ISOSCALAR GIANT MONOPOLE RESONANCE IN THE Ca ISOTOPES

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Outline

Introduction

1. Isoscalar giant monopole resonances (ISGMR) centroid $\bar{E}$ is connected with the nuclear matter incompressibility. The incompressibility or the compression modulus is important in several physical contexts such as:

a) prompt supernova explosions;

***H. A. Bethe*, Rev. Mod. Phys. 62, 801 (1990).**

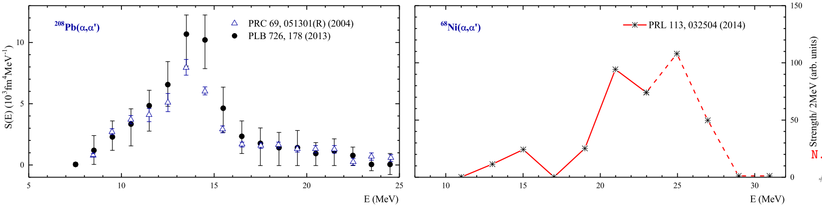
b) the interiors of neutron stars;

***J. M. Lattimer and M. Prakash*, Astrophys. J. 550, 426 (2001).**

c) heavy-ion collisions at intermediate and high energies;

***H. Stöcker and W. Greiner*, Phys. Rep. 137, 277 (1986).**

2. The properties of the ISGMR in stable and radioactive atomic nuclei have been extensively investigated in many experiments.



***U. Garg and G. Colò*, Prog. Part. Nucl. Phys. 101, 55 (2018).**

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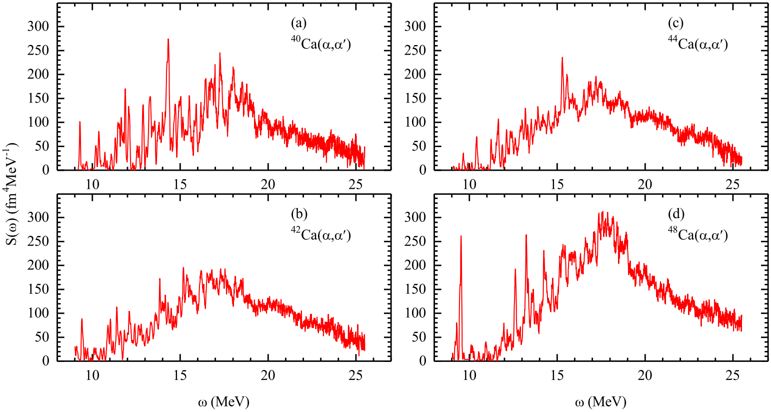
Introduction

3. The ISGMR strength distributions were measured using the high energy-resolution capabilities at iThemba LABS. This allows for the observation of pronounced fine structure.



***S. D. Olorunfunmi et al.*, Phys. Rev. C105, 054319 (2022).**

MAIN INGREDIENTS OF THE MODEL

Realization of QRPA: the FRSA model

The starting point of the method is the HF-BCS calculations of the ground state, where spherical symmetry is assumed for the ground states. The continuous part of the single-particle spectrum is discretized by diagonalizing the HF Hamiltonian on a harmonic oscillator basis.

J. P. Blaizot and D. Gogny, **Nucl. Phys.** A284, **429 (1977)**.

The residual interaction in the particle-hole channel V_{res}^{ph} and in the particle-particle channel V_{res}^{pp} can be obtained as the second derivative of the energy density functional $\mathcal{H}$ with respect to the particle density ρ and the pair density $\tilde{\rho}$, respectively.

$$V_{res}^{\mathrm{ph}} \sim \frac{\delta^2\mathcal{H}}{\delta\rho_1\delta\rho_2} \quad V_{res}^{\mathrm{pp}} \sim \frac{\delta^2\mathcal{H}}{\delta\tilde{\rho}_1\delta\tilde{\rho}_2} .$$

G. T. Bertsch and S. F. Tsai, **Phys. Rep.** 18, **125 (1975)**.

Realization of QRPA: the FRSA model

We employ the effective Skyrme interaction in the particle-hole channel

$$V(\vec{r}_1,\vec{r}_2)=t_0\left(1+x_0\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)+\frac{t_1}{2}\left(1+x_1\hat{P}_\sigma\right)\left[\delta(\vec{r}_1-\vec{r}_2)\vec{k}^2+\vec{k}'^2\delta(\vec{r}_1-\vec{r}_2)\right]$$
$$+t_2\left(1+x_2\hat{P}_\sigma\right)\vec{k}'\cdot\delta(\vec{r}_1-\vec{r}_2)\vec{k}+\frac{t_3}{6}\left(1+x_3\hat{P}_\sigma\right)\delta(\vec{r}_1-\vec{r}_2)\rho^\alpha\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)$$
$$+iW_0(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\vec{k}'\times\delta(\vec{r}_1-\vec{r}_2)\vec{k}\right].$$

T. H. R. Skyrme, Nucl. Phys. 9, 615 (1959).

D. Vautherin and D. M. Brink, Phys. Rev. C5, 626 (1972).

The Hamiltonian includes the pairing correlations are generated by the density-dependent zero-range force in the particle-particle channel

$$V_{pair}(\vec{r}_1,\vec{r}_2)=V_0\left(1-\eta\frac{\rho(r_1)}{\rho_c}\right)\delta(\vec{r}_1-\vec{r}_2),$$

where ρ_c is the nuclear saturation density; η and V_0 are model parameters. For example, $\eta=0$ and $\eta=1$ are the case of a volume interaction and a surface-peaked interaction, respectively.

A. P. Severyukhin, V. V. Voronov, N. V. Giai, Phys. Rev. C77, 024322 (2008).

Realization of QRPA: the FRSA model

We introduce the phonon creation operators

$$Q^{+}_{\lambda\mu i}=\frac{1}{2}\sum_{jj'}\left[X^{\lambda i}_{jj'}\,A^{+}(jj';\lambda\mu)-(-1)^{\lambda-\mu}Y^{\lambda i}_{jj'}\,A(jj';\lambda-\mu)\right],$$

$$A^{+}(jj';\lambda\mu)=\sum_{mm'}C^{\lambda\mu}_{jm\,j'm'}\alpha^{+}_{jm}\alpha^{+}_{j'm'}.$$

The index λ denotes total angular momentum and μ is its z-projection in the laboratory system. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and one-phonon excited states are $Q^{+}_{\lambda\mu i}|0\rangle$ with the normalization condition

$$\langle 0|[Q_{\lambda\mu i},Q^{+}_{\lambda\mu i'}]|0\rangle=\delta_{ii'}.$$

Making use of the linearized equation-of-motion approach one can get the QRPA equations

$$\begin{pmatrix}\mathcal{A} & \mathcal{B}\\ -\mathcal{B} & -\mathcal{A}\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}=\omega\begin{pmatrix}X\\ Y\end{pmatrix}.$$

Solutions of this set of linear equations yield the one-phonon energies ω and the amplitudes X, Y of the excited states.

D. J. Rowe, **Nuclear Collective Motion (Methuen, London 1970).**

Phonon-phonon coupling (PPC)

To take into account the effects of the phonon-phonon coupling (PPC) in the simplest case one can write the wave functions of excited states as a linear combination of one-, two- (PPC+2ph) and three-phonon (PPC+3ph) configurations

$$\Psi_\nu(JM) = \Big\{ \sum_i R_i(J\nu) Q^+_{JMi} + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \left[Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2} \right]_{JM}$$

$$+ \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 J'}} T^{\lambda_1 i_1 \, \lambda_2 i_2}_{J' \, \lambda_3 i_3}(J\nu) \left[\left[Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2} \right]_{J'} Q^+_{\lambda_3 \mu_3 i_3} \right]_{JM} \Big\} |0\rangle$$

with the normalization condition

$$\sum_i R_i^2(J\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} \left[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) \right]^2 + 6 \sum_{\substack{\lambda_1 i_1 \lambda_2 i_2 \\ \lambda_3 i_3 \, J'}} \left[T^{\lambda_1 i_1 \, \lambda_2 i_2}_{J' \, \lambda_3 i_3}(J\nu) \right]^2 = 1.$$

V. G. Soloviev, Theory of Atomic Nuclei: Quasiparticles and Phonons (Inst. of Phys., Bristol 1992).

M. Grinberg and Ch. Stoyanov, Nucl. Phys. A573, 231 (1994).

Phonon-phonon coupling (**PPC**)

The variational principle leads to a set of linear equations for the unknown amplitudes $R_i(J\nu)$, $P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)$ and $T^{\lambda_1 i_1 \, \lambda_2 i_2}_{J' \, \lambda_3 i_3}(J\nu)$:

$$R_i(J\nu)(\omega_{Ji} - E_\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu) U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = 0;$$

$$P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu) + 3 \sum_{\lambda'_1 i'_1 \lambda'_2 i'_2} T^{\lambda'_1 i'_1 \, \lambda'_2 i'_2}_{\lambda_1 \, \lambda_2 i_2}(J\nu) U^{\lambda'_1 i'_1}_{\lambda'_2 i'_2}(\lambda_1 i_1)$$

$$+ \frac{1}{2} \sum_{i'} U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji') R_{i'}(J\nu) = 0;$$

$$T^{\lambda'_1 i'_1 \, \lambda'_2 i'_2}_{J' \, \lambda'_3 i'_3}(J\nu) \left(\omega_{\lambda'_1 i'_1} + \omega_{\lambda'_2 i'_2} + \omega_{\lambda'_3 i'_3} - E_\nu\right) + \sum_i P^{J' i}_{\lambda'_3 i'_3}(J\nu) U^{\lambda'_1 i'_1}_{\lambda'_2 i'_2}(J' i) = 0.$$

$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ is the matrix element coupling:

$$U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji) = \langle 0 | Q_{Ji} \mathcal{H} \left[Q^+_{\lambda_1 i_1} Q^+_{\lambda_2 i_2} \right]_J | 0 \rangle.$$

These equations have the same form as the QPM equations, but the single-particle spectrum and the parameters of the residual interaction are calculated with the Skyrme forces.

A. P. Severyukhin, N. N. Arsenyev, N. Pietralla, **Phys. Rev. C104, 024310 (2021).**

RESULTS AND DISCUSSION

Details of calculations

The isoscalar monopole operator can be defined as

$$\hat{M}_{\lambda=0} = \sum_{i=1}^{A} r_i^2 .$$

The integral characteristics of the ISGMR are the centroid energy E_c and the spreading width Γ

$$E_c = \frac{m_1}{m_0} \quad \text{and} \quad \Gamma = 2.35\sqrt{\frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2} ,$$

where $m_k = \sum_\nu (E_\nu)^k \left| \langle 0_\nu^+ | \hat{M}_{\lambda=0} | 0_{g.s.}^+ \rangle \right|^2$ are the energy-weighted moments. Of special importance are the non-energy weighted sum rule (NEWSR) that is simply the moment m_0, and the energy-weighted sum rule (EWSR) m_1.

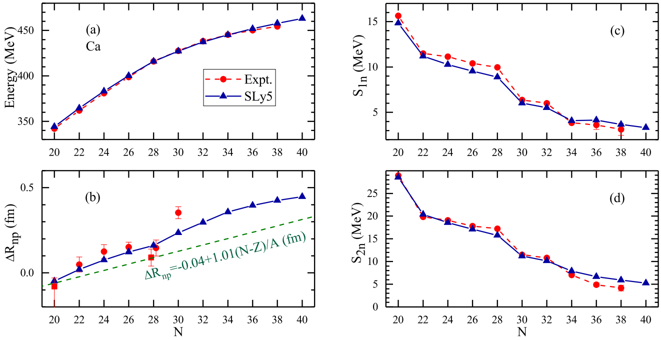
The ISGMR strength distribution is averaged out by a Lorentzian distribution with a width of Δ as follows:

$$S(\omega) = \sum_\nu \left| \langle 0_\nu^+ | \hat{M}_{\lambda=0} | 0_{g.s.}^+ \rangle \right|^2 \frac{1}{2\pi} \frac{\Delta}{(\omega - E_\nu)^2 + \Delta^2/4} .$$

Details of calculations

In the case of the Ca isotopes, we use the Skyrme interaction SLy5. The pairing strength V_0=-270 MeV·fm^3 is fitted to reproduce the experimental neutron pairing energies near ^{50}Ca.

***E. Chabanat et al.*, Nucl. Phys. A635, 231 (1998).**



***N. N. Arsenyev et al.*, Phys. Rev. C95, 054312 (2017).**

***M. Wang et al.*, Chin. Phys. C41, 030003 (2017).**

***M. Tanaka et al.*, PRL 124, 102501 (2020).**

***A. Trzcińska et al.*, PRL 87, 082501 (2001).**

Details of calculations

To construct the wave functions of the 0^+ states, in the present study we take into account all two- and three-phonon configurations below 30 MeV that are built from the phonons with different multipoles $\lambda^\pi = 0^+, 1^-, 2^+, 3^-, 4^+$ and 5^- coupled to 0^+. We have checked that extending the configuration space plays a minor role in our calculations.

RPA low-lying states: 2_1^+, 3_1^-, 4_1^+ and 5_1^- of ^{48}Ca

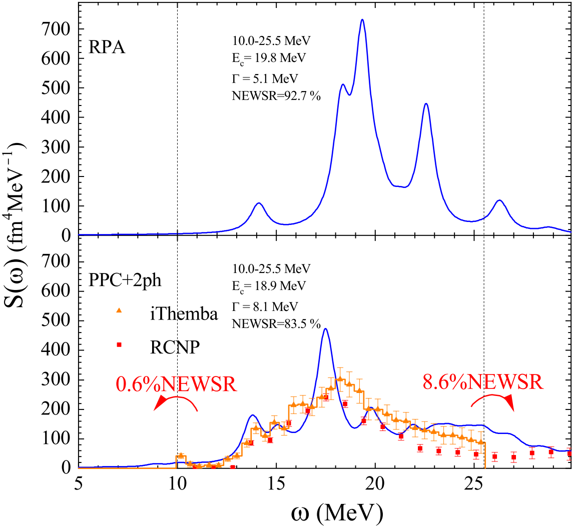
λ_1^π	Energy, MeV		$B(E\lambda; \lambda_1^\pi \to 0_{gs}^+)$, W.u.	
	Expt.	RPA	Expt.	RPA
2_1^+	3.832	3.19	$1.84^{+0.17}_{-0.14}$	1.3
3_1^-	4.507	4.47	$8.4^{+4.3}_{-3.5}$	4.1
4_1^+	4.503	3.51		2.1
5_1^-	5.729	4.52		8.6

***N. N. Arsenyev and A. P. Severyukhin*, Phys. At. Nucl. 85, 912 (2022); 86, 466 (2023).**

***J. Chen*, Nucl. Data Sheets 179, 1 (2022).**

ISGMR in ^{48}Ca

SLy5



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N. N. Arsenyev and A. P. Severyukhin, Phys. At. Nucl. 85, 912 (2022).

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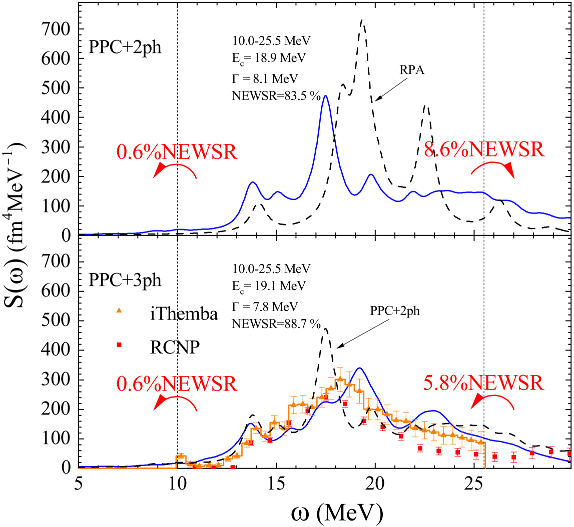
ISGMR in ^{48}Ca

SLy5



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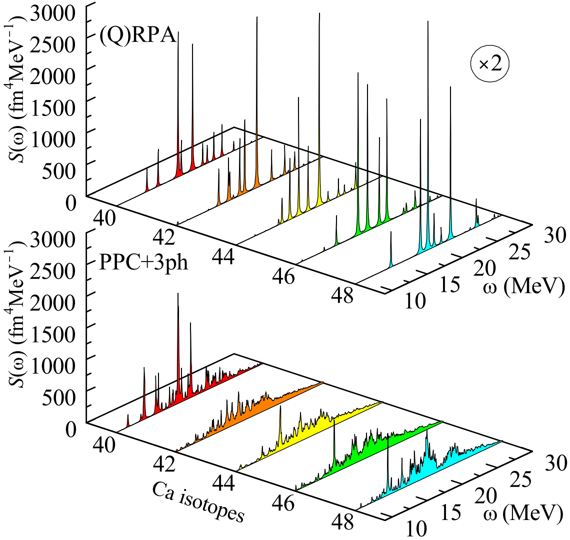
N. N. Arsenyev and A. P. Severyukhin, Phys. At. Nucl. 85, 912 (2022).

S. D. Olorunfunmi et al., Phys. Rev. C105, 054319 (2022).

ISGMR in ^{40,42,44,46,48}Ca SLy5



***N. N. Arsenyev and A. P. Severyukhin*, Phys. At. Nucl. 86, 466 (2023).**

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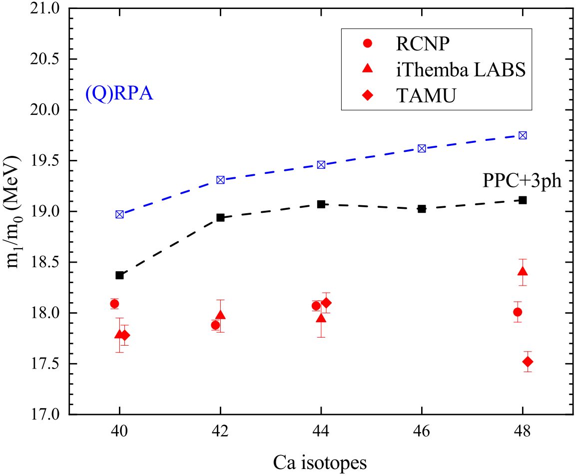
Moment ratios of the ISGMR in ^{40,42,44,46,48}Ca SLy5



Here, $m_k = \sum_\nu (E_\nu)^k \left|\langle 0_\nu^+ | \hat{M}_{\lambda=0} | 0_{g.s.}^+ \rangle\right|^2$ are the energy-weighted moments.

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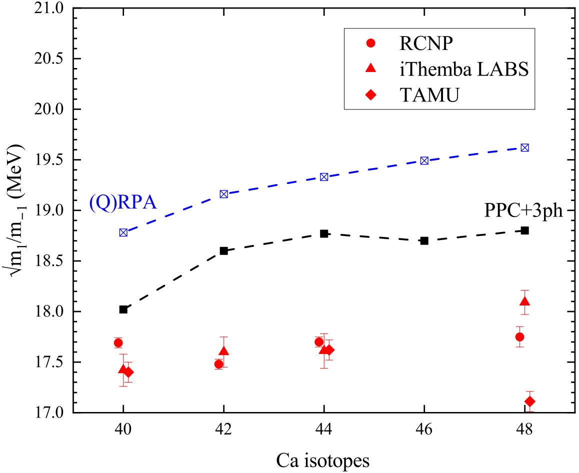
N. N. Arsenyev and A. P. Severyukhin, Phys. At. Nucl. 86, 466 (2023).

S. D. Olorunfunmi et al., Phys. Rev. C105, 054319 (2022).

Moment ratios of the ISGMR in ^{40,42,44,46,48}Ca SLy5



Here, $m_k = \sum_\nu (E_\nu)^k \left| \langle 0^+_\nu | \hat{M}_{\lambda=0} | 0^+_{g.s.} \rangle \right|^2$ are the energy-weighted moments.

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N. N. Arsenyev and A. P. Severyukhin, **Phys. At. Nucl. 86, 466 (2023).**

S. D. Olorunfunmi et al., **Phys. Rev. C105, 054319 (2022).**

Conclusions

Starting from the Skyrme EDF, we have studied the effects of the phonon-phonon coupling on the properties of the ISGMR for the ^{40,42,44,46,48}Ca isotopes. The inclusion of the coupling between one-, two- and three-phonon terms in the wave functions of excited 0^+ states leads to a redistribution of the strength of the ISGMR. This coupling causes spreading and an overall shift of the ISGMR centroid energy down with respect to its value obtained in the QRPA, which can amount up to 700 keV. We have clearly demonstrated that the inclusion of the PPC effects is crucial in order to achieve a unified description of the fine structure of the ISGMR. A reasonable agreement with the newest experimental results for the properties of the ISGMR in ^{40,42,44,48}Ca has been obtained.

Investigations of fine structure of the ISGMR in ^{40,42,44,48}Ca used a wavelet analysis are underway. The preliminary studies were performed in ^{48}Ca, e.g.,

– ***S. D. Olorunfunmi et al.***, **Jour. Phys.: CS** 1643, **012154 (2020)**

Acknowledgments

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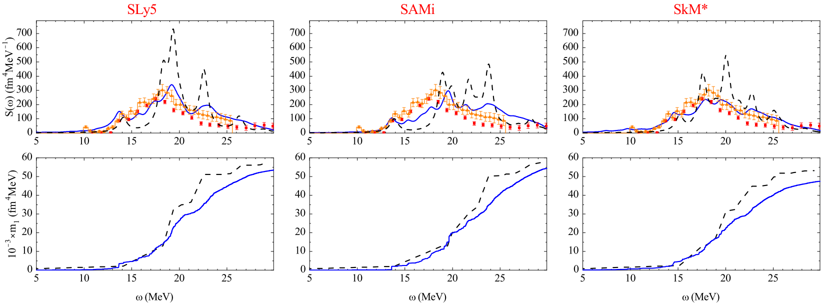
THE END

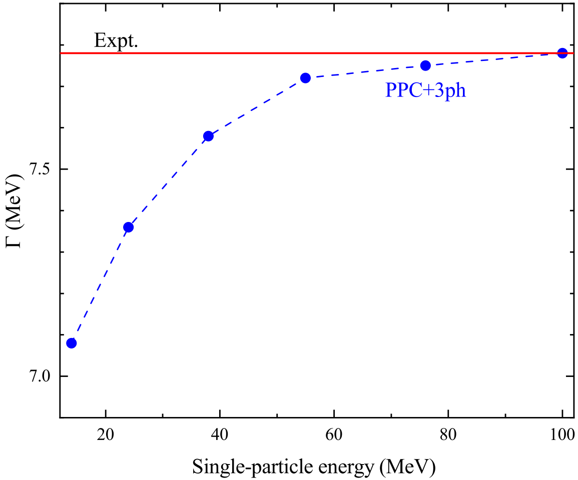
BACKUP SLIDES

ISGMR in ^{48}Ca SLy5 vs SAMi vs SkM*



***N. N. Arsenyev and A. P. Severyukhin*, in preparation.**

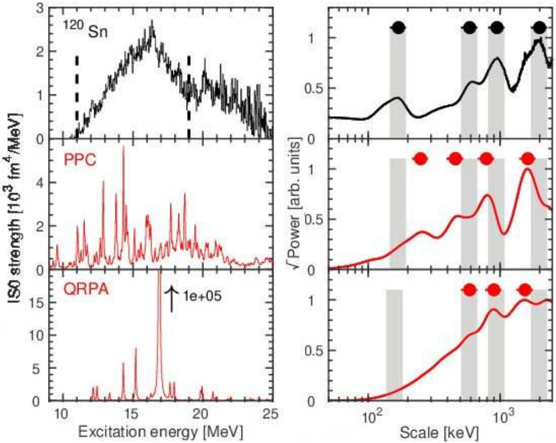
ISGMR width for ^{48}Ca SLy5



N. N. Arsenyev and A. P. Severyukhin, Phys. At. Nucl. 86, 466 (2023).

S. D. Olorunfunmi, PhD Thesis (University Wits, Johannesburg 2020).

Fine structure of the ISGMR in ^{120}Sn SLy4



A. Bahini et al., in preparation.

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