_0\hat{n}(\hat{b}+\hat{b}^\dagger)}_{\text{Interaction}} + 4E_C(n_g + n_r(\hat{b}+\hat{b}^\dagger))^2 + \underbrace{\hbar\omega_r\hat{b}^\dagger\hat{b}}_{\text{Resonator}}.$$

Approximation I *(Two-level systems):*

$$\hat{H}_{\mathrm{cQED}}/\hbar \longrightarrow \hat{H}_{\mathrm{QRM}}/\hbar = \frac{\omega_0}{2}\hat{\sigma}_z - g(\hat{b}+\hat{b}^\dagger)\hat{\sigma}_x + \omega_r\hat{b}^\dagger\hat{b}.$$

Approximation II *(Duffing-Harmonic coupled oscillators):*

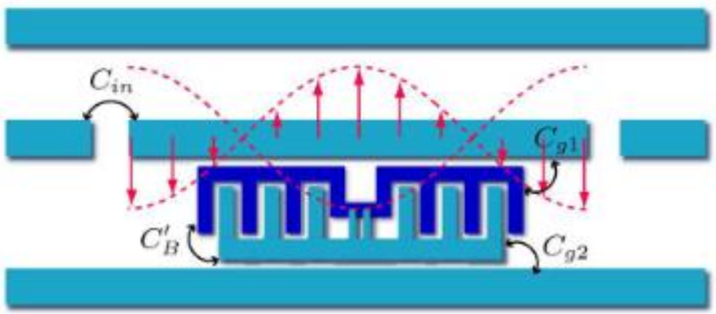
$$\hat{H}_{\mathrm{cQED}}/\hbar \longrightarrow \hat{H}_{\mathrm{D\text{-}H}}/\hbar = \omega_{\mathrm{t}}\hat{a}^\dagger\hat{a} + \omega_r\hat{b}^\dagger\hat{b} - g(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger) - \frac{E_C}{12}(\hat{a}+\hat{a}^\dagger)^4.$$

KRISS

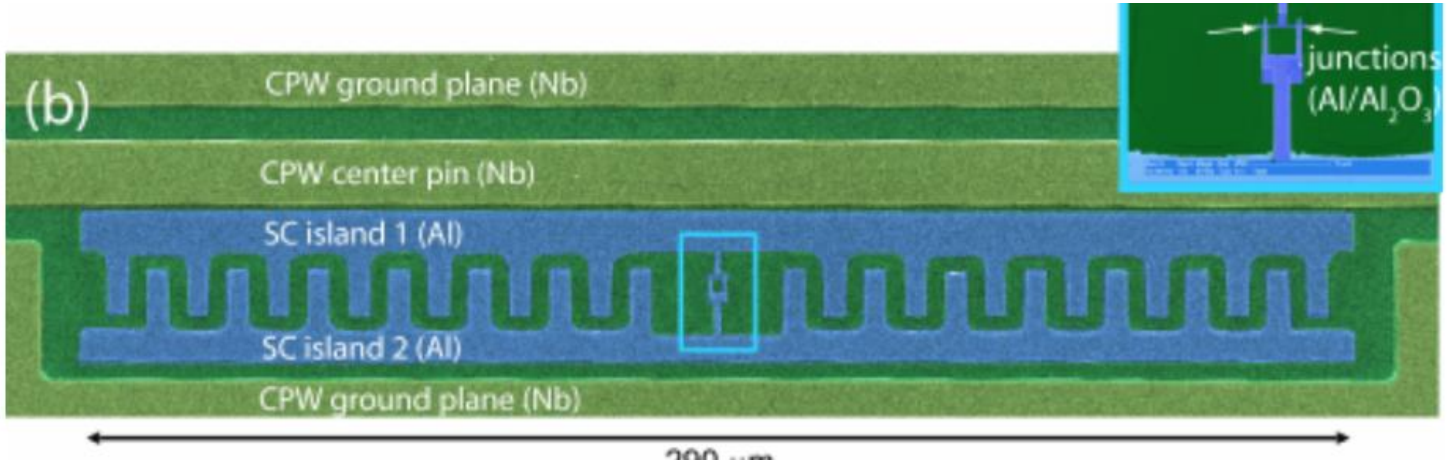
Part I - SCQ : Transmon - etymology.

Transmission line shunted **plasma oscillation** qubit.

**Old-fashion designs :*



PRA, ***76,*** *042319 (2007).*



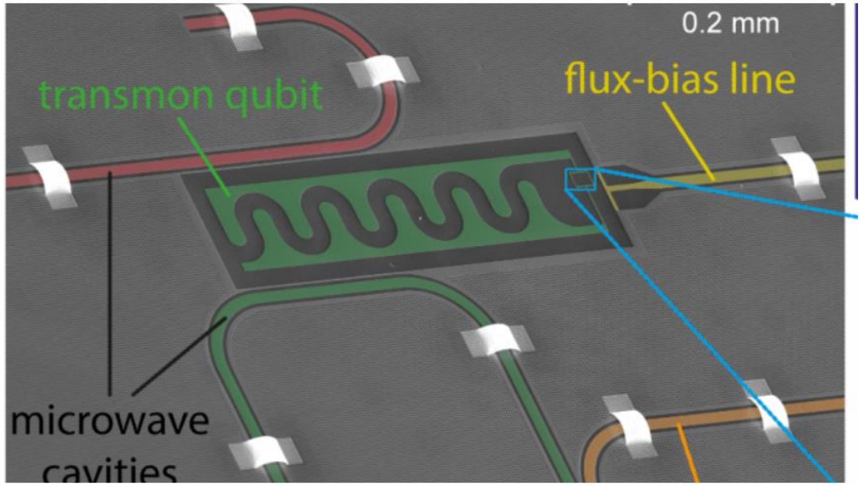
Quantum Inf Process **8**, 105–115 (2009)

Etymology sounds reasonable but…

Part I - SCQ : Transmon - etymology.

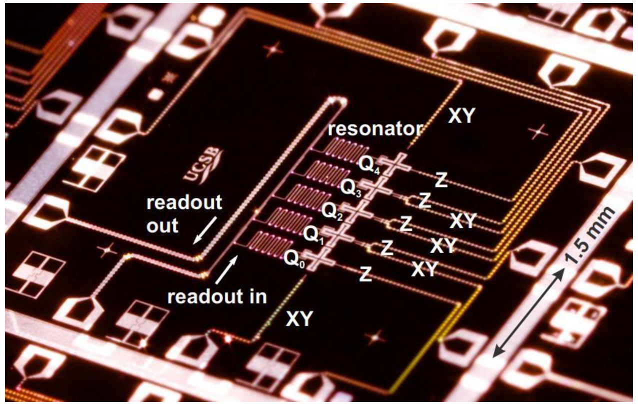
Transmission line shunted **plasma oscillation** qubit ?

**Nowadays :*

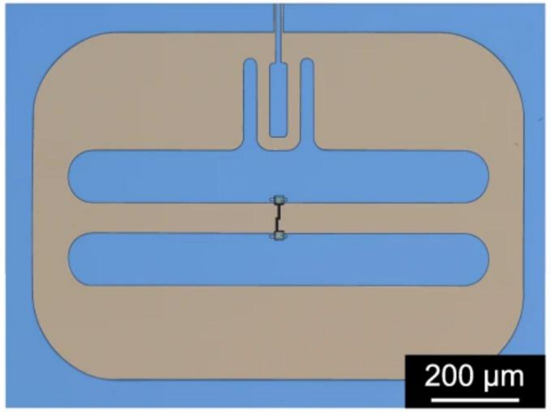


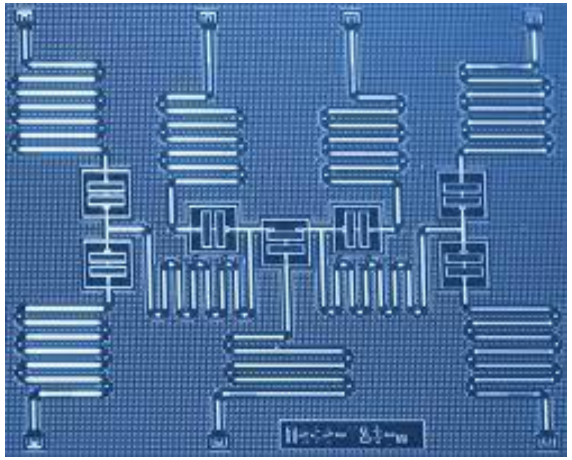
(QuTech).

(UCSB).

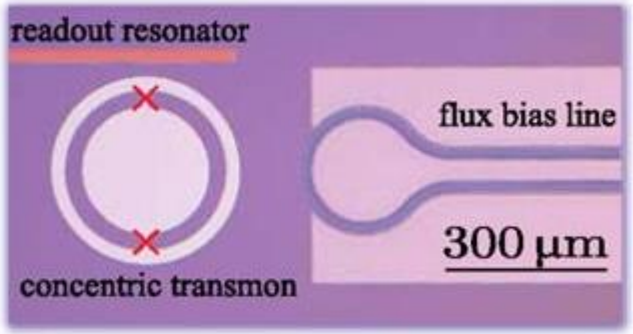


(Princeton).



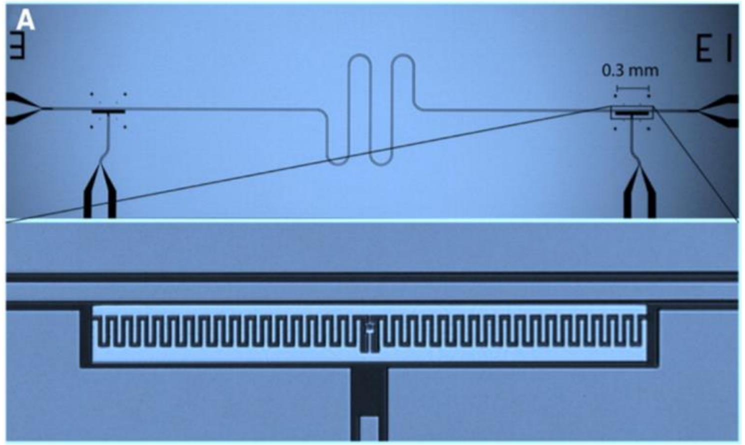


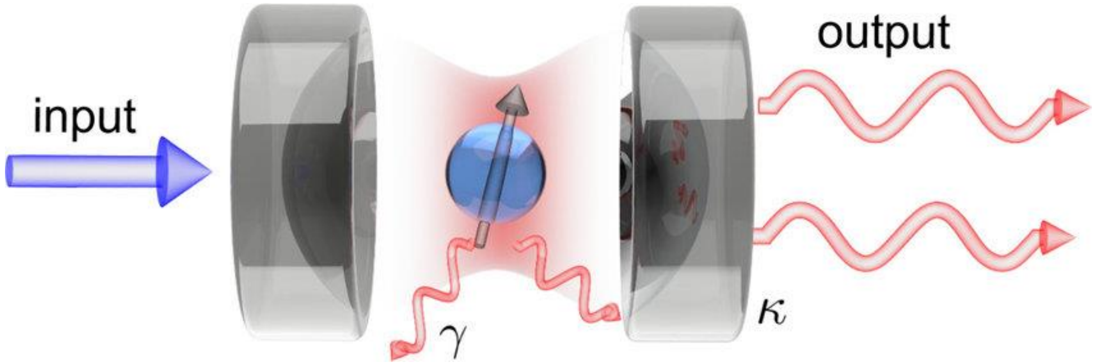
(IBM).



(NIST).

Part I –Circuit vs Cavity QED (transmon case only).





	Circuit	Cavity
Canonical relation	$[\Phi, Q] = i$	$[x, p] = i$
Dipole operaotr	Q	x
Frecquency scale	GHz	THz
Coupling scale	$g/2\pi \sim 10 - 100\text{MHz}$	$g/2\pi \sim 1$ MHz
Two-state approximation	Limited	Excellent

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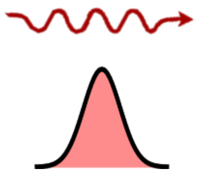
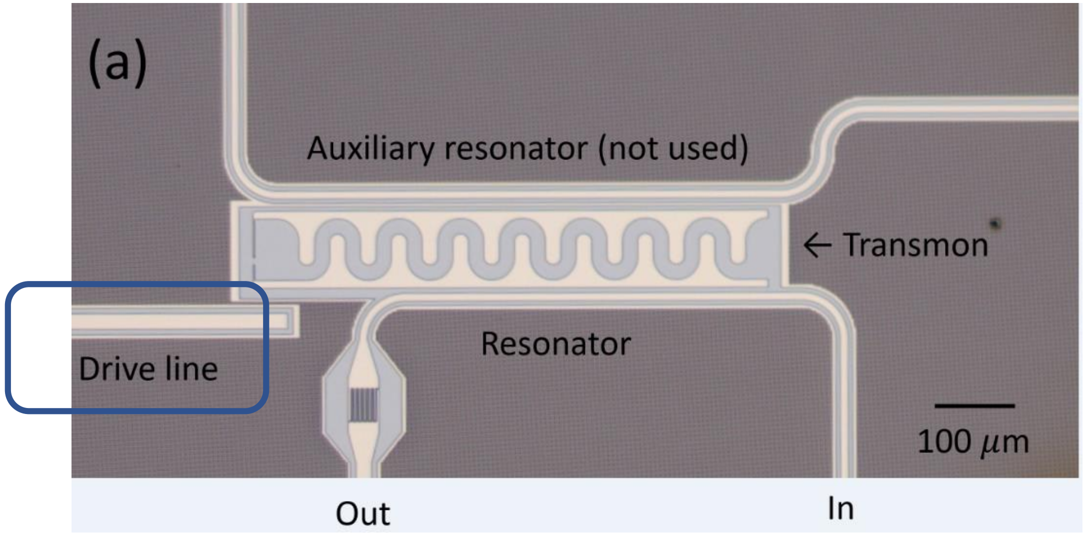
Part I – circuit QED : Qubit state control.

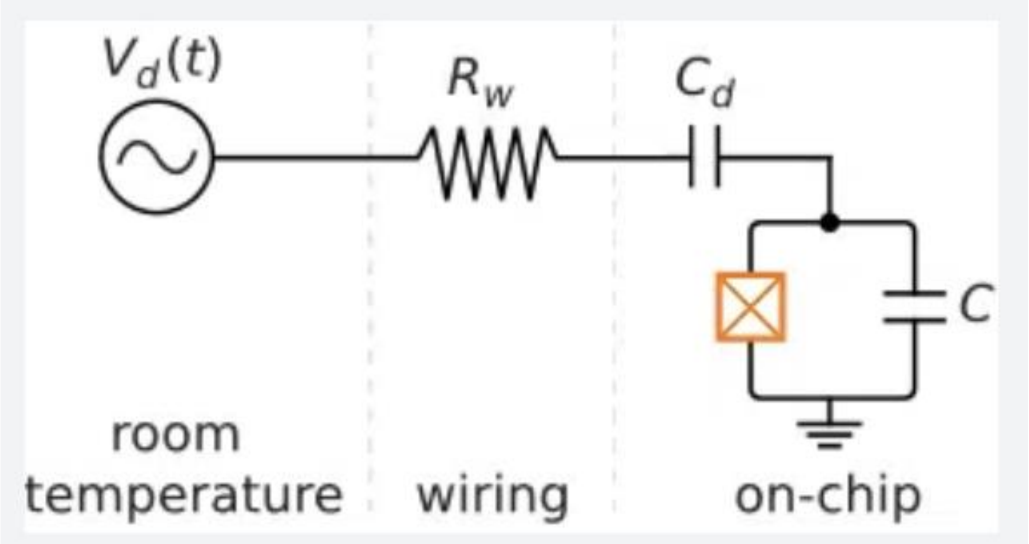
Drive line on a circuit.



Optical microscope.



Circuitary.



 CW, for spectroscopy.

 pulse, for state control.

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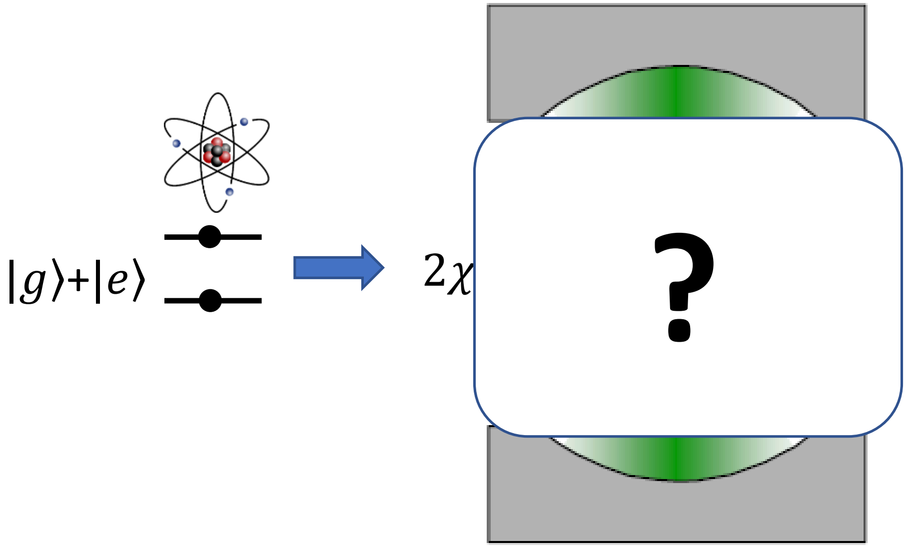
Part I – circuit QED : Qubit state readout.

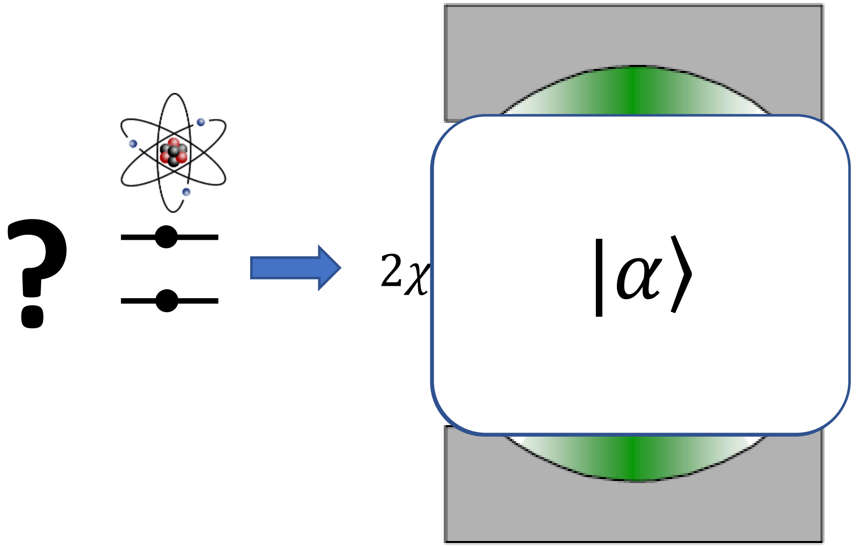
Atomic physics analogy.

Recall,
$\boldsymbol{H_I \sim \chi\, \hat{a}^{+}\hat{a}\, \hat{\sigma}_z}$**.**

Haroche experiment.



Our goal.



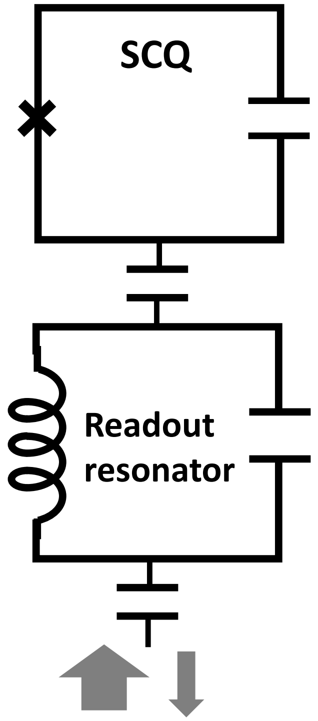
$|0, 1\rangle$ **: photon # state.**

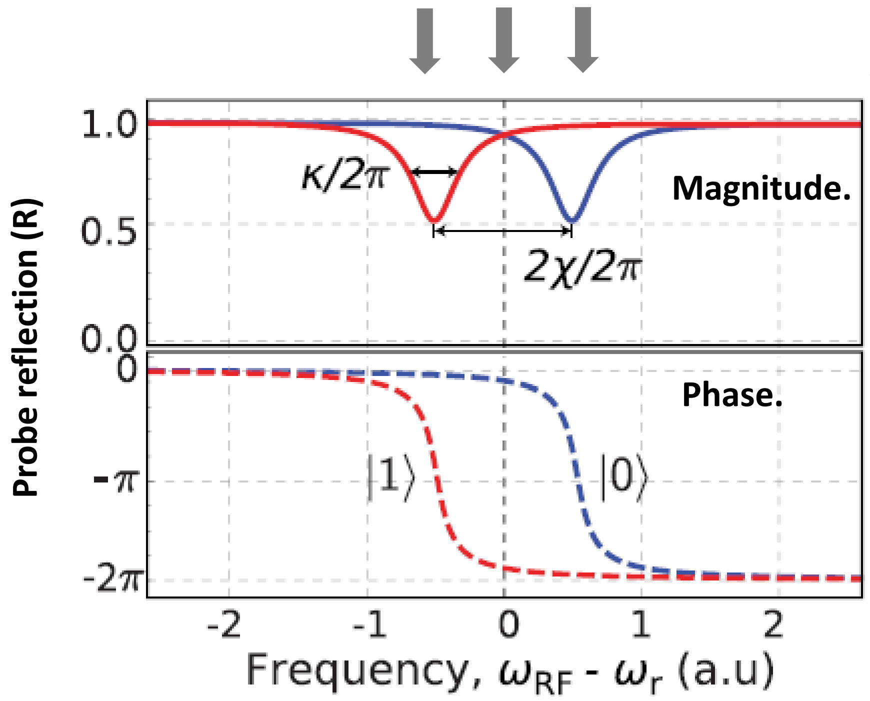
$|\alpha\rangle$ **: coherent state.**

Part I – circuit QED : Qubit state readout.

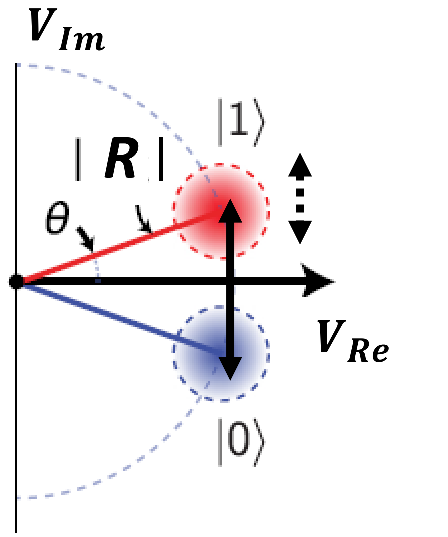
Normally use 'readout resonator's dispersively coupled to SCQs.

Recall, $H_I \sim \chi\,\hat{a}^{+}\hat{a}\,\hat{\sigma}_z$.





Applied Physics Reviews ***6,*** *021318 (2019).*



reflection ($|R|$) $\sim \sqrt{V_{Re}^2 + V_{Im}^2}$

Single shot condition: $\updownarrow \gg \updownarrow$

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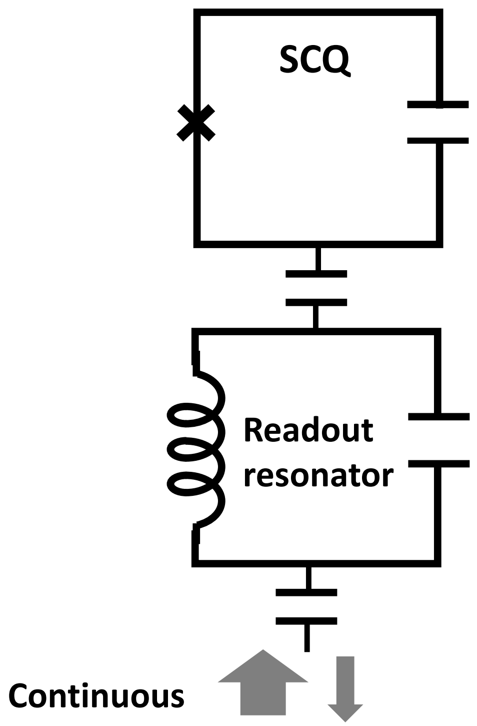
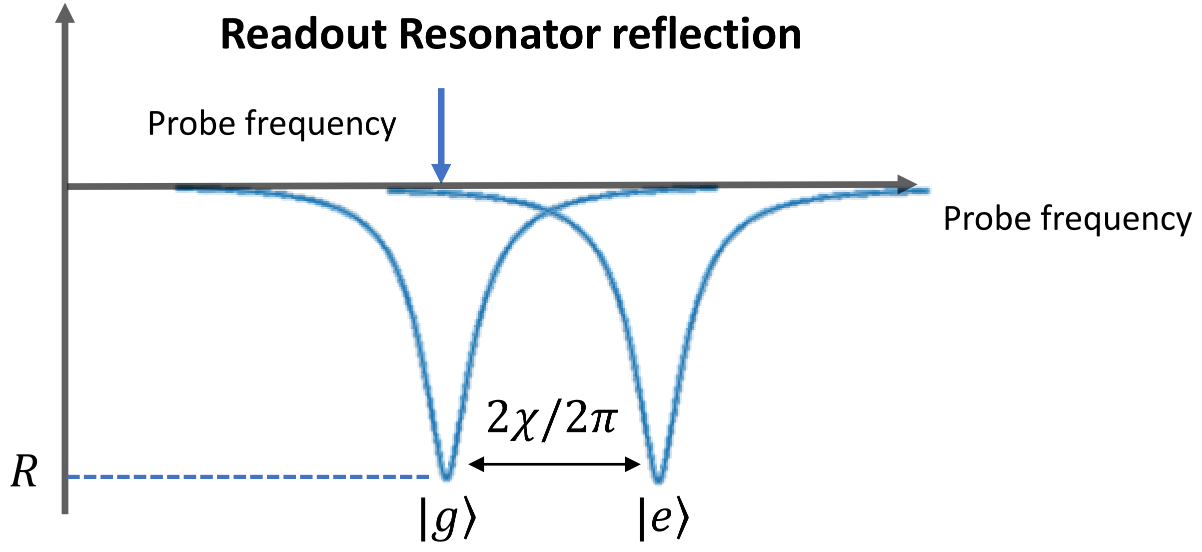
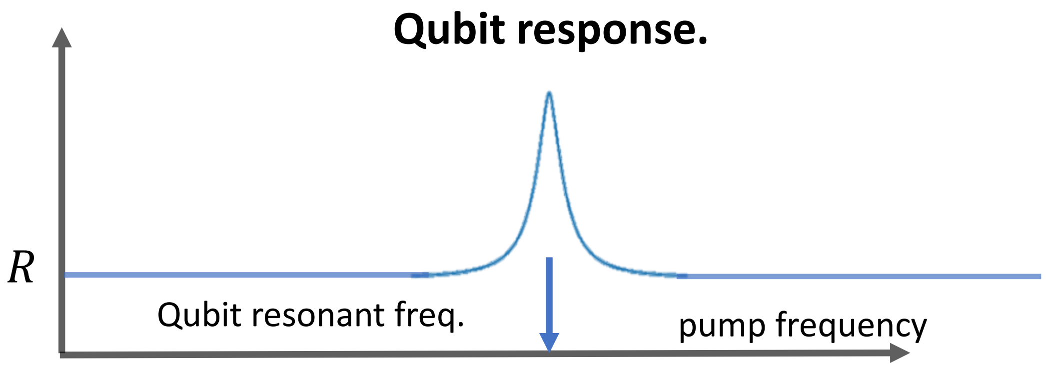
Part I – circuit QED : Two-tone spectroscopy.

Recall, $H_I \sim \chi\, \hat{a}^{+}\hat{a}\, \hat{\sigma}_z$.





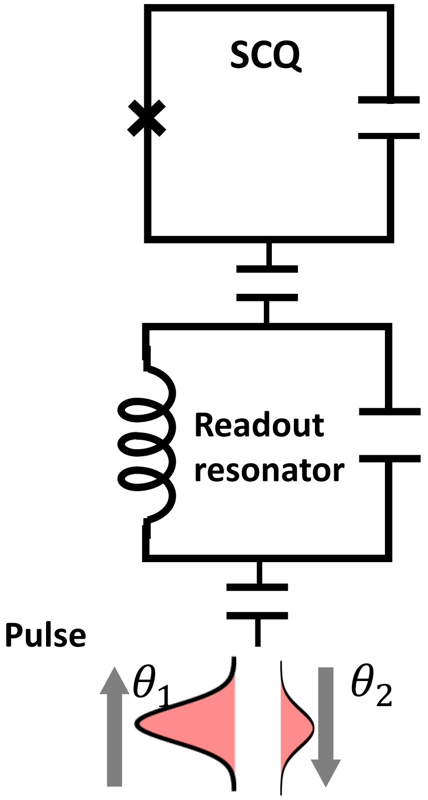


It is not non-demolition measurement.
too luxurious at this step.

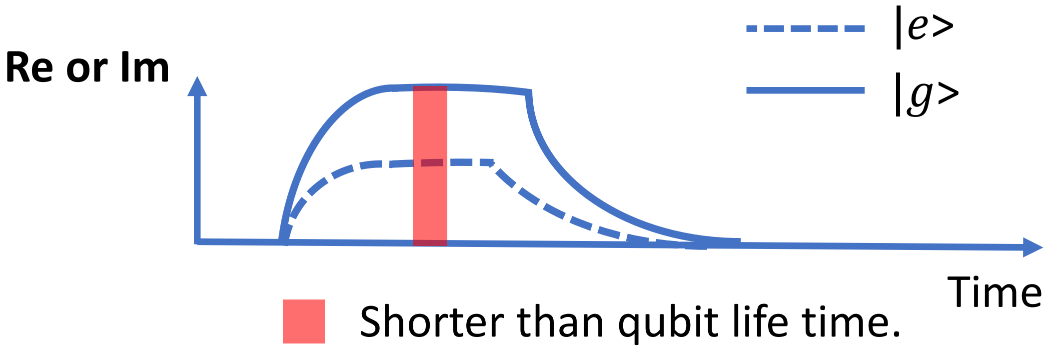
*Reading list :
PRA, ***74,*** *042318 (2006).*

Part I – circuit QED : Pulsed measurement.

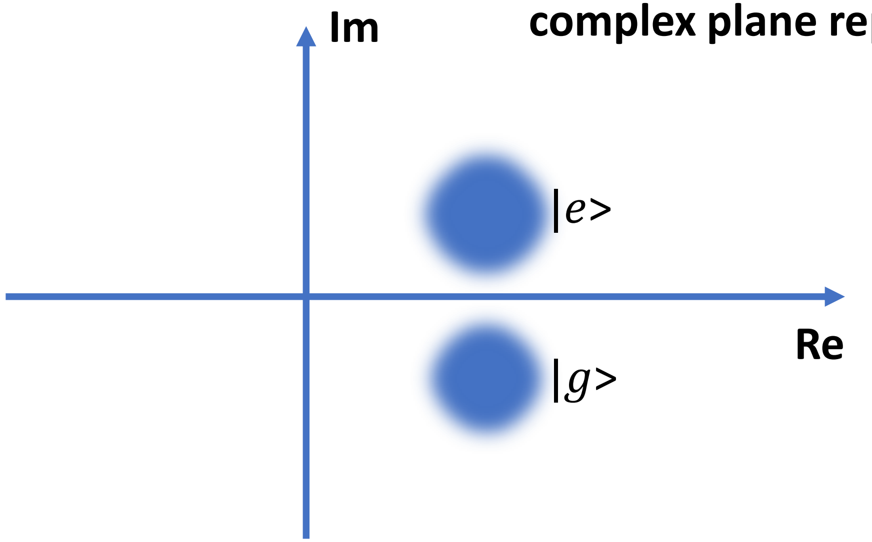
Recall, $H_I \sim \chi\, \hat{a}^{+}\hat{a}\, \hat{\sigma}_z$.



Reflected readout pulse.



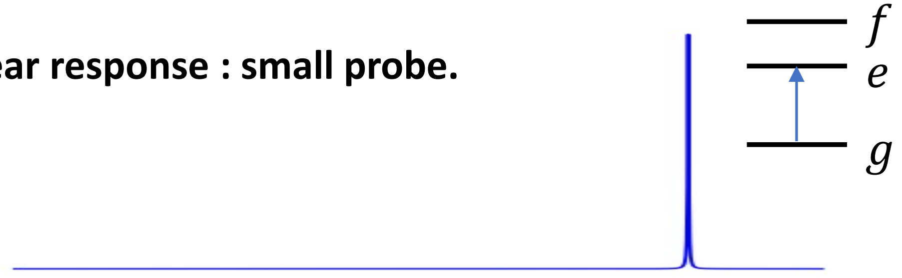
complex plane representation.



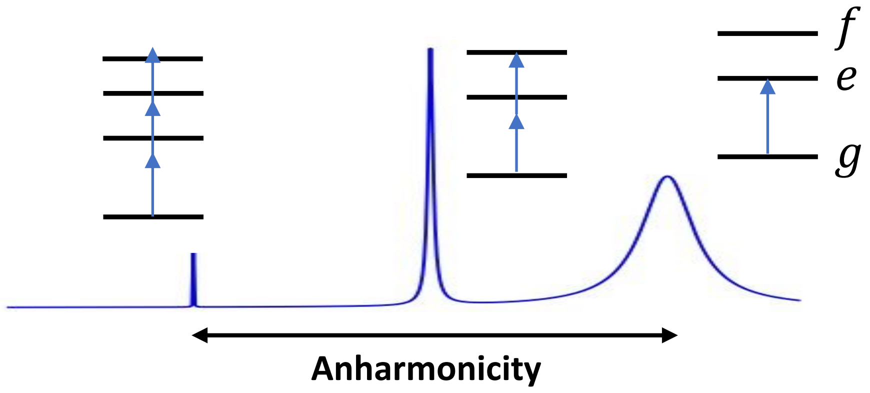
Part I – circuit QED : Elementary experiments.

Exploring transmon energy level structure.

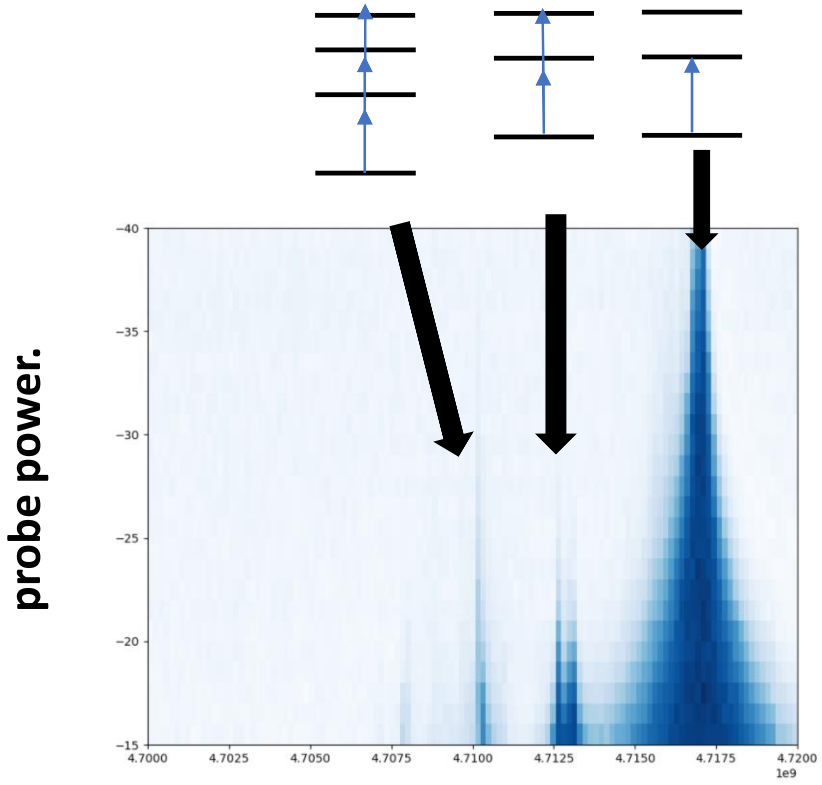
Linear response : small probe.



Nonlienar response : large probe.



$\sim E_c$ **: 150-300 MHz.**



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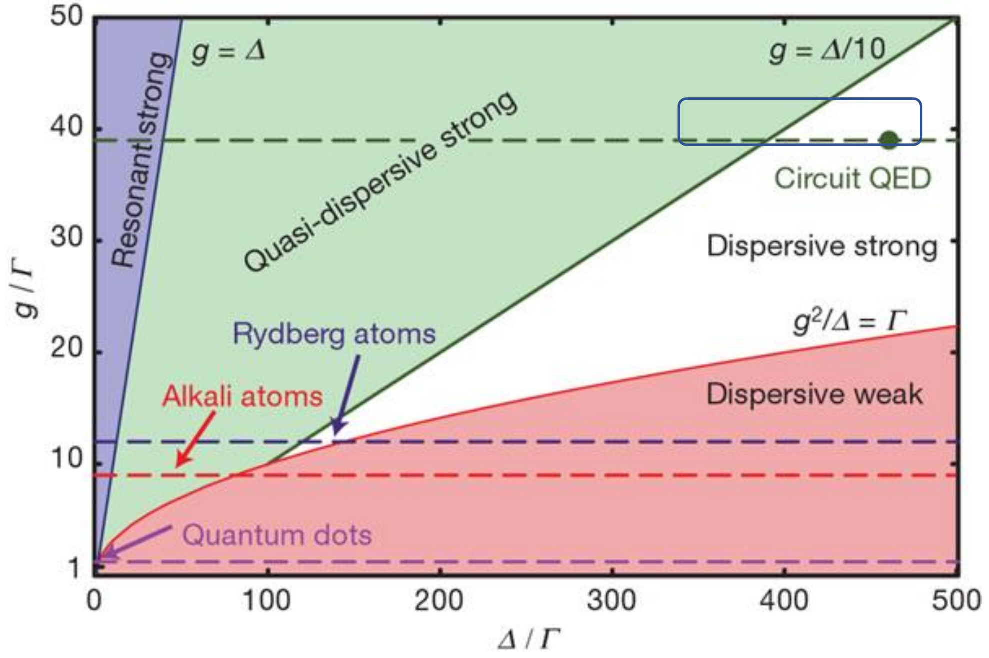
Reading list : PRA, **76, 042319 (2007).*

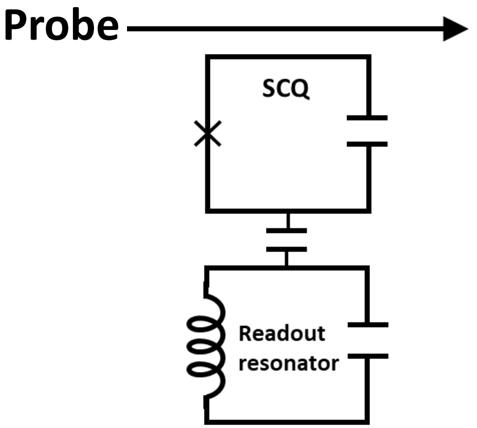
Part I – circuit QED : Elementary experiments.

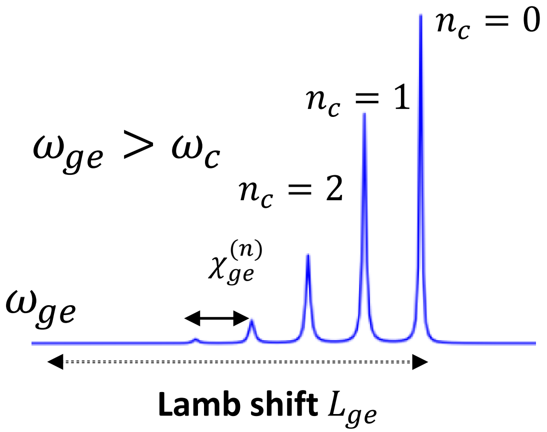
Photon number splitting with strong dispersive coupling.

Cavity/Circuit QED Phase Diagram.

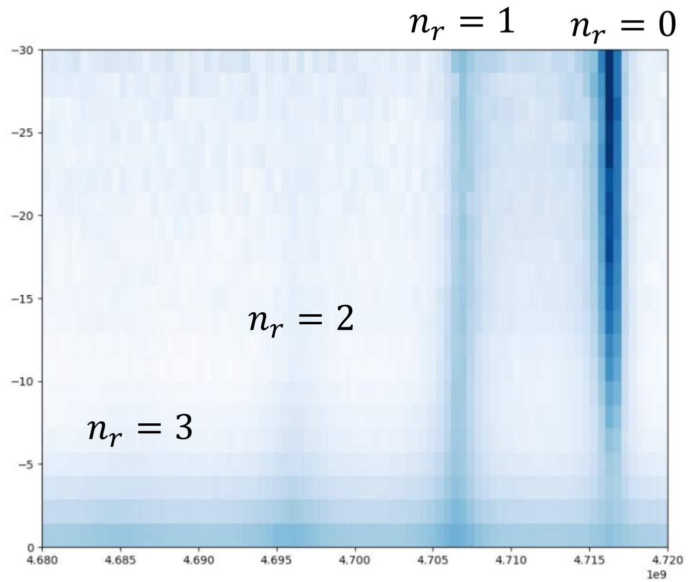


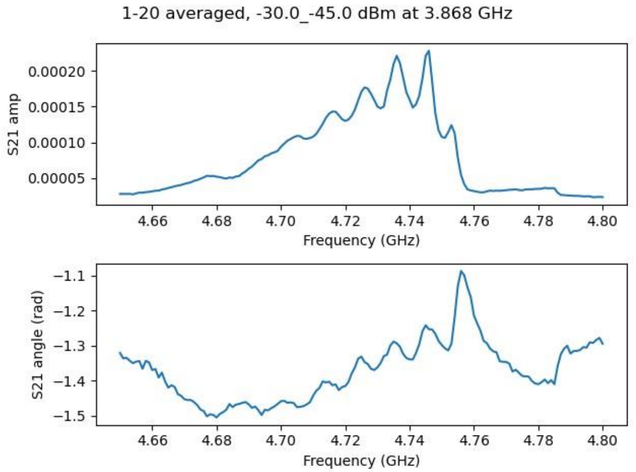
(Schuster *et al.*)





(qubit spectrum)





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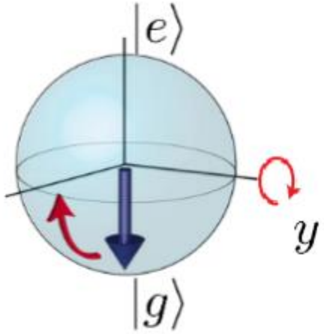
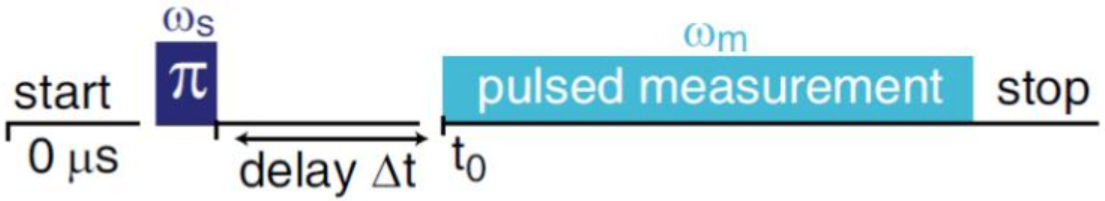
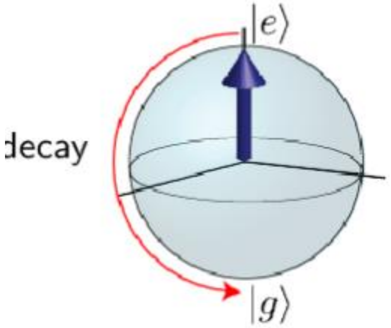
Part I – circuit QED : Elementary experiments (T1 meas).

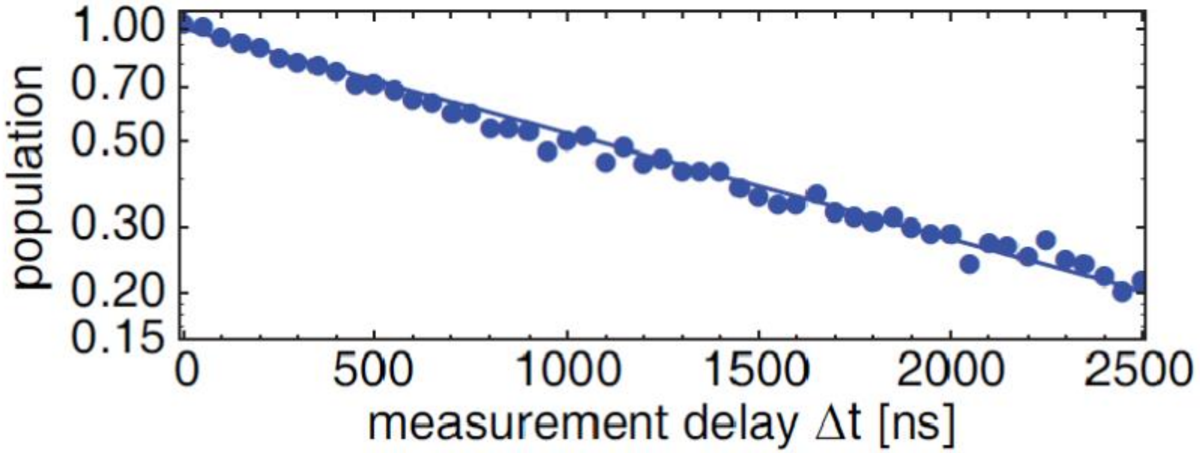
pulse scheme:





T_1=1.2 us





R. Bianchetti, QUDEV, ETH Zurich (2010)

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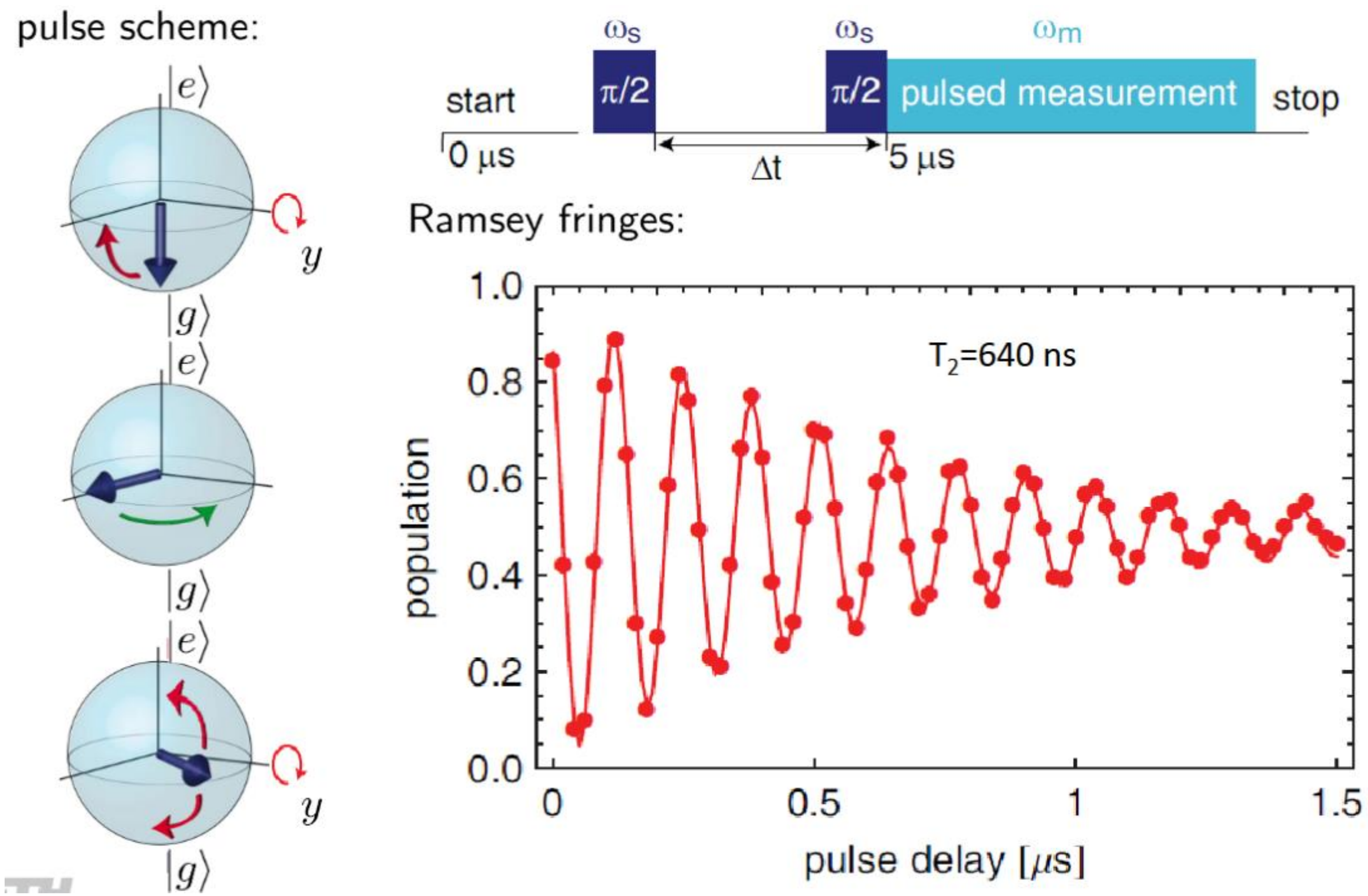
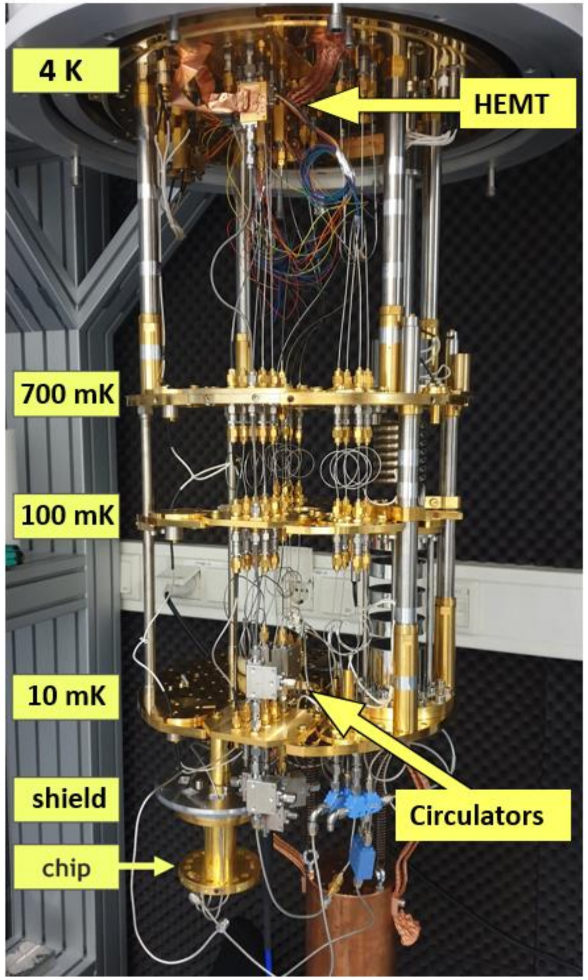
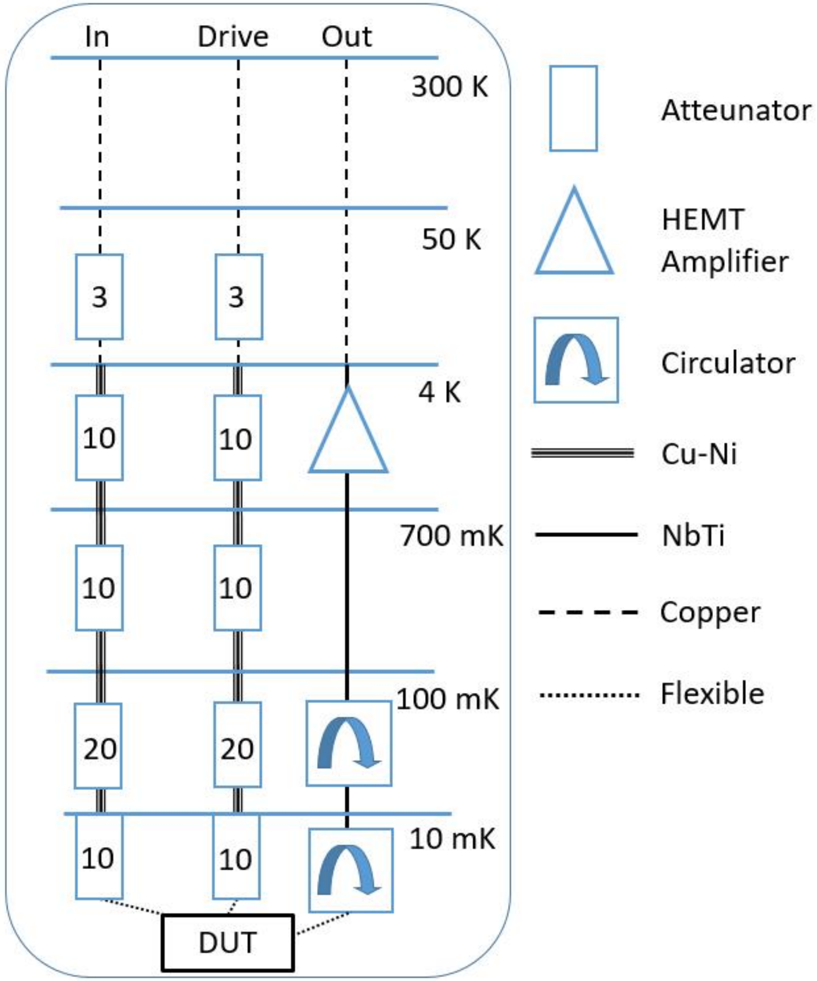
Part I – circuit QED : Elementary experiments (T2 meas).



R. Bianchetti, QUDEV, ETH Zurich (2010)

Part I – circuit QED : Real experiments – setup.





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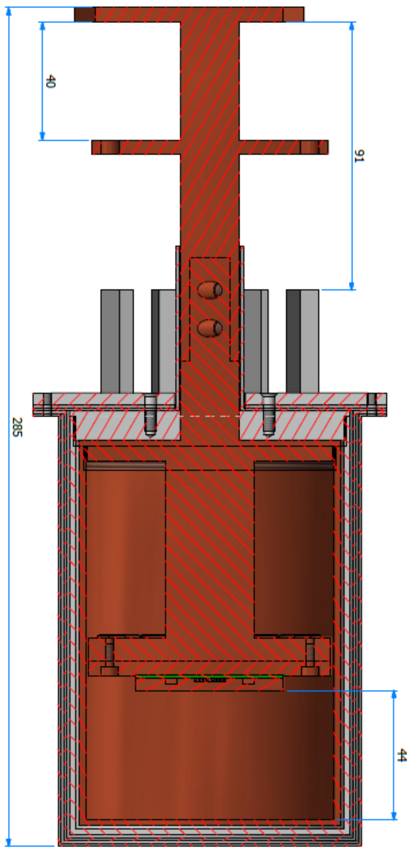
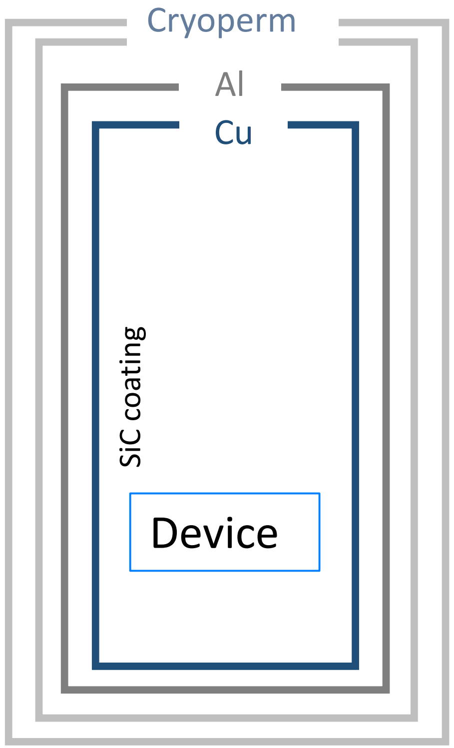
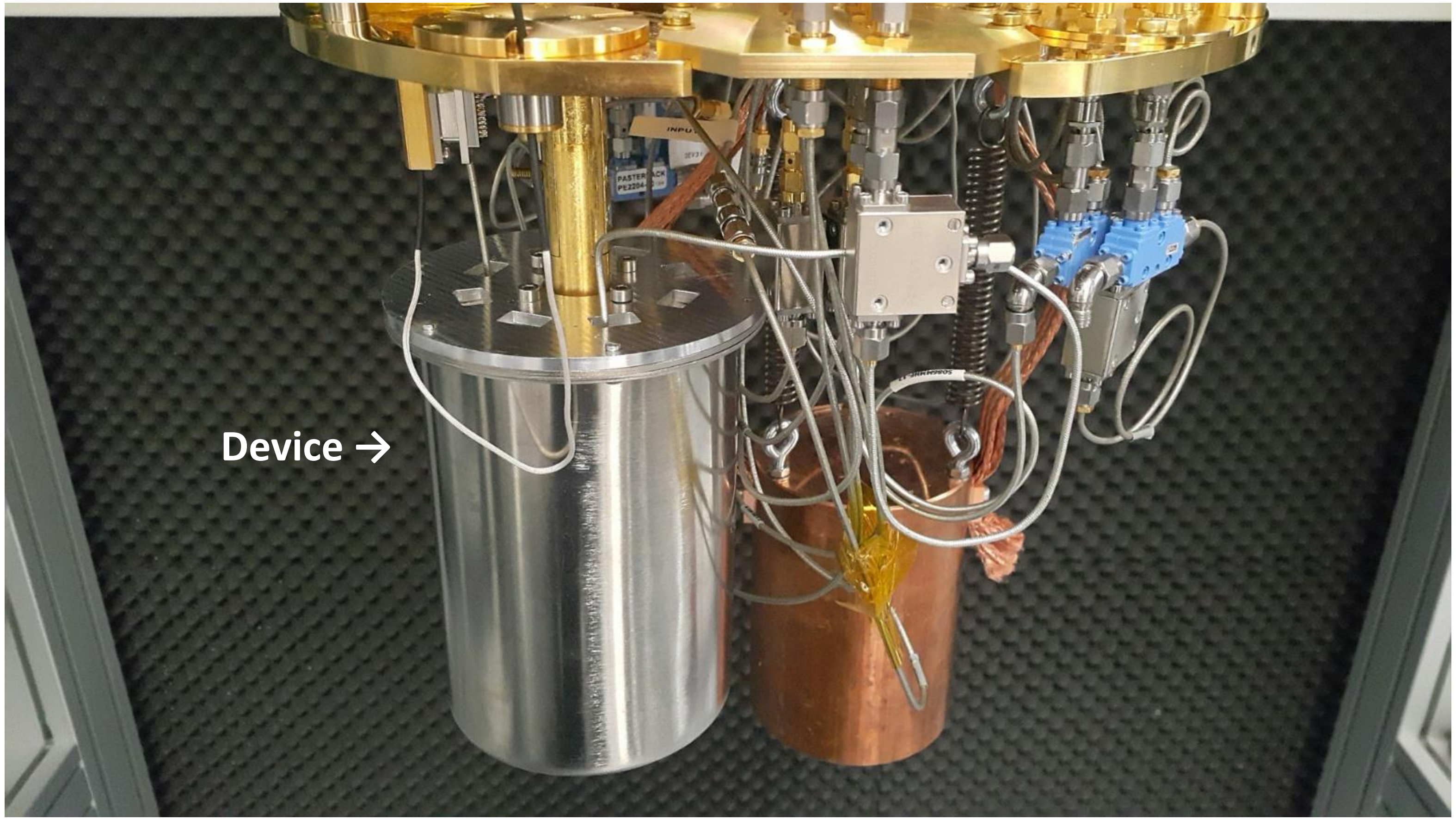
Part I – circuit QED : Real experiments – setup.







To protect the device from radiation and magnetic noise.

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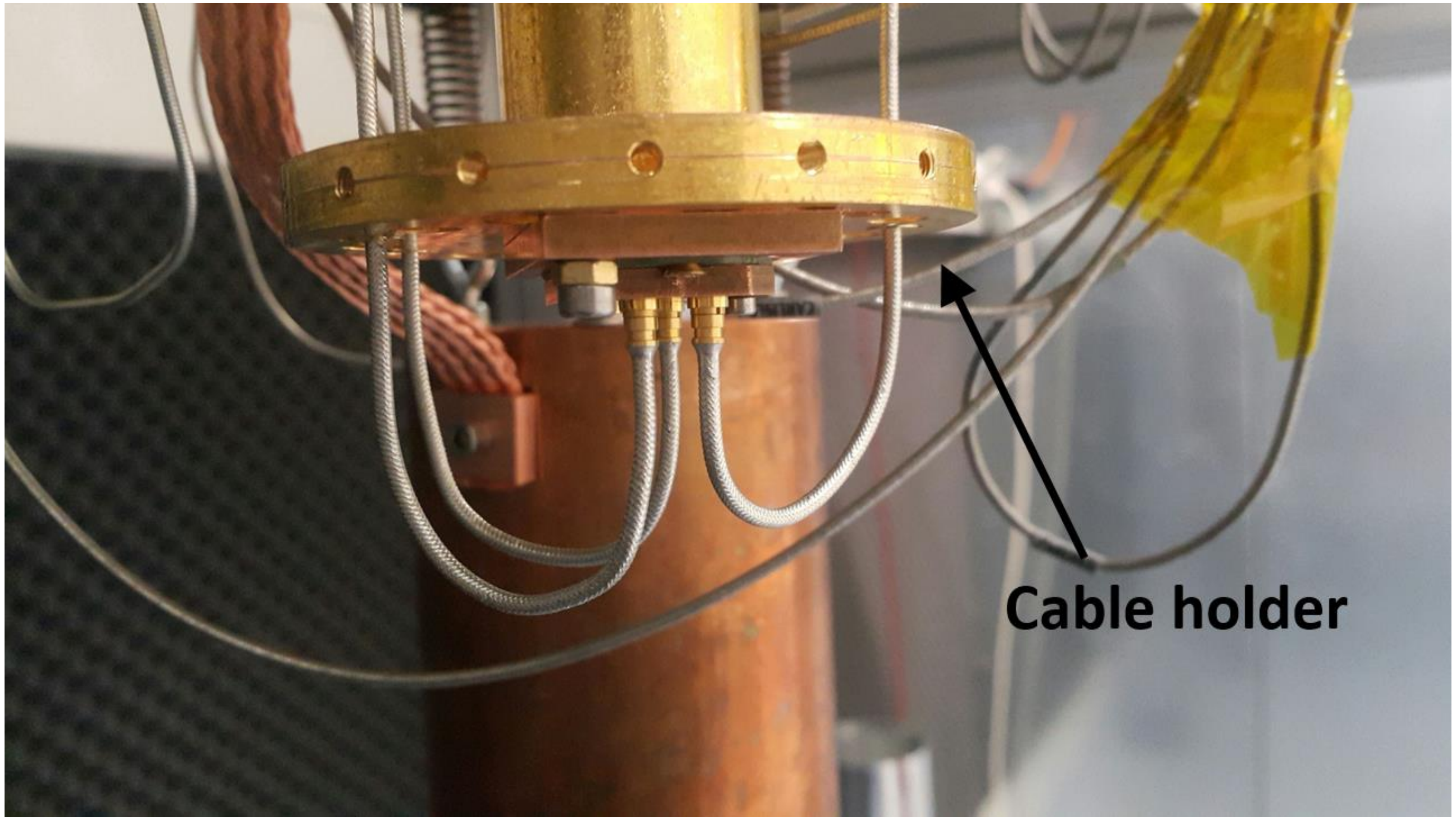
Part I – circuit QED : Real experiments – setup.

Inside



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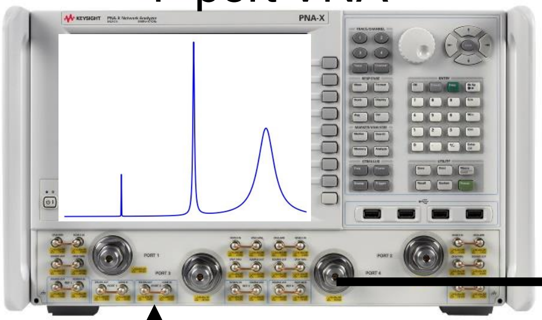
Part I – circuit QED : Real experiments – frec domain meas.

4 -port VNA

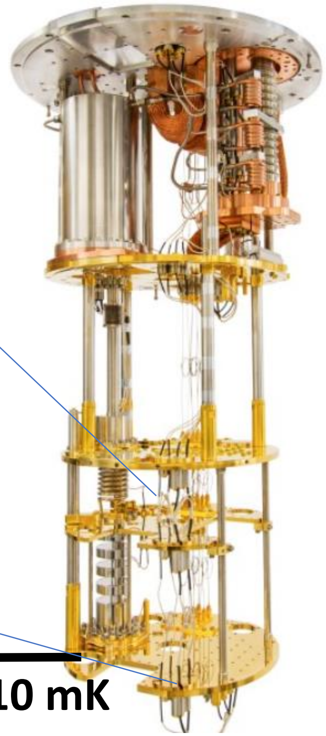


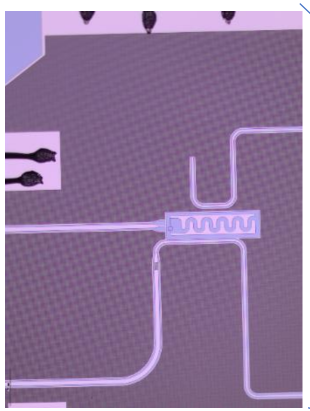
receiver

sources

Probe and spectroscopy signal.

Cryogenic fridge

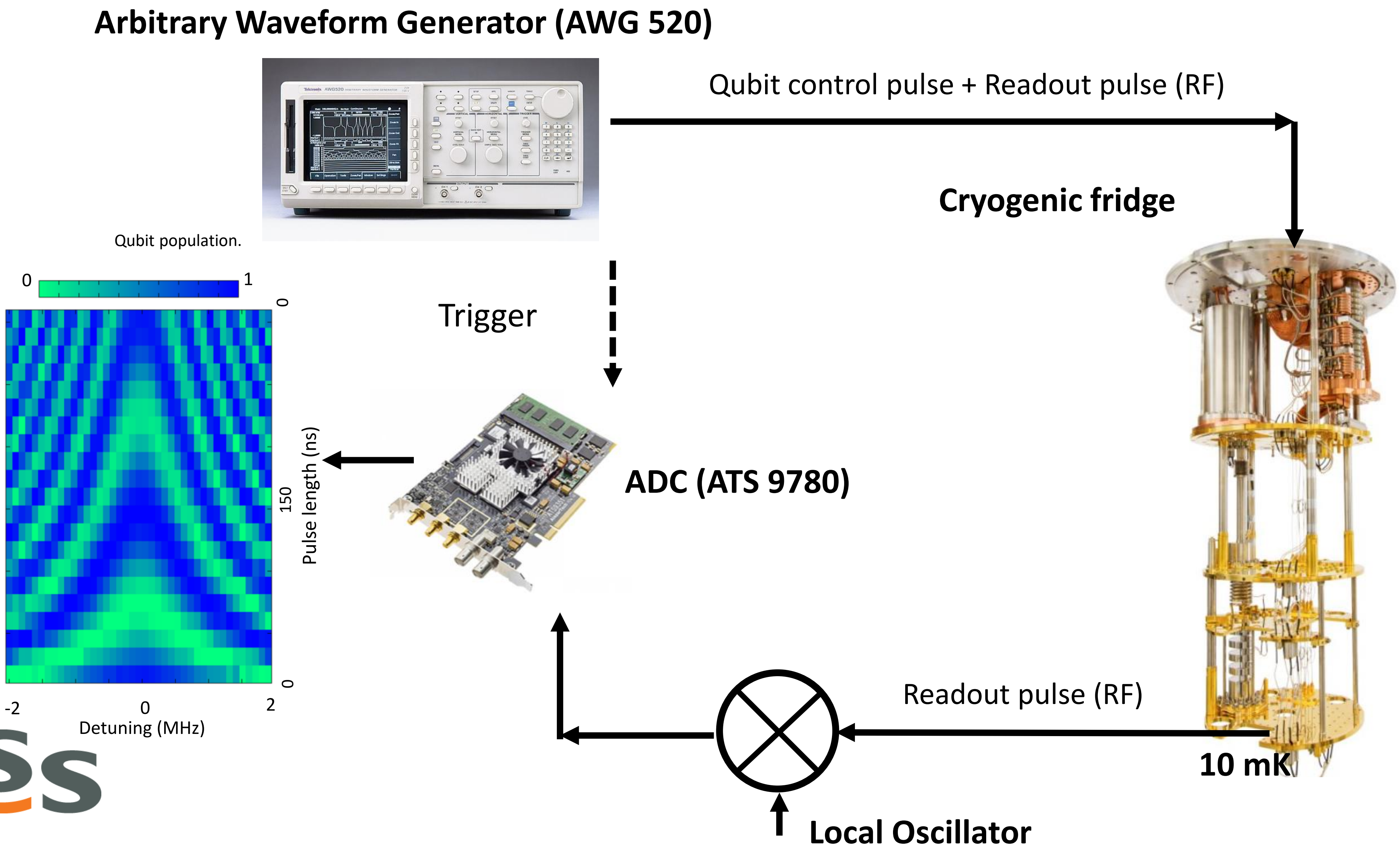




Qubit device

10 mK

Part I – circuit QED : Real experiments – time domain meas.



Outline

Part I - Fundamentals.

Backgrounds : Light-matter interaction.

Introduction to Cavity-QED.

Cavity-QED on circuits : Circuit-QED.

Experimental milestones in circuit QED.

Current trend in circuit QED.

Part II - Methods.

Analytical methods.

Rotating frame.

Rotating wave approximation.

Perturbative diagonalization.

Numerical methods.

QuTip

QuCAT (Quantum Circuit Analysis Tool).

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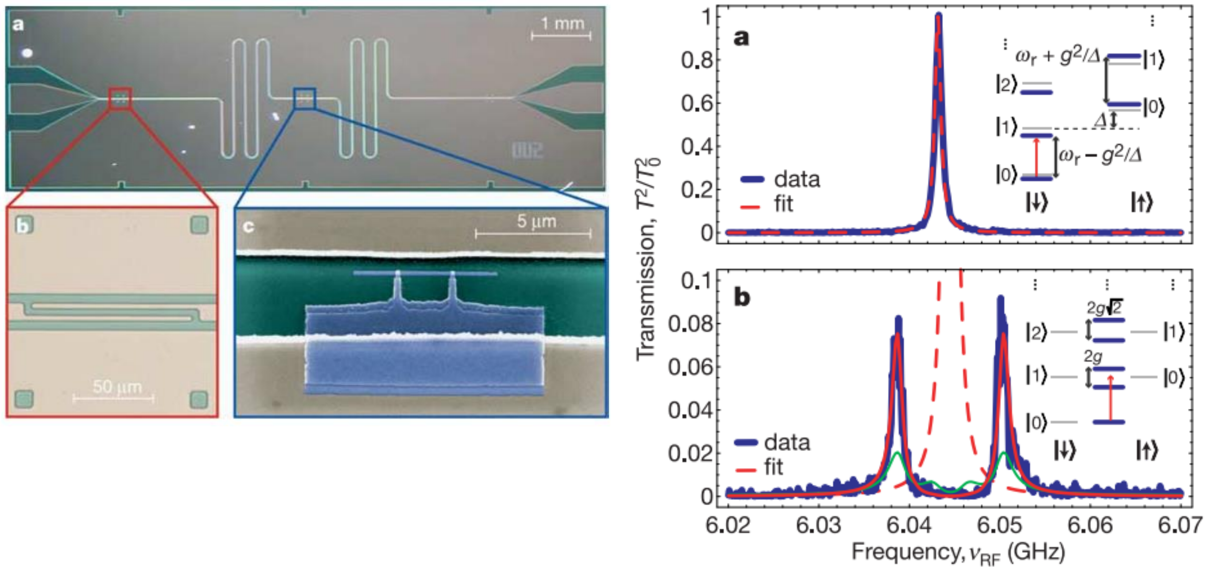
Part I – Experimental milestones in circuit QED.

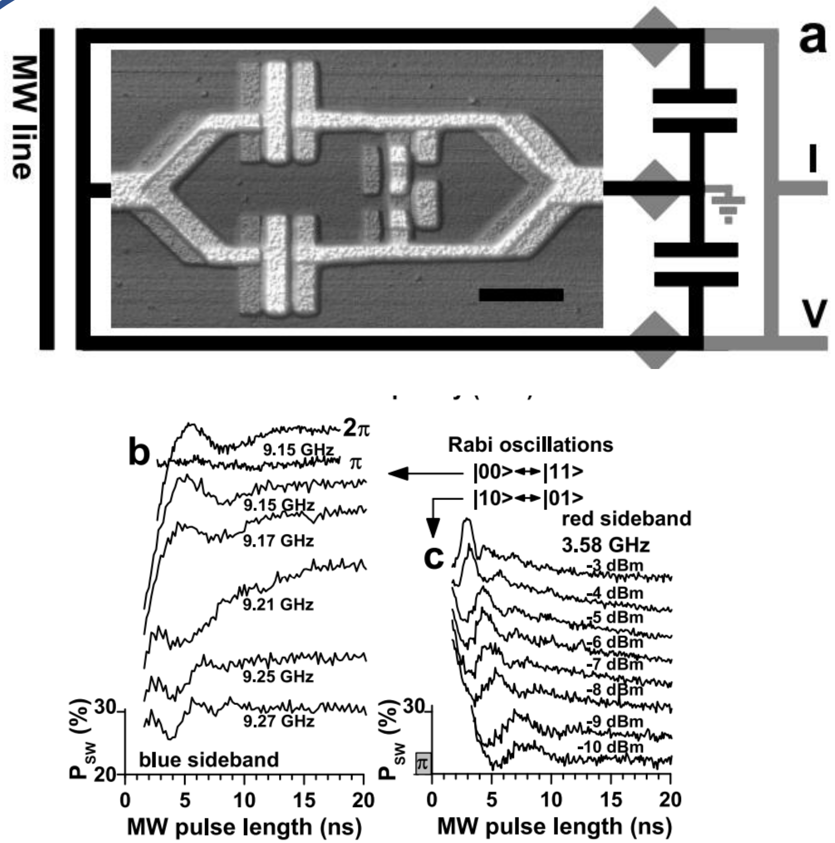
Entering strong coupling regime.

Charge-based qubit (CPB).



Nature ***431***, *162 (2004).*

Flux qubit



Nature ***431***, *159 (2004).*

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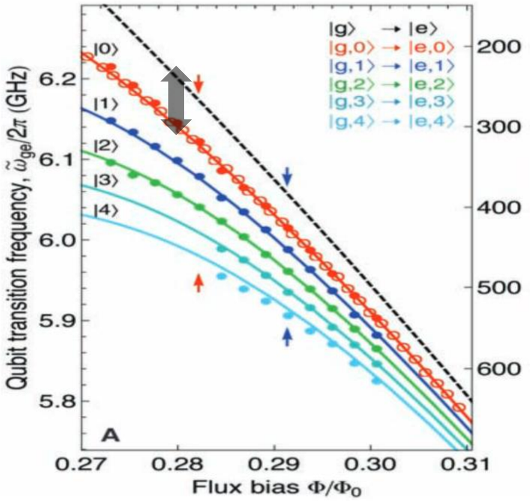
Part I – Experimental milestones in circuit QED.

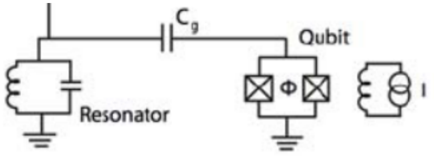
Exploring fundamental vacuum effects.

Lamb shift (transmon).



Observed Lamb shift ~ 50 MHz.
(>> atomic cases).

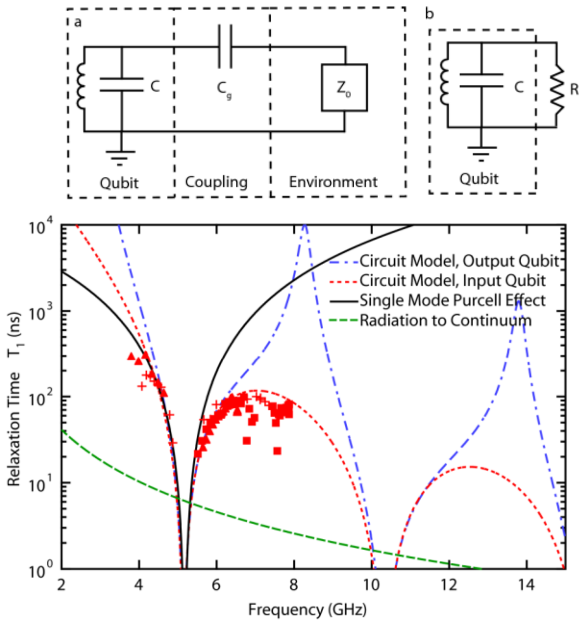
5% of qubit frec.



Flux tunable transmon.

Science **322**, 1357 (2008).

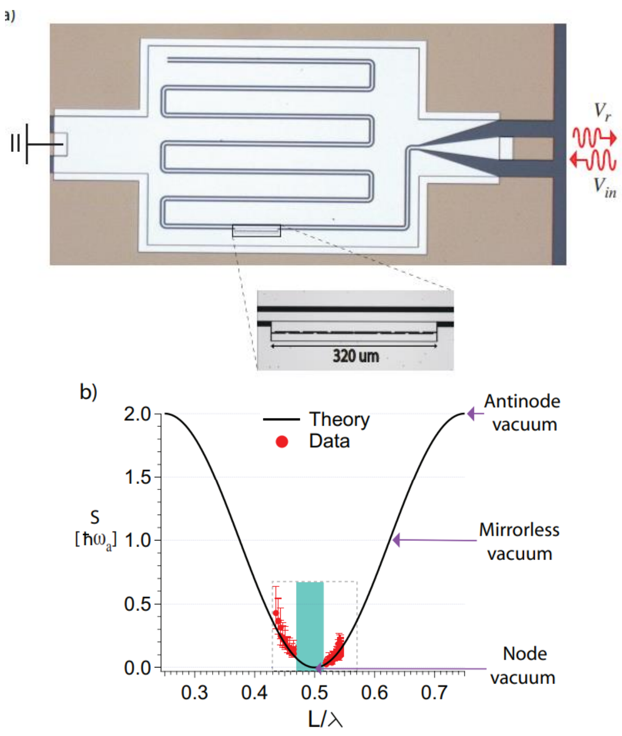
Purcell effect



Controlling Purcell effects by modulating qubit frec (flux tunable).

PRL **101**, 080502 (2008).

Probing vacuum mode



Nat. Phys **11**, 1045 (2015)

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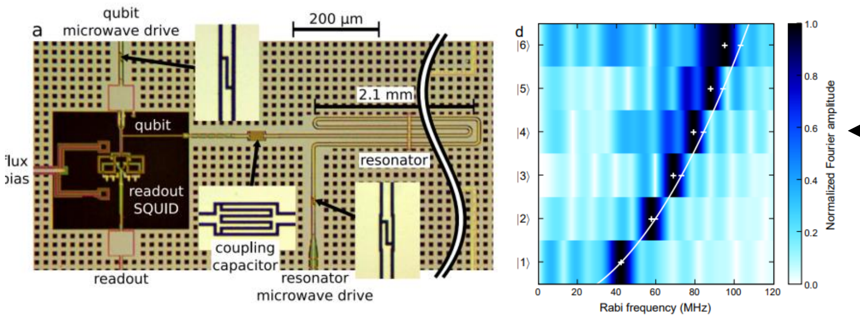
Part I – Experimental milestones in circuit QED.

Microwave quantum optics

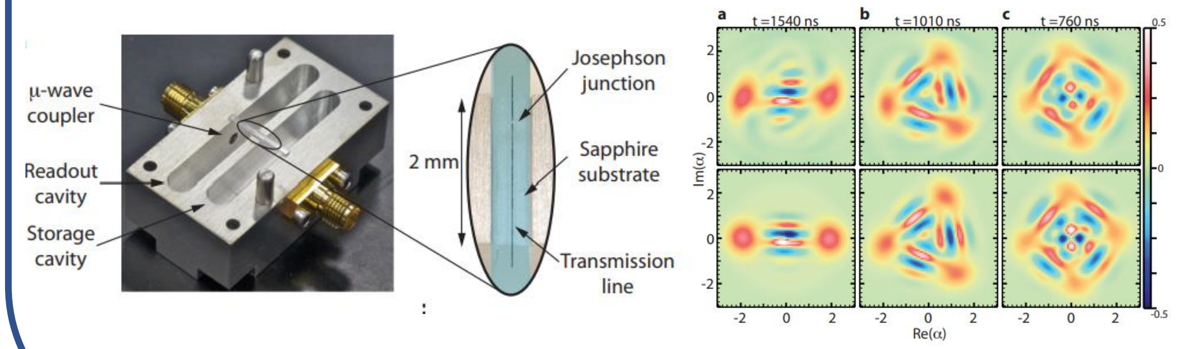
Non-classical microwave (standing).



← Fock state.

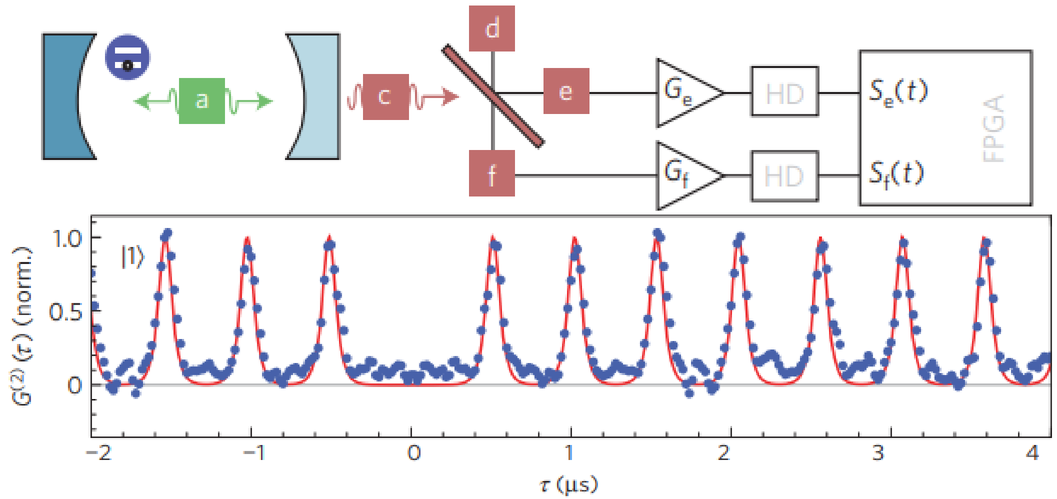
Nature ***454***, 310 (2008)

Cat state.

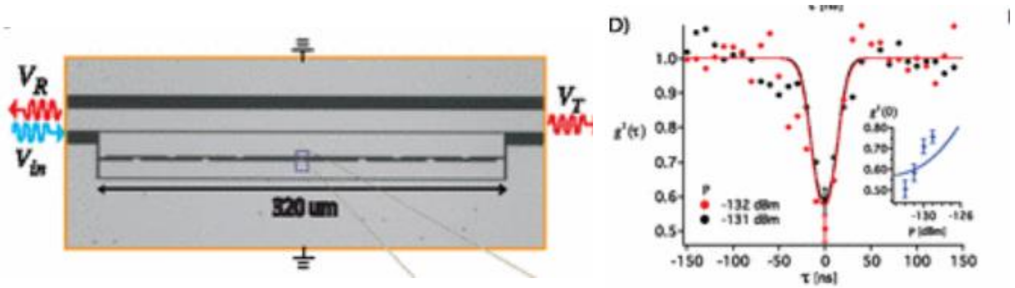


Nature ***495***, 205 (2013)

Non-classical microwave (propagating).



Nat. Phys **7**, 154 (2011)

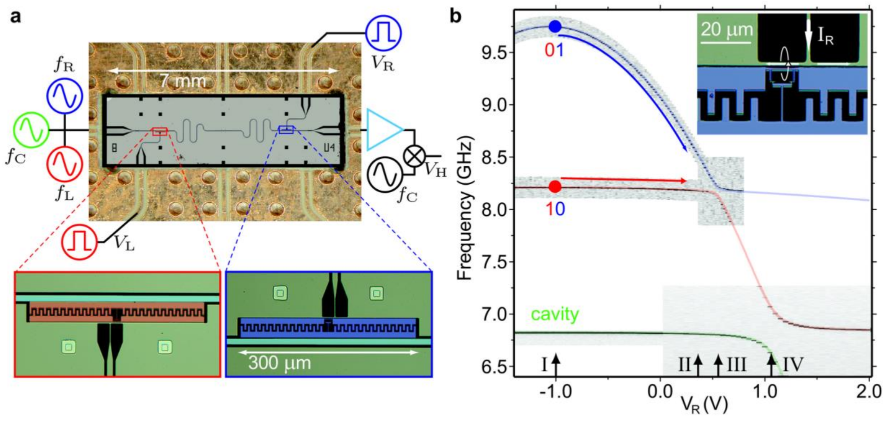


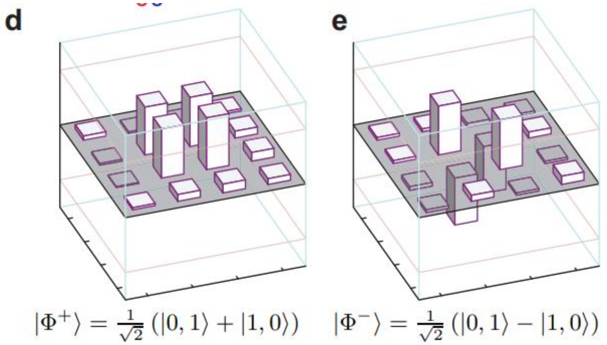
PRL **7**, 263601 (2012)

Part I – Experimental milestones in circuit QED.

Quantum gate operation

Two-qubit gates

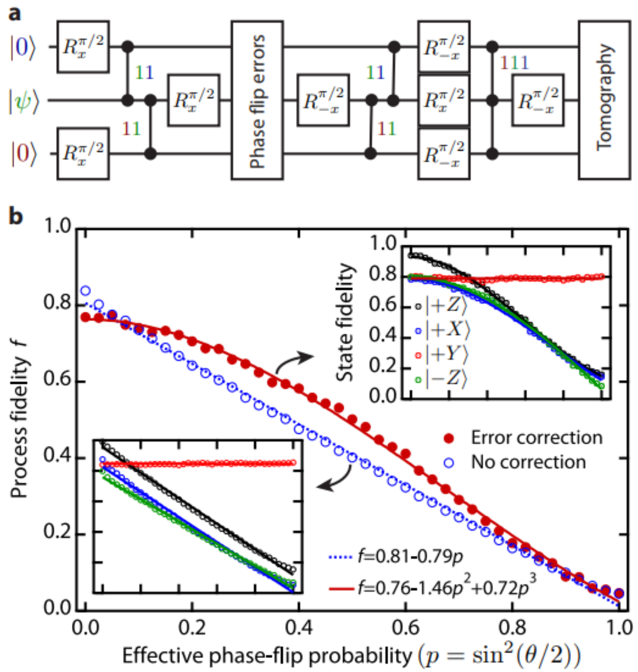




Bell state tomography

Nature ***460****, 240 (2009)*

Error correction code



Nature ***482****, 382 (2012)*

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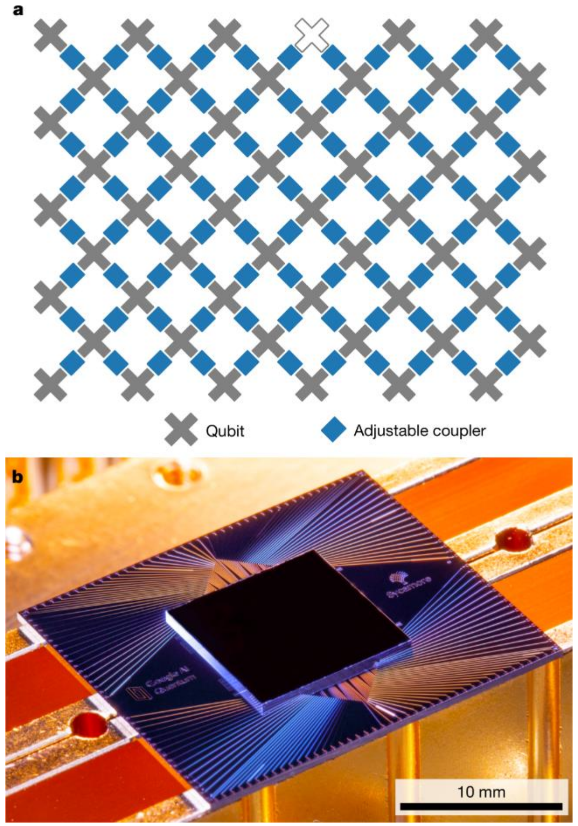
Part I – Current trends in circuit QED.

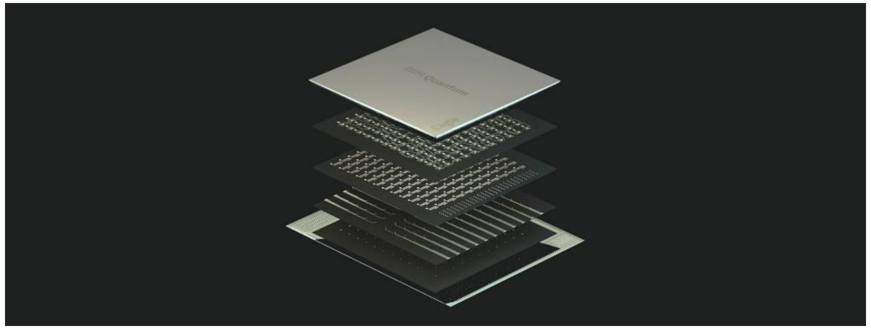
Scaling up qubit numbers.

53 qubits (S)



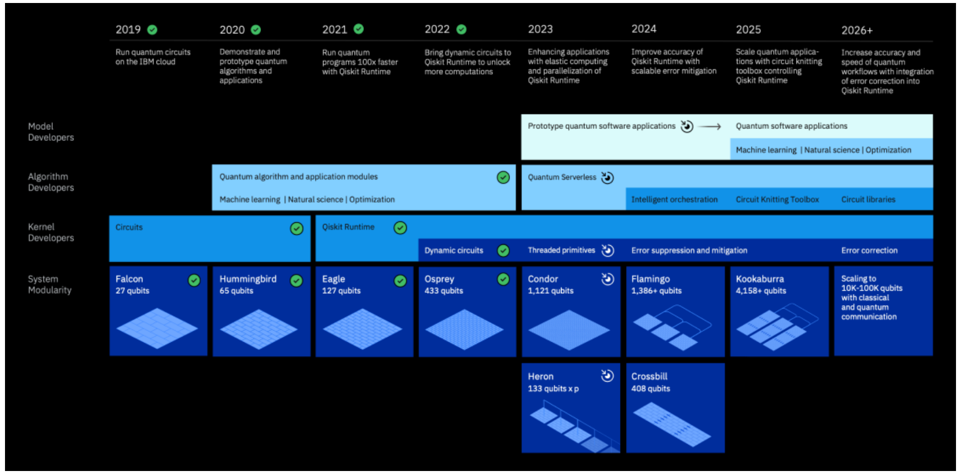
Nature ***574***, *505(2019).*

127 qubits (IBM Eagle).



https://research.ibm.com/quantum-computing

Roadmaps toward 100k qubits.



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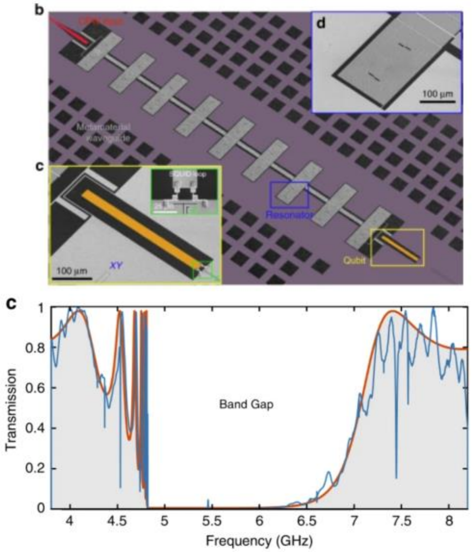
Part I – Current trends in circuit QED.

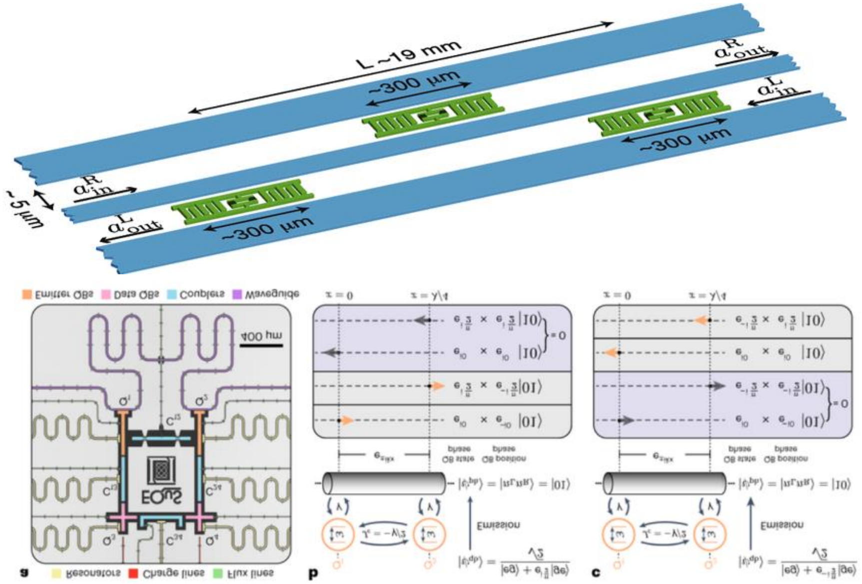
Replacing resonators with others.

Metamaterials



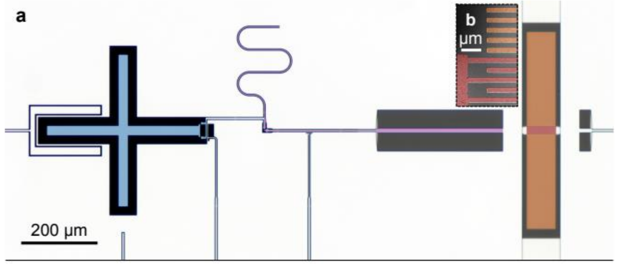
Nat. Comm. ***9***, *3706 (2018).*

Waveguides

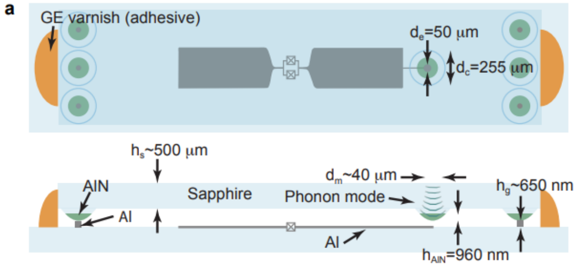


PRA 88, 043806 (2013).
Nat. Phys ***19***, *1 (2023).*

Acoustic waves resonators



Nature ***563***, *661(2018).*

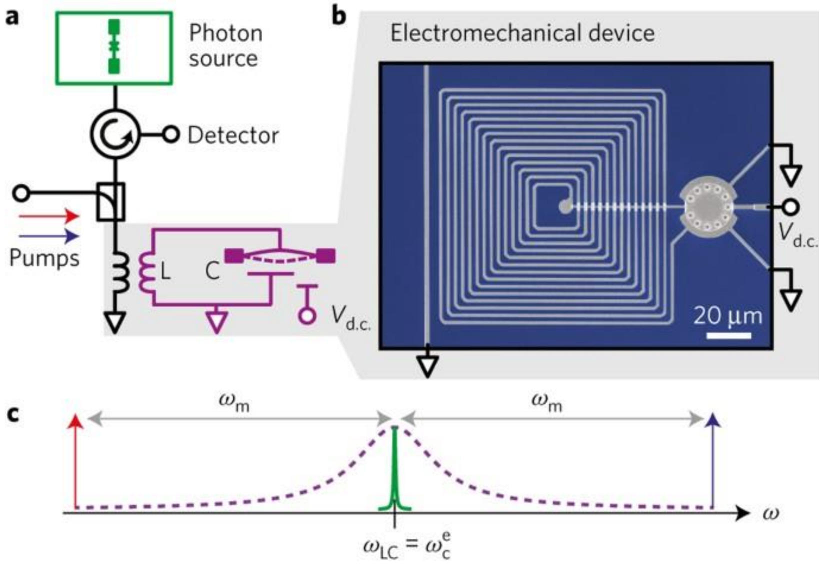


Nature ***563***, *666 (2018).*

Part I – Current trends in circuit QED.

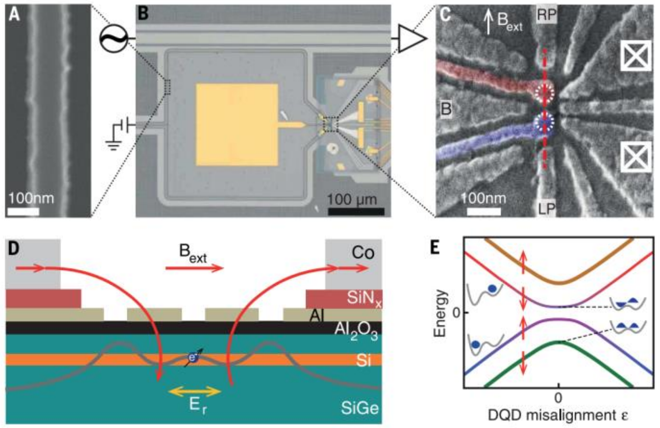
Hybrid circuit QED

cQED with mechanics.



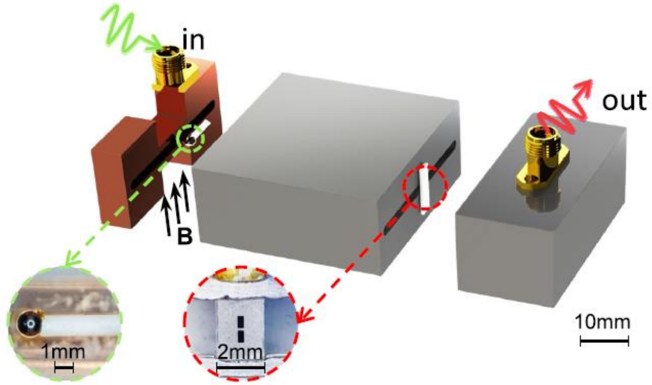
Nat. Phys ***13***, *1163 (2018).*

cQED with QD.



Science ***359***, *1123 (2018).*

cQED with Magnons.



PRL ***130***, *193603 (2023).*

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Outline

Part I - Fundamentals.

Backgrounds : Light-matter interaction.

Introduction to Cavity-QED.

Cavity-QED on circuits : Circuit-QED.

Experimental milestones in circuit QED.

Current trend in circuit QED.

Part II - Methods.

Analytical methods.

- Rotating frame.
- Rotating wave approximation.
- Perturbative diagonalization.

Numerical methods.

- QuTip
- QuCAT (Quantum Circuit Analysis Tool).

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Part II – Analytical methods.

In cQED research, you will often encounter Hamiltonian like,

$$\underbrace{\hat{H} = \frac{\omega_q}{2}\hat{\sigma}_z + \omega_c \hat{a}^\dagger \hat{a} + g(\hat{a}^\dagger + \hat{a})\hat{\sigma}_x}_{\hat{H}_{\mathrm{QRM}}} \underbrace{+ \sum_i \Omega_d^{(i)} \hat{\sigma}_x \cos(\omega_d^{(i)} t)}_{\hat{H}_{\mathrm{drive}}}.$$

$$(\omega_t^{(1)} + \chi_t)\hat{a}^\dagger \hat{a} + \omega_r^{(1)} \hat{b}^\dagger \hat{b} - \frac{1}{12}\left[\chi_t^{1/4}(\hat{a} + \hat{a}^\dagger) + \chi_r^{1/4}(\hat{b} + \hat{b}^\dagger)\right]^4 + \Omega_d \cos\omega_d t(\hat{a} + \hat{a}^\dagger)$$

→ ***Composed of sigma, ladder operators with oscillating time dependence...***

→ ***Multi-mode, off-diagonal, rapid time-dependent...***

Also, typically dissipation process is not negligible in the experiments.

→ ***Thus, Schrodinger equation is not enough.***

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Part II – Open quantum system : Master equation.

Master equation :

$$\frac{d\hat{\rho}_s}{dt} = \underbrace{-i\left[\hat{H}_s(t), \hat{\rho}_s(t)\right]}_{\text{Liouvillian, unitary evolution}}$$

$$\underbrace{+\sum_n \frac{1}{2}\left(2\hat{C}_n\hat{\rho}(t)\hat{C}_n^\dagger - \hat{\rho}(t)\hat{C}_n^\dagger\hat{C}_n - \hat{C}_n^\dagger\hat{C}_n\hat{\rho}(t)\right).}_{\text{Lindbladian, non-unitary evolution}}$$

$$\hat{C}_1 = \frac{1}{\sqrt{T_1}}\left|0\right\rangle\left\langle 1\right|$$

$$\hat{C}_2 = \frac{1}{\sqrt{T_2^*}}\left(\left|0\right\rangle\left\langle 0\right| - \left|1\right\rangle\left\langle 1\right|\right)$$

For two-level atom.

→ Solving master equations with time-dependent Hamiltonian takes very long time even in numerical manners.

→ Fitting the data with master equation model with fast time-dependence?

Practically impossible in normal cases.

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****For derivation and detailed information in the general systems :***

D. Steck, quantum-optics lectures.

https://atomoptics.uoregon.edu/~dsteck/teaching/quantum-optics/quantum-optics-notes.pdf

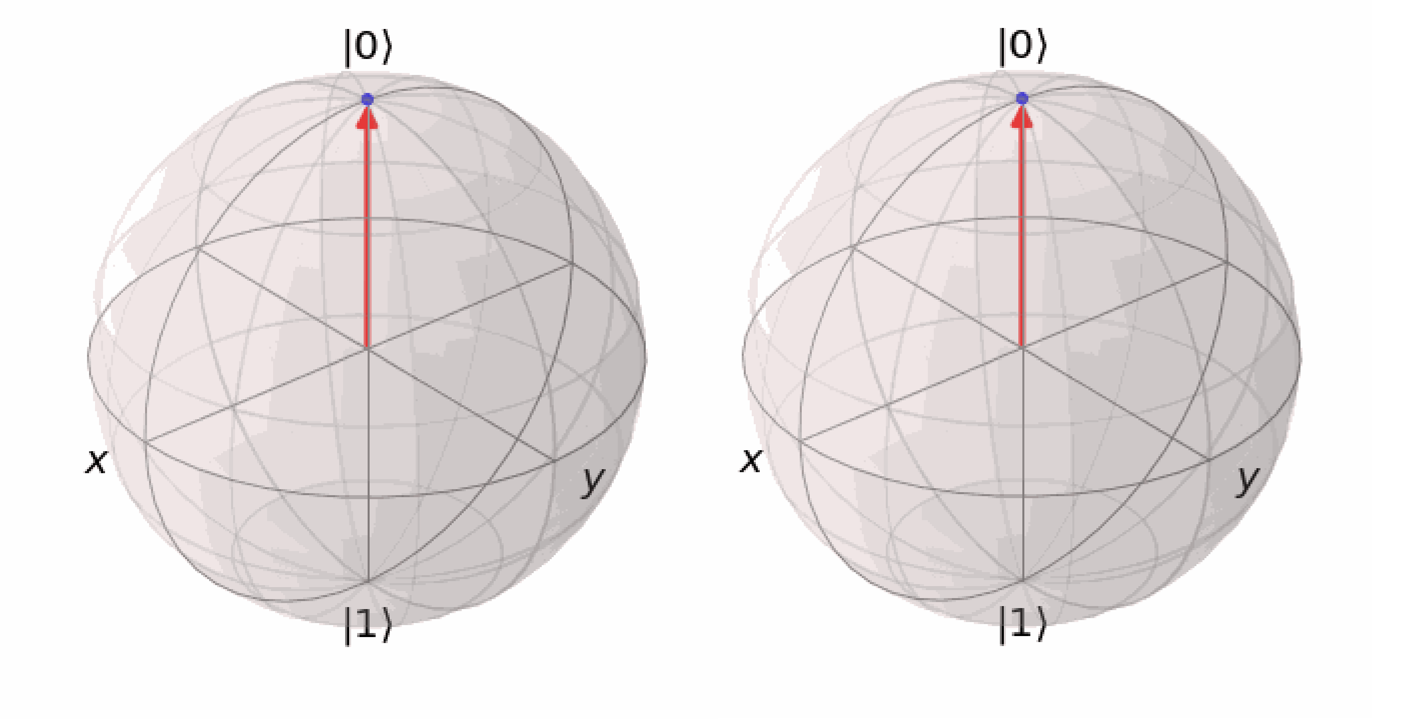
Part II – Rotating frame : Getting rid of time-dependence.

Simplifying Hamiltonian by moving together with oscillating terms.

$$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \frac{\Omega_d}{2} e^{i\omega_q t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{-i\omega_q t}\,\hat{\sigma}_+$$

$$\Downarrow$$

$$\hat{H}_{Rot} = \cancel{\frac{\omega_q - \omega_q}{2}}\hat{\sigma}_z + \frac{\Omega_d}{2}\hat{\sigma}_- + \frac{\Omega_d}{2}\,\hat{\sigma}_+$$



(Lab frame). (Rotating frame).

Dynamics looks much simpler in the rotating frame.

No approximation yet !

Part II – Rotating frame.

Rotating transformation :

$$\hat{H}_{Rot} = \hat{U}_R(t)\hat{H}_{Lab}(t)\hat{U}_R^+(t) + i\hat{U}_R^+(t)\partial_t\hat{U}_R(t)$$

Generators :

Two-state systems $\hat{U}_R^{TLS} = e^{-i\omega_d t*\hat{\sigma}_z/2}$

Harmonic oscillators $\hat{U}_R^{HO} = e^{-i\omega_d t*\hat{a}^+\hat{a}}$

Rules :

Do not touch already existing diagonal terms.

$$\boldsymbol{i\hat{U}_R^+(t)\partial_t\hat{U}_R(t) = -\omega_d\hat{a}^+\hat{a}} \quad \textit{or} \quad \boldsymbol{-\frac{\omega_d}{2}\hat{\sigma}_z}$$

$\hat{a} \rightarrow \hat{a}\, e^{-i\omega_d t}$

or

$\hat{\sigma}_- \rightarrow \hat{\sigma}_- e^{-i\omega_d t}$

Destruction operator become more destructive and vice versa.

Let's check.

$$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \frac{\Omega_d}{2}e^{i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2}e^{-i\omega_d t}\,\hat{\sigma}_+$$

$$\hat{H}_{Rot} = \frac{\omega_q-\omega_d}{2}\hat{\sigma}_z + \frac{\Omega_d}{2}\hat{\sigma}_- + \frac{\Omega_d}{2}\,\hat{\sigma}_+$$

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Part II – Doubly Rotating frame.

Multi-mode situation :

$$
\begin{aligned}
\hat{\mathscr{H}} &= \frac{\omega_q - \omega_p}{2}\hat{\sigma}_z + \cancelto{(\omega_c + 2\omega_d - \omega_p)}{\omega_c}\, \hat{a}^\dagger \hat{a} - 2\chi_{qc}\hat{\sigma}_z \hat{a}^\dagger \hat{a} \\
&+ \frac{\Omega_{sb}}{2}\left(\hat{a}\hat{\sigma}_+ e^{\cancelto{i(\omega_p - 2\omega_d)t}{-2i\omega_d t}} + \hat{a}^\dagger \hat{\sigma}_- e^{\cancelto{-i(\omega_p - 2\omega_d)t}{+2i\omega_d t}}\right) \\
&+ \frac{\Omega_p}{2}\left(\hat{\sigma}_+ e^{\cancel{-i\omega_p t}} + \hat{\sigma}_- e^{\cancel{+i\omega_p t}}\right),
\end{aligned}
$$

$$\hat{U}_R^D = \boxed{e^{-i(\omega_p - 2\omega_d)t\cdot\hat{a}^+\hat{a}}} \cdot \boxed{e^{-i\omega_p t\cdot\hat{\sigma}_z/2}}$$

$$
\begin{aligned}
\hat{\mathscr{H}}' &= \frac{(\omega_q - \omega_p)}{2}\hat{\sigma}_z + (\omega_c + 2\omega_d - \omega_p)\hat{a}^\dagger \hat{a} - 2\chi_{qt}\hat{\sigma}_z \hat{a}^\dagger \hat{a} \\
&+ \frac{\Omega_{sb}}{2}\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-\right) \\
&+ \frac{\Omega_p}{2}\left(\hat{\sigma}_+ + \hat{\sigma}_-\right).
\end{aligned}
$$

Part II – Rotating wave approximation : When rotating frame method fails.

$$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \boxed{\Omega_d \cos\omega_d t\, \hat{\sigma}_x}$$

$$= \frac{\Omega_d}{2} e^{i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{-i\omega_d t}\,\hat{\sigma}_+ + \frac{\Omega_d}{2} e^{-i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{i\omega_d t}\,\hat{\sigma}_+$$

Let's move on to rotating frame.

co-rotating *counter-rotating*

$$\rightarrow \frac{\Omega_d}{2}\hat{\sigma}_- + \frac{\Omega_d}{2}\hat{\sigma}_+ + \frac{\Omega_d}{2} e^{-2i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{2i\omega_d t}\,\hat{\sigma}_+$$

We still have some oscillating terms...

Rotating wave approximation :

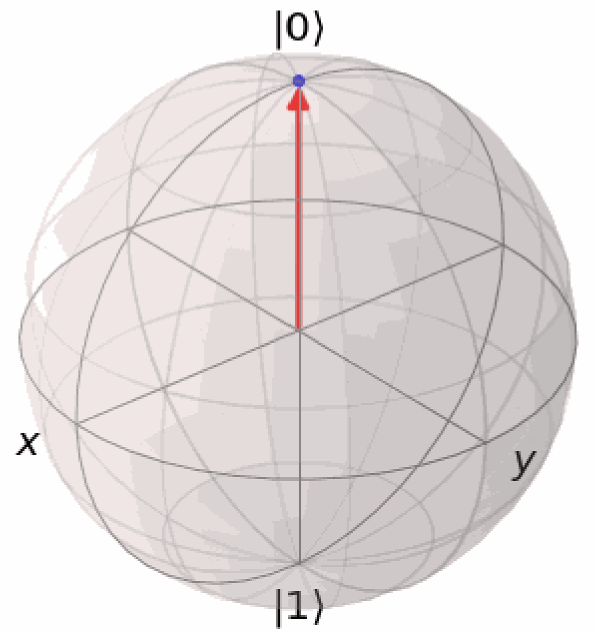
Neglecting counter-rotating components as long as $\omega_q, \omega_d \gg \Omega_d, \omega_q - \omega_d$ are satisfied.

Part II – Rotating wave approximation (RWA).

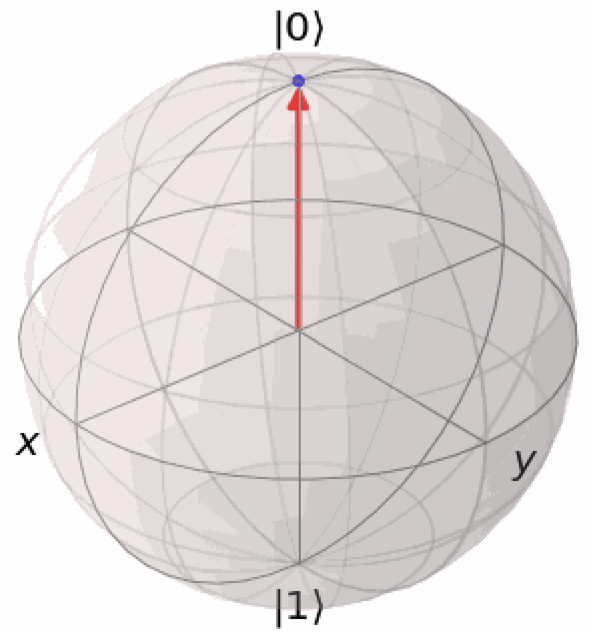
$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \Omega_d \cos\omega_d t\,\hat{\sigma}_x,$ $\quad \omega_q = \omega_d = 2\pi \cdot 6\text{ GHz},$ $\quad \Omega_d = 2\pi \cdot 500\text{ MHz}$

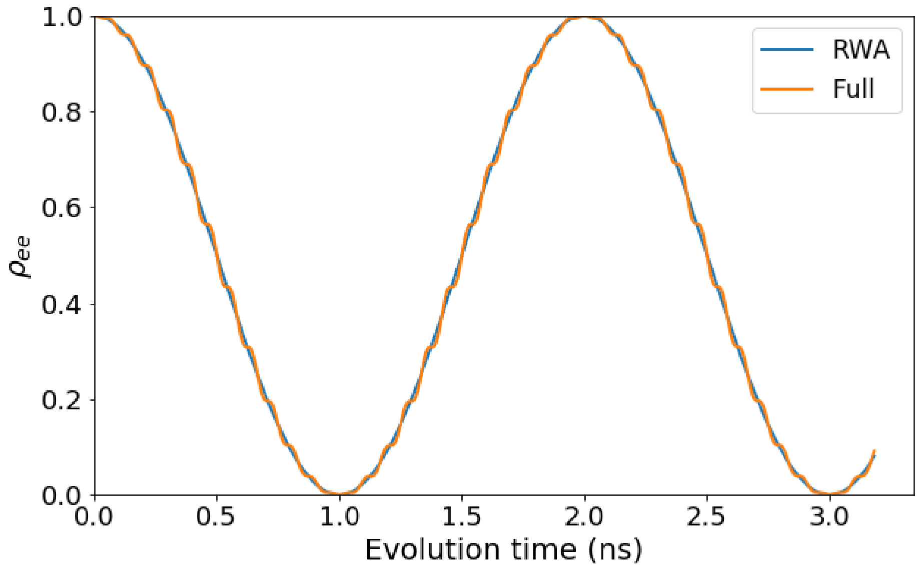
$\omega_q, \omega_d \gg \Omega_d, \omega_q - \omega_d$

(Without RWA).



(With RWA).





(Rotating frame).

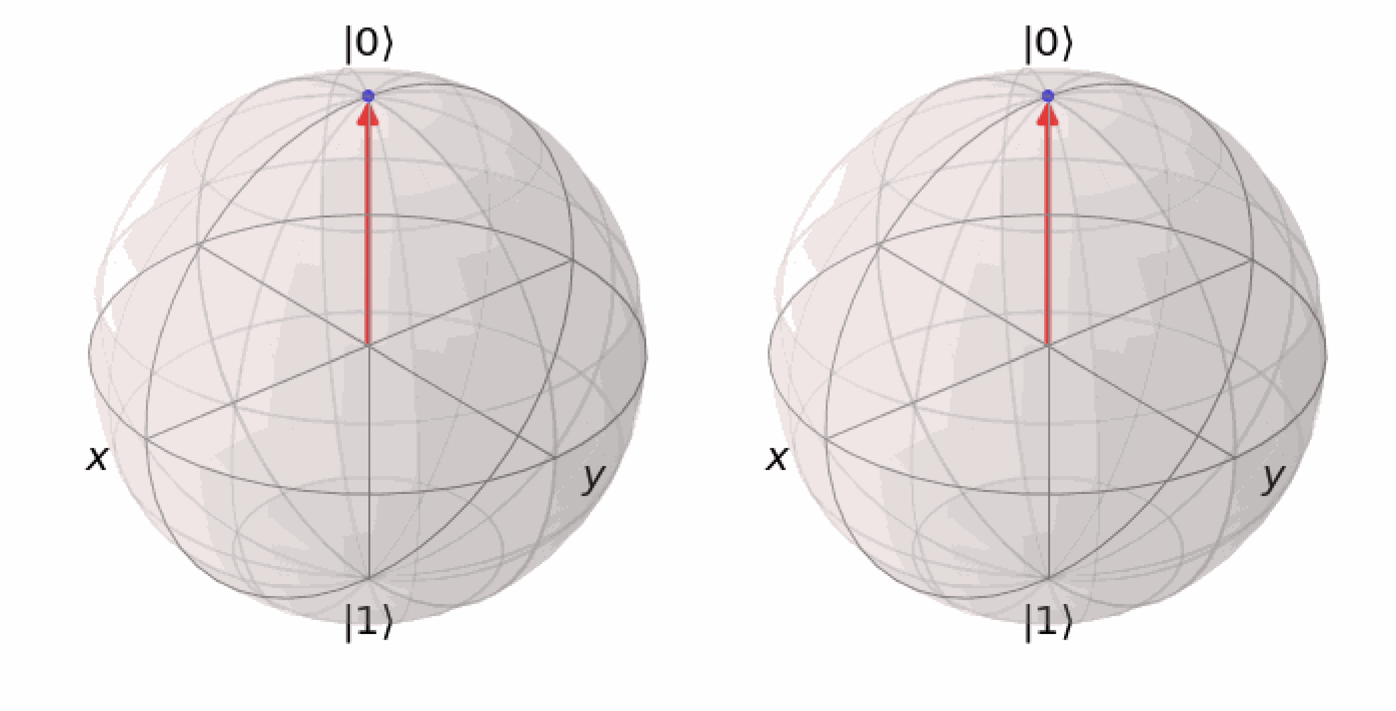
Capturing only the slow and useful dynamics of the systems.

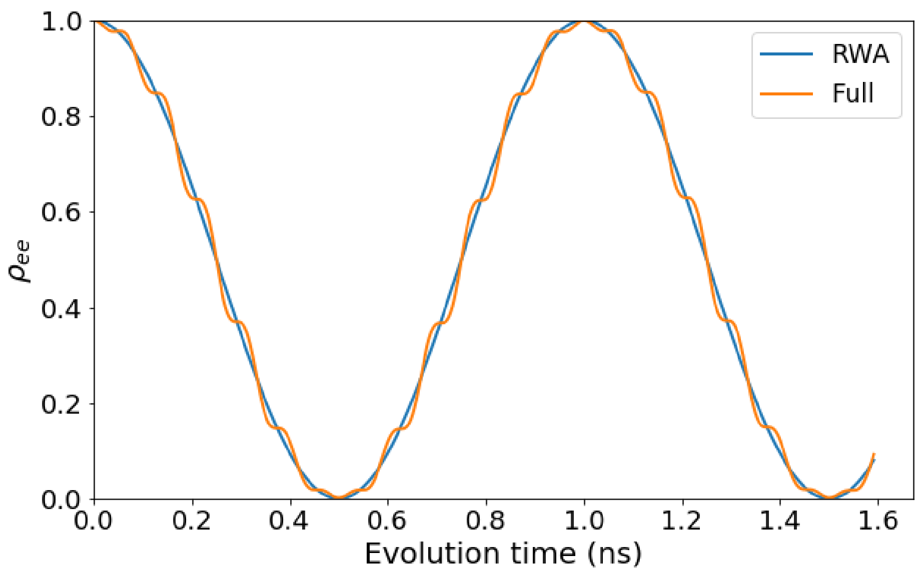
Part II – Rotating wave approximation - breakdown.

$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \Omega_d \cos\omega_d t\,\hat{\sigma}_x$, $\omega_q = \omega_d = 2\pi\cdot 6$ GHz, $\Omega_d = 2\pi\cdot 1$ GHz

$\omega_q, \omega_d \gg \Omega_d, \omega_q - \omega_d$

(Without RWA). (With RWA).





Discrepancy is getting larger...

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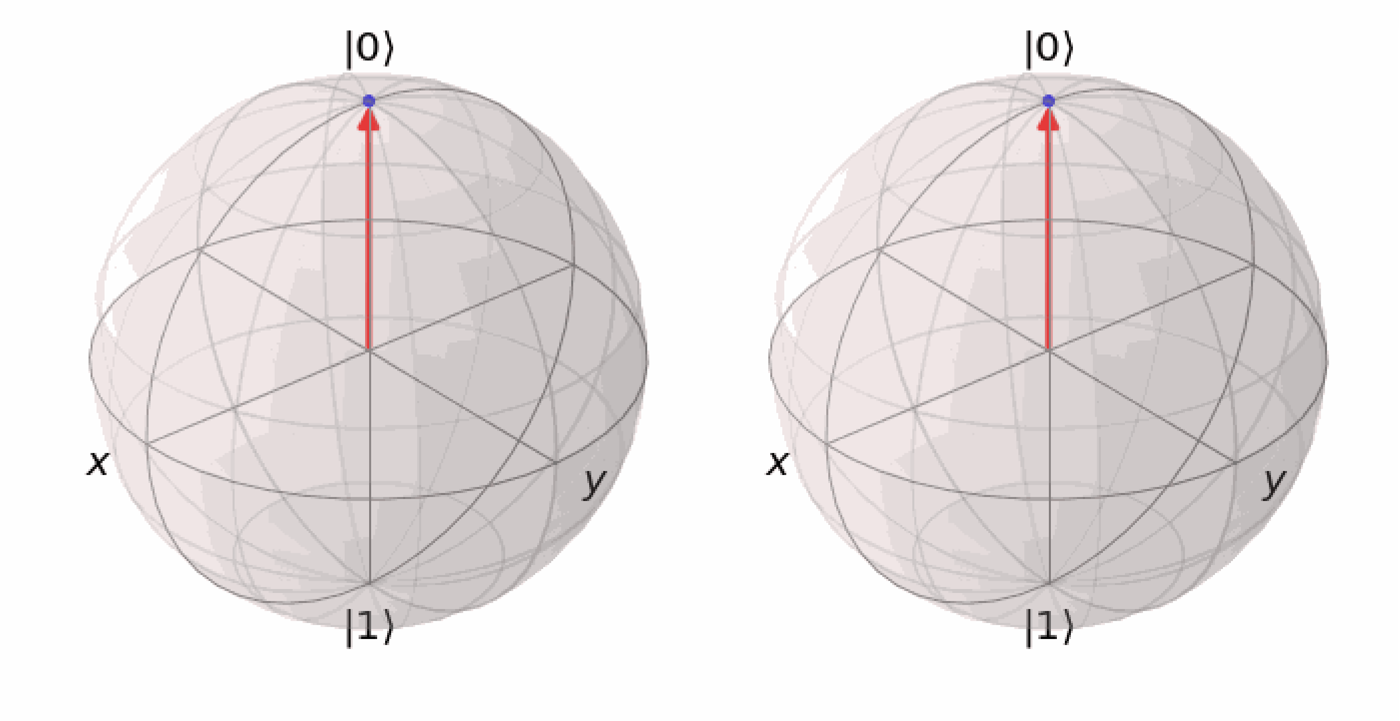
(Rotating frame).

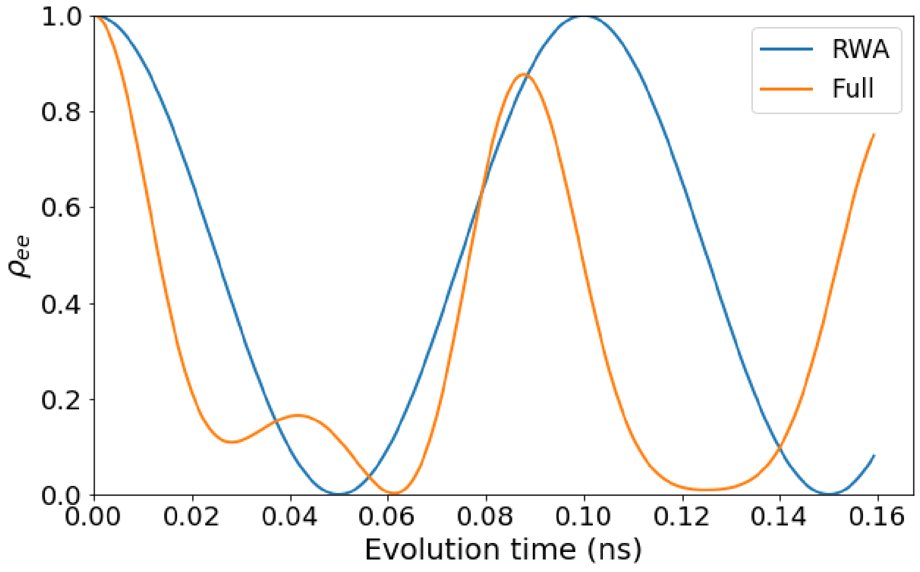
Part II – Rotating wave approximation - breakdown.

$\hat{H}_{Lab} = \frac{\omega_q}{2}\hat{\sigma}_z + \Omega_d \cos\omega_d t\,\hat{\sigma}_x,$ $\omega_q = \omega_d = 2\pi \cdot 6$ GHz, $\Omega_d = 2\pi \cdot 10$ GHz

$\omega_q, \omega_d \gg \Omega_d, \omega_q - \omega_d$

(Without RWA). (With RWA).





(Rotating frame).

Entirely deviate...

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Part II – Rotating wave approximation – general application.

Classifying the terms that can be neglected.

Discriminator : $\dfrac{\textit{Magnitude of terms.}}{\textit{Asymptotic rotating speed of terms.}}$

quantitative corrections. *quantitative & qualitative influence.*

Negligible $0 \longleftrightarrow \infty$ *Indispensable*

**Caution – should compare terms of the same form.*

Tips :

$\hat{\sigma}_- \sim \hat{\sigma}_- \, e^{-i\omega_q t}$ $\qquad \hat{a}_- \sim \hat{a}_- \, e^{-i\omega_r t}$

$\hat{\sigma}_+ \sim \hat{\sigma}_+ \, e^{i\omega_q t}$ $\qquad \hat{a}_+ \sim \hat{a}_+ \, e^{i\omega_r t}$

For two-level systems. *For oscillators.*

**Caution – Lab frame POV.*

Practice :

$$\frac{\omega_q}{2}\hat{\sigma}_z + \frac{\Omega_d}{2} e^{i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{-i\omega_d t}\,\hat{\sigma}_+ + \frac{\Omega_d}{2} e^{-i\omega_d t}\,\hat{\sigma}_- + \frac{\Omega_d}{2} e^{i\omega_d t}\,\hat{\sigma}_+$$

$$\omega_t \hat{a}^\dagger \hat{a} + \omega_r \hat{b}^\dagger \hat{b} - g(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

$$\omega_r \hat{a}^\dagger \hat{a} + \frac{\omega''_{a_j}}{2}\hat{\sigma}_{z_j} + g\left(\hat{\sigma}_{+_j} e^{-i\omega_{d_k} t} + \hat{\sigma}_{-_j} e^{+i\omega_{d_k} t}\right)\left(\hat{a}^\dagger + \hat{a}\right)$$

Part II – Perturbative diagonalization.

In cQED studies, system Hamiltonians often take the below form.

$$H = H_0 + V$$

perturbative & off-diagonal.

$$H_{\rm JC} = \underbrace{\hbar\omega\hat{a}^\dagger\hat{a} + \hbar\omega_0\frac{\hat{\sigma}_z}{2}}_{H_0} + \underbrace{\hbar g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-\right)}_{V}$$

Goal :

Obtain approximate eigenvalues in *<u>analytical forms.</u>*

(S. Girvin,
Circuit QED: Superconducting Qubits Coupled to Microwave Photons).

Strategy.

Let's define, $U = e^{\hat{\eta}};\ U^\dagger = e^{\hat{\eta}^\dagger} = e^{-\hat{\eta}}$

$$\tilde{H} \approx H_0 + V + [\hat{\eta}, H_0] + [\hat{\eta}, V] + \cancelto{\approx 0}{\frac{1}{2}[\hat{\eta}, [\hat{\eta}, H_0]]}$$

If $[\hat{\eta}, H_0] = -V$ *is satisfied,*

$$\Rightarrow \quad \tilde{H} = H_0 + \frac{1}{2}\boxed{[\hat{\eta}, V]}$$

normally diagonal.

'Schrieffer-Wolff transformation.'

**Today's tutorial is applicable for time-independent cases only.*
**See the following papers for time-dependent cases.*

Phys. Rev. Res ***4****, 013005 (2022).*

Phys. Rev. Appl ***18****, 024009 (2022).*

Part II – Perturbative diagonalization.

Revisit : dispersive interaction.

$$H_{\mathrm{JC}} = \underbrace{\hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\omega_0\frac{\hat{\sigma}_z}{2}}_{H_0} + \underbrace{\hbar g\left(\hat{a}\hat{\sigma}_+ + \hat{a}^{\dagger}\hat{\sigma}_-\right)}_{V}$$

$$\tilde{H} = H_0 + \chi\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)\sigma^z$$

The proper transform generator is :

$$\hat{\eta} = \frac{g}{\Delta}(\hat{a}\sigma^{+} - \hat{a}^{\dagger}\sigma^{-})$$

$$\chi \equiv \hbar\frac{g^2}{\Delta}.$$

**Please recall that non-demolition measurement scheme fails for large cavity photon numbers.*

** The neglected higher-order terms have off-diagonal components,*

→ Induce qubit-cavity energy exchanges when the approximation breakdowns.

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Cavity-QED on circuits : Circuit-QED.

Experimental milestones in circuit QED.

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- Rotating wave approximation.
- Perturbative diagonalization.

Numerical methods.

- **QuTip**
- **QuCAT** (**Qu**antum **C**ircuit **A**nalysis **T**ool).

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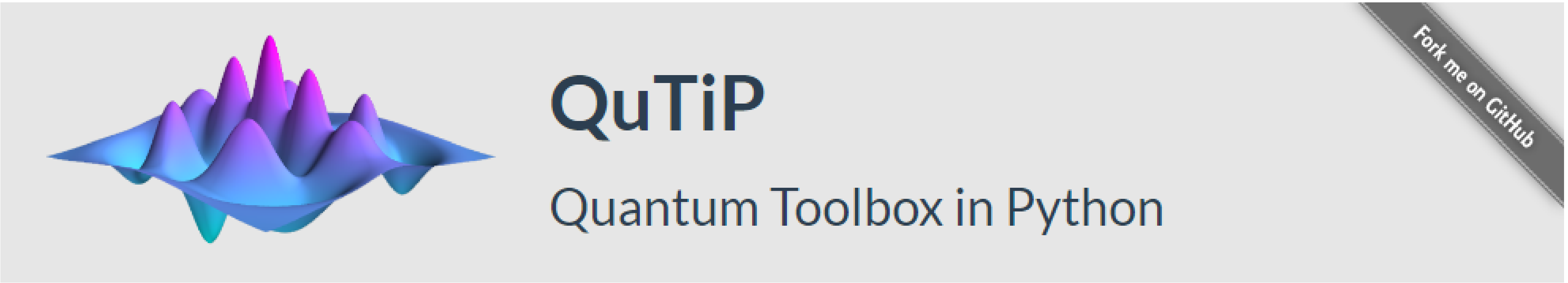
Part II – QuTip.



Opensource software for simulating the dynamics of open quantum systems.

https://qutip.org/

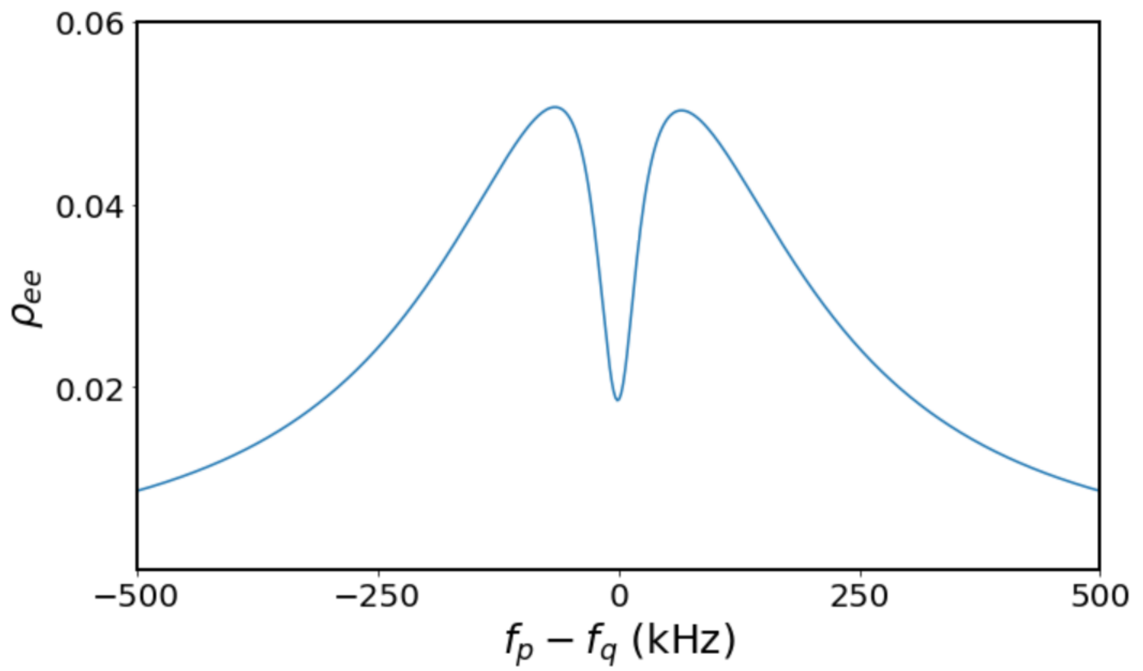
J. R. Johansson, P. D. Nation, and F. Nori, Comp. Phys. Comm. **184**, 1234 (2013)
J. R. Johansson, P. D. Nation, and F. Nori, Comp. Phys. Comm. **183**, 1760–1772 (2012)

Part II – QuTip.

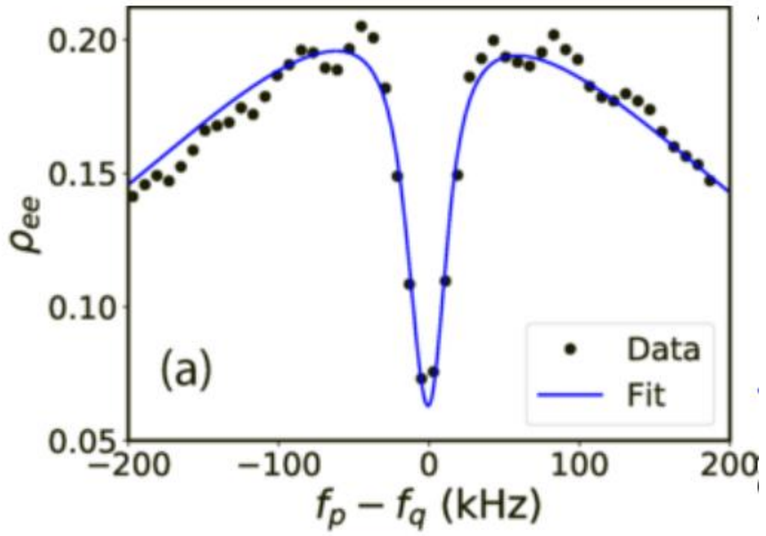
```
def spectroscopy_eps(wq, Aq, gamma, Nq, wc, kappa, Nc, eps, wd_list, chi, g):


    a = tensor(destroy(Nq), qeye(Nc))
    b = tensor(qeye(Nq), destroy(Nc))
    num_b = b.dag()*b
    num_a = a.dag()*a

    r=[]

    H0 = (wq)*num_a + (wc)*num_b - 0.5*Aq*a.dag()*a.dag()*a*a - g*(a.dag()*b + b.dag()*a)
    for wp in wp_list:

        H = H0
        H -= wp*num_a
        H -= wp*num_b
        H += +eps*1j*(a.dag()-a)
        H -= 2*pi*chi*num_a*num_b

        c_ops = []
        c_ops.append(np.sqrt(gamma*(1.))*a)
        c_ops.append(np.sqrt(kappa*(1.))*b)

        rho_ss = steadystate(H, c_ops)
        r.append([expect(a.dag()*a,rho_ss)])
    return np.array(r)
```

Nq = 2, $\hat{a}$ = $\hat{\sigma}$_effectively.

$$\hat{\mathscr{H}} = \frac{\omega_q}{2}\hat{\sigma}_z + \omega_t \hat{a}^\dagger \hat{a} - 2\chi_{qc}\hat{\sigma}_z \hat{a}^\dagger \hat{a} + \frac{\Omega_{sb}}{2}\left(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-\right) + \frac{\Omega_p}{2}\left(\hat{\sigma}_+ e^{-i\omega_p t} + \hat{\sigma}_- e^{+i\omega_p t}\right),$$

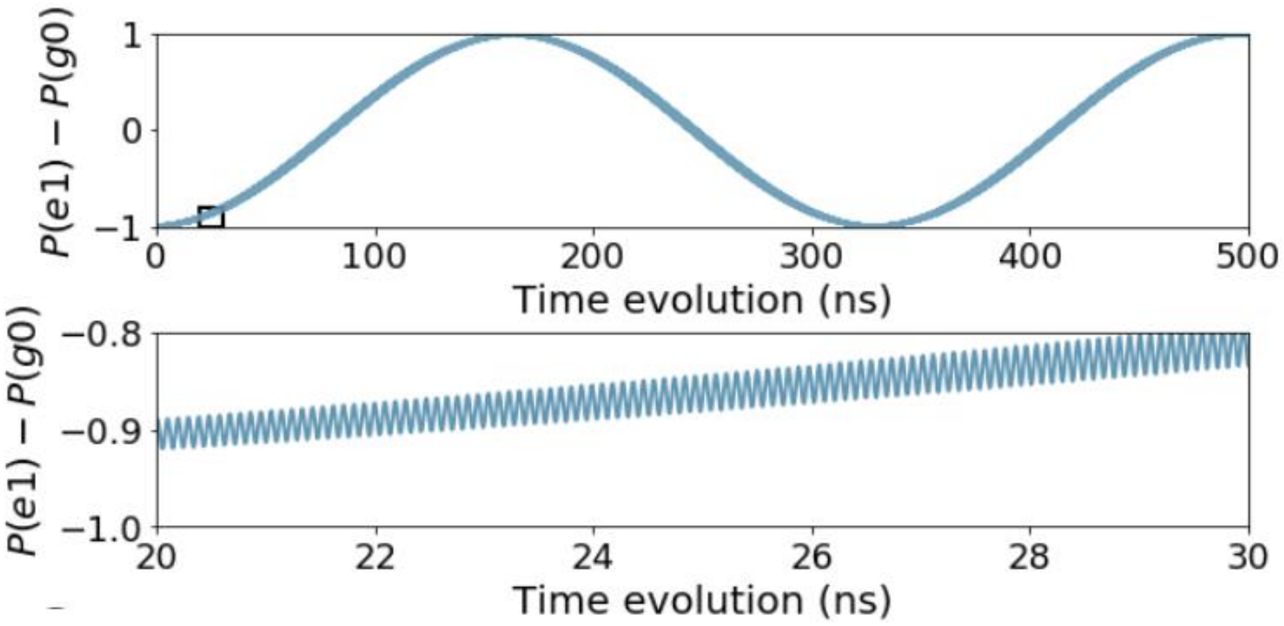


Spectrum simulation of electromagnetically induced transparency.
I used this simulation to fit the experimental data.



Part II – QuTip : overcoming memory issue in time-dependent problem solving.

When you should solve time-dependent problem, the below situation requires huge overheads.



Macro dynamic time scale : $1/\Omega_{gate}$

Micro dynamics time scale : $1/\omega_d$

If $\Omega_{gate} \ll \omega_d$,

→ *Need very small timestep and a long evolution time.*

→ *Deplete RAM.*

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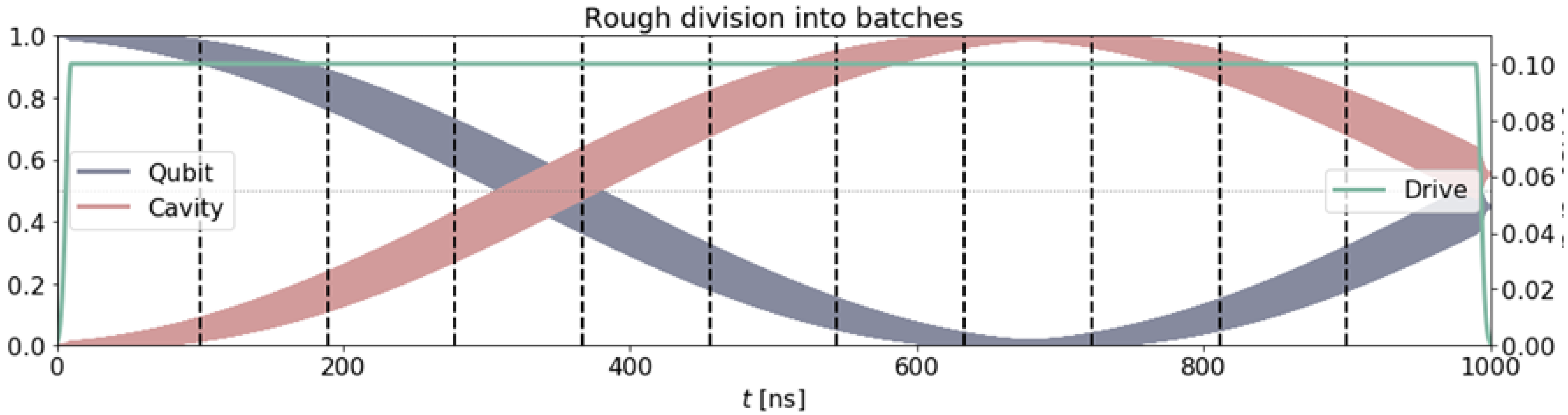
Part II – QuTip : overcoming memory issue in time-dependent problem solving.

Segment + combine method.



Calculation :

Calculate 1 → Save data to hard disk → Clean RAM → Calculate 2 → ⋯

Data plot :

Load data from hard disk.

Selectively choose the segments.

Loading all data at once might have your computer frozen.

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https://github.com/bann-01/2-photon-sideband-modules

Part II – QuCAT.

QuCAT (**Qu**antum **C**ircuit **A**nalysis **T**ool) – *developed by Dr. Mario Gely (my previous PhD groupmate).*

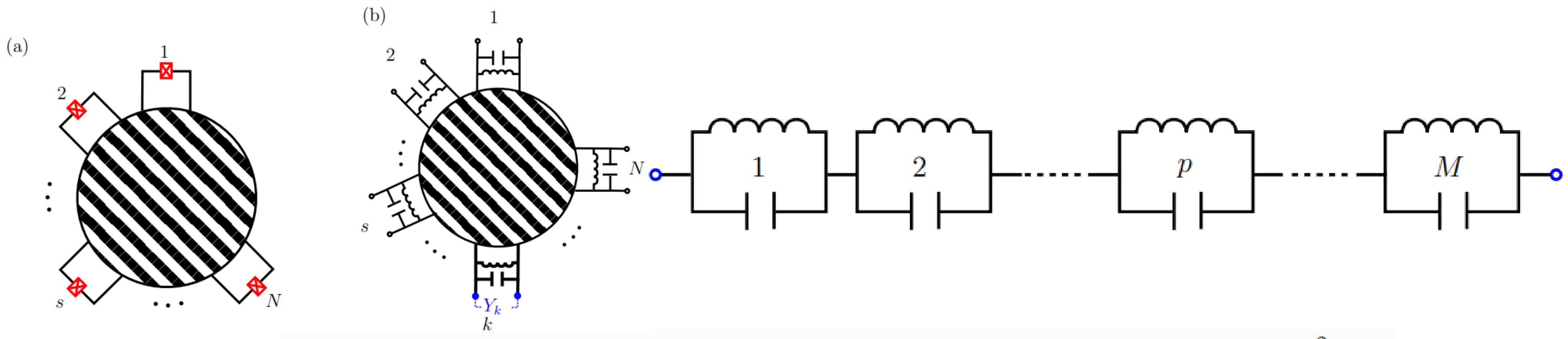
https://qucat.org/

New J. Phys. **22** 013025 (2020).

Target systems :

Weakly nonlinear circuits comprised of multiple Josephson junctions & linear lumped elements.

Principle : Black-box circuit quantization (*PRL* ***108****, 240502 (2012).*



$$\hat{H} = \sum_{m\in\text{modes}} \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m + \sum_j \sum_{2n\leq\text{taylor}} E_j \frac{(-1)^{n+1}}{(2n)!} \left(\frac{\phi_{zpf,m,j}}{\phi_0} (\hat{a}_m^\dagger + \hat{a}_m) \right)^{2n}$$

Part II – QuCAT.

https://qucat.org/

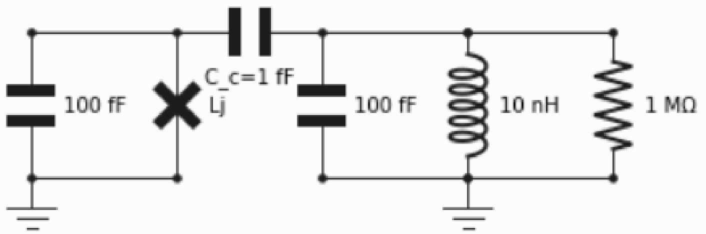
New J. Phys. **22** 013025 (2020).

Features:

Circuit object.

GUI interface.

```
cir = GUI('circuits/basics.txt', # location of the circuit file
        edit=True, # open the GUI to edit the circuit
        plot=True, # plot the circuit after having edited it
        print_network=True # print the network
       )
```



You can draw whatever circuits you want to simulate.

but distributed elements are not supported...

Parameters should meet 'weakly anharmonic' condition.

for example, not applicable to fluxoniums.

Intuitive outputs.

```
f,k,A,chi = cir.f_k_A_chi(pretty_print=True,Lj = 8e-9)
```

mode	freq.	diss.	anha.
0	5.01 GHz	1.57 MHz	583 Hz
1	5.6 GHz	3.84 kHz	191 MHz

Kerr coefficients (diagonal = Kerr, off-diagonal = cross-Kerr)

mode	0	1
0	583 Hz	
1	667 kHz	191 MHz

$$\hat{H} = \sum_m \sum_{n\neq m} (\hbar\omega_m - A_m - \frac{\chi_{mn}}{2})\hat{a}_m^\dagger \hat{a}_m - \frac{A_m}{2}\hat{a}_m^\dagger \hat{a}_m^\dagger \hat{a}_m \hat{a}_m - \chi_{mn}\hat{a}_m^\dagger \hat{a}_m \hat{a}_n^\dagger \hat{a}_n$$

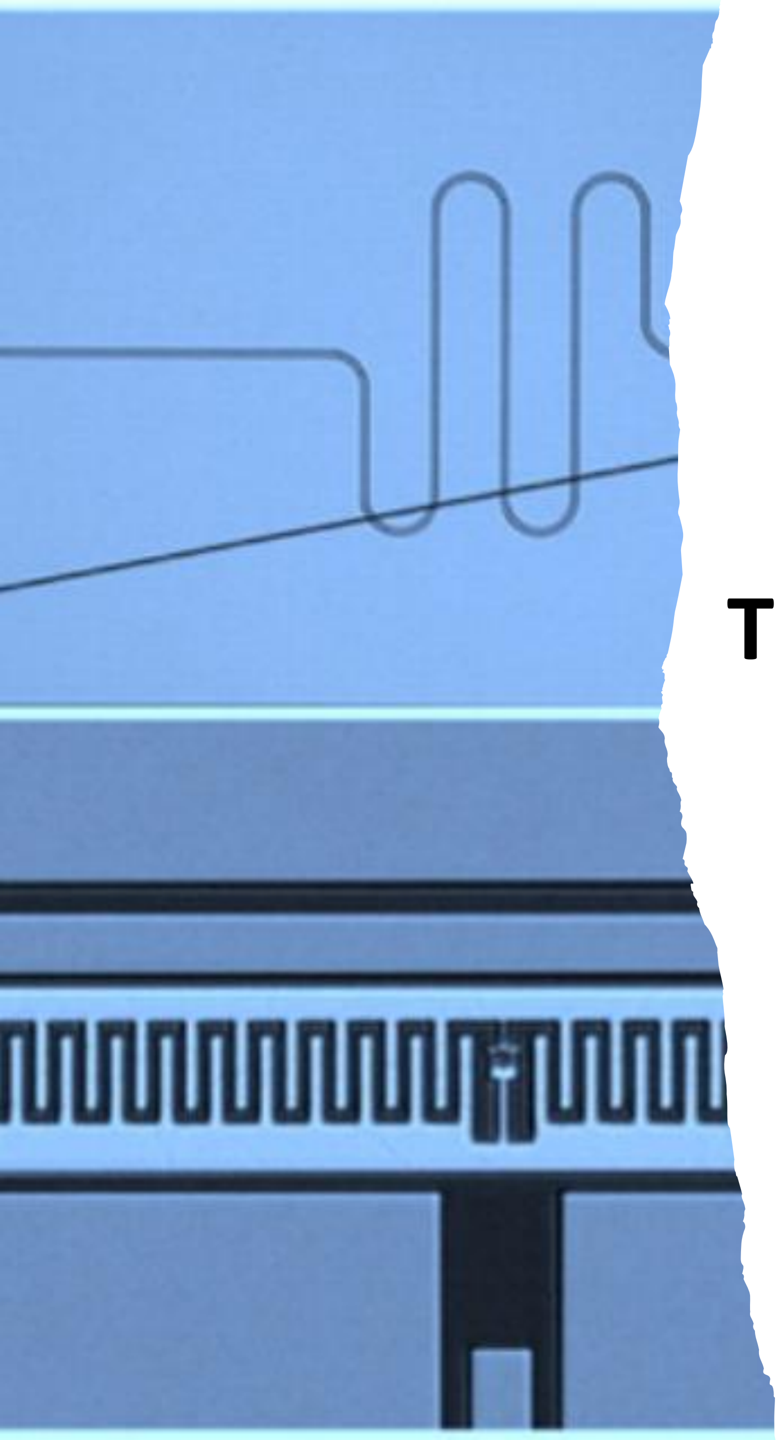
Qutip-compatible.

Qutip's quantum object.

```
# Compute hamiltonian (for h=1, so all energies are expressed in frequency units, not angular)
H = cir.hamiltonian(
    modes = [0,1],# Include modes 0 and 1
    taylor = 4,# Taylor the Josephson potential to the power 4
    excitations = [8,10],# Consider 8 excitations in mode 0, 10 for mode 1
    Lj = 8e-9)# set any component values that were not fixed when building the circuit

# QuTiP method which return the eigenergies of the system
ee = H.eigenenergies()
```

You can play with results as if using qutip.



Thanks for your attention.